THE DESIGN OF TWO RADIAL INFLOW TURBINES

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ABSTRACT

Nozzle wakes are a common source of fatigue failures in radial inflow turbines. This paper describes an approach to avoiding these problems, without sacrificing performance or manufacturability, by using a small number of carefully designed nozzle vanes and tuning the rotor blades to avoid nozzle passing resonances. Two radial inflow turbines designed via this approach are reviewed, including predicted and measured aerodynamic performance and predicted and measured natural frequencies - all of which are in good agreement. One of the turbines is small, and the other is large, but both have no nozzle passing resonances anywhere in their wide operating ranges, and both have measured efficiencies above 84 percent.

INTRODUCTION

Fatigue failure from nozzle wake excited vibration of blades and bladed disks is a well known hazard in both axial and radial inflow turbines. In the case of axial turbines, effective methods for avoiding and suppressing nozzle passing resonances have existed for some time. By contrast, the problem has been characteristically far more difficult to deal with in radial inflow turbines.

Three factors are responsible for this. First, once vibration problems arise in radial inflow turbines, they are difficult to eliminate. Techniques for damping used in axial turbines, such as loose lacing wires from blade to blade, are less suited to radial inflow configurations. Eliminating a vibration problem therefore often requires redesign of the rotor, which is costly. Second, at least until recently, the natural frequencies of radial flow blades and bladed disks have been difficult to predict. Easy to use, accurate methods for calculating the natural frequencies of axial blades have been available since the 1950's. But only in the last few years have dependable methods emerged for analyzing the structurally irregular geometries of radial flow blading. As a consequence, radial inflow turbines could not be carefully tuned during design to avoid nozzle passing resonances in the way that axial turbines have been. Third, higher order modes -- e.g. modes with two or more nodal points along the shroud line -- can be excitable in radial inflow turbines. The nozzle wake disturbance propagates along the length of the rotor blade, creating a phase relation between the unsteady pressure at the trailing edge and the unsteady pressure at the leading edge. If this phase relation in the excitation acting along the length of the blade correlates with the mode shape, then in resonance the mode will be responsive, and thus a hazard, even though it is not one of the principal modes of the blades.

This paper describes an approach we have used for dealing with nozzle wake induced vibration in the design of radial inflow turbines. The basic idea is to attack the problem in the same way that it is customarily attacked in axial turbines. During design, the rotor blades are carefully tuned to avoid resonances by modifying their thickness distribution and trailing edge contour -- two variables that aerodynamic performance is comparatively less sensitive to. Finite-element methods are used to predict natural frequencies for purposes of tuning, and modal analysis tests are subsequently used to confirm them. To make the blade tuning problem tractable, we use a smaller number of nozzle vanes than found in most radial inflow turbine designs. This is the distinctive feature of the approach, the feature most apparent to the naked eye. By using a small number of nozzles, we entirely avoid potential resonances in higher order modes, and we keep the excitation frequency in a range where the tuning problem is most solvable. The nozzle vanes have to be designed with care in order to avoid sacrificing aerodynamic performance. But careful aerodynamic design of the vanes has the added advantage of minimizing their wakes, thereby tending to reduce the excitation levels on the blades, especially in the second harmonic of the nozzle passing disturbance.

Here, we will review two radial inflow turbine designs illustrating this approach. Both designs are for high efficiency, low horsepower applications, but otherwise they are quite different. One is a steam turbine, with a 15 cm diameter wheel. The other is
driven by exhaust from a chemical process and has a 56 cm diameter wheel. The steam turbine is an original design, a prototype for a family of steam turbines. The process turbine is a one-of-a-kind design, replacing a turbine that had suffered nozzle wake induced fatigue failures in the field.

DESIGN METHODS

The overall approach allows for iteration between aerodynamic design and vibrational design, while at the same time maintaining a blade geometry that is easy to manufacture. The first step is to establish a basic aerodynamic design, assuming a rotor blade thickness distribution and a number of nozzles based on previous experience. Once the basic rotor is fully defined, steady state stresses are computed to verify that it is mechanically viable. Then the natural frequencies are computed, and a preferred number of nozzles is chosen - i.e. the number of nozzles that offers the best prospect for avoiding resonances in the operating range once the rotor is tuned. The next step is to tune the rotor blades so that no natural frequencies encroach on any excitation frequency of concern anywhere in the operating range. If the tuning problem proves unsolvable without major revision of the blade shape, then we revert to the first step and develop a new basic aerodynamic design in the light of the vibrational constraints that have emerged. As soon as the tuning problem appears to be solvable without revising the basic aerodynamic design, work begins on the final aerodynamic design of the nozzles. When the rotor is fully tuned analytically, its aerodynamic performance is computed again to verify that the changes made to the blade thickness distribution and trailing edge contour during tuning have not unacceptably compromised its aerodynamic design.

Aerodynamic Design Methods

The aerodynamic design involves two phases, preliminary design and detailed design, each with its own governing computer programs. The program used in preliminary design performs a mean line flow analysis in which losses are computed for each component, using semi-empirical correlations. The program is used iteratively to arrive at an optimal design, subject to externally imposed constraints. The basic dimensions of each component are determined, and then design and off-design performance is predicted for the preferred configuration. In the process of defining this configuration, trade-off data are generated giving the performance penalty associated with various constraints. These data are useful for anticipating any aerodynamic consequences of changes made during the vibration design.

The detailed rotor aerodynamic design is done using an interactive master program, COMIG, applicable to radial flow compressors and pumps as well as turbines (Ref 1). Figure 1 displays the functional organization of this program. In addition to rotor aerodynamic design, it provides automatic mesh generation for finite-element programs, inputs for computerized drafting, and tapes for numerically controlled machining - all using a common data base. COMIG performs two functions in the aerodynamic design of the rotor: it determines detailed blade shapes, and it analyzes the flow in the blade passage.

COMIG uses a straight-line-element generating procedure to define the blade surfaces. The inputs are the hub and shroud contours, the blade thickness distribution along the hub and shroud, the blade angle variation along the shroud, and the angular orientation of the straight-line elements that generate the mean camber surface. The program assumes linearly tapered thicknesses from hub to shroud. Hence, the suction and pressure surfaces are defined by a series of straight-line elements, as illustrated in Figure 2. The use of such elements to form the blade surface greatly simplifies the rest of COMIG. These ruled surfaces are entirely capable of providing excellent aerodynamic performance, and as discussed below, they offer decisive advantages for numerically controlled machining.

The detailed aerodynamic design of the rotor proceeds iteratively. After the blades are specified, the flow in the blade passages is analyzed. The program for doing this employs a streamline curvature method from hub to shroud and a finite difference method from blade to blade. It computes flow...
Vibration Design Methods

The vibration design of the turbines discussed here used two publically accessible, finite element programs, ANSYS (Ref 4) and SUPERB (Ref 5). Neither of these programs - at least in the form available to us when we were designing these turbines - is ideally suited to bladed disk vibration analysis. SUPERB cannot treat centrifugal stiffening of the blades and disk, and ANSYS can treat centrifugal stiffening only if certain types of elements are used, which are inappropriate for radial inflow turbine disks. Fortunately, in the designs discussed here centrifugal stiffening could be compensated for without having to calculate its precise effects on the natural frequencies. But this is not always the case. Also, neither ANSYS or SUPERB allows for a systematic phase lag between adjacent vibrating blades around the circumference. Hence, to treat the blade disk properly, the entire disk has to be modeled, which is excessively expensive. We adopted a more simple approach for the turbines discussed here. In the case of the steam turbine, natural frequencies were calculated for the blade alone, fixed to a rigid disk. In the case of the process turbine, natural frequencies were calculated both for the blade alone and for the blade coupled to a (cyclically symmetric) sector of the disk. In both cases, the frequencies were subsequently measured by means of modal analysis tests.

The vibration design must satisfy two requirements. First, the steady state stresses must leave an adequate margin for vibratory stress on a Goodman diagram for the material - in the case of steel, a minimum of 0.40 GPa vibratory stress margin. This requirement, which can be met without undue penalty in industrial applications, reduces the vibration design problem to one concerned only with strongly responsive resonances. Second, an adequate margin must be maintained between the blade dominated natural frequencies and the excitation frequencies of concern everywhere across the operating range. The margin we required for the radial inflow turbines discussed here was twice the margin we require for axial turbine blades.

A STEAM TURBINE DESIGN

The steam turbine design is a prototype for a family of single and multi-stage steam turbines. These turbines will be of a variety of sizes, operating over a broad range of speeds and inlet pressures and temperatures. But all of them will be in the low-volume-flow-path and low-power regime in which radial inflow turbines can outperform axial turbines. The prototype rotor, shown in Figure 3, is 152 mm in diameter, with 13 full blades and 13 splitter blades. A 2:1 area ratio axial diffuser follows it. It was designed for a wide speed range - from 26,500 to 52,500 rpm - thus making the vibration design problem more difficult. Because the design was only a prototype, however, it did not have to meet such stringent mechanical requirements as it would have, had it been a production design. In particular, we chose to ignore the second harmonic of the nozzle wake excitation in its design.

FIG. 3 - STEAM TURBINE ROTOR

Design Requirements

The design operating conditions for the prototype stage were as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Total Temperature (deg K)</td>
<td>505.98</td>
</tr>
<tr>
<td>Inlet Total Pressure (kPa)</td>
<td>801.86</td>
</tr>
<tr>
<td>Rotational Speed (rpm)</td>
<td>52,500</td>
</tr>
<tr>
<td>Mass Flow Rate (kg/sec)</td>
<td>1.81</td>
</tr>
<tr>
<td>Total-to-Static Expansion Ratio</td>
<td>2.34</td>
</tr>
<tr>
<td>Design Expansion power (kW)</td>
<td>280.08</td>
</tr>
<tr>
<td>Target Total-to-Static Efficiency</td>
<td>0.89</td>
</tr>
</tbody>
</table>

The minimum value of the trailing edge thicknesses allowed on the basis of manufacturing considerations was 1 mm. This was the only externally imposed constraint that had an impact on the turbine's performance.
Figure 5 gives the blade thickness distribution along the hub and shroud. The hub-to-shroud thickness ratio is 1.5 at the inlet, 3.0 at the estimated location of the maximum stress, and 2.0 at the exit.

The results of the flow analysis are given in Figure 6. Blade surface velocities were calculated for different numbers of rotor blades. With 13 full blades and no splitters, the calculated pressure surface velocities are negative for the hub and mean streamtubes. Splitter blades were added to avoid these negative surface velocities and to keep the exit relative velocity as low as possible. The maximum suction surface diffusion (along the hub streamtube) is roughly 0.30, and the diffusion rate is 1.05.

Diffusion along the hub streamtubes is difficult to eliminate; of all the blade geometries analyzed, the final design had the least.

Aerodynamic Design

Rotor Design. Figure 4 shows the meridional flow paths of the rotor for the maximum and the design flow capacities. Usually, the rotor blading is designed for the maximum flow capacity anticipated in any application of the design. In this case, however, the blading was selected on the basis of the flow analysis of the design flow capacity configuration.

The meridional flow path shown in the figure was arrived at in a sequence of steps. First, an initial flow path was modified to yield a uniform accelerating flow with respect to the average meridional velocity distribution. Then the final flow path was selected on the basis of the meridional velocity distribution along the hub and shroud streamtubes, as well as the blade surface loading distribution. In the present case, a 15 degree slope at the hub exit was found to minimize the diffusion and diffusion rates better than the initial choice of 0 degrees. The 15 degree slope also reduces the exit losses or sudden dump effect associated with the annular diffuser downstream of the rotor.

Nozzle Design. The inlet and exit radii, the axial width, and the original number of nozzle vanes were determined during preliminary design. A 16 vane design was completed before results were available from the vibration analyses. These results dictated a shift to 9 vanes. This was accomplished by scaling the coordinates of the corresponding axial blade shape by the ratio of the number of vanes and then converting to the equivalent radial cascade. This approach left the two-dimensional aerodynamic design parameters of the vanes unchanged – i.e. the pitch-chord ratio, the trailing edge blockage, and the blade loading. However, it increased the nozzle inlet radius by about 28 percent.

The final nozzle vanes are shown in Figure 7, and the velocities along the vane surfaces are given in Figure 8. The velocities are strictly accelerating, with no diffusion on either surface.

Vibration Design

Both the blades and the splitters have to be tuned, but we will concentrate here on the more difficult blade tuning problem. Figure 9 shows the calculated Campbell diagram for the blades defined in the original aerodynamic design. The natural frequencies were obtained using ANSYS, with the blade modeled by 66 linear shell elements and the disk assumed rigid. As the Campbell diagram makes clear, if the 16 nozzles used in the original aerodynamic design are retained, the blade tuning problem will be
difficult. Almost certainly, a new blade configuration will be needed to eliminate all three of the resonances indicated in the operating range.

To complete the definition of the blade tuning problem, safety margins have to be added to the natural frequencies. Earlier, in an effort to gain a better idea of what the margin should be, we compared different solutions for the natural frequencies of a related blade, using ANSYS and SUPERB. The blade was one from a somewhat larger rotor in the same family of steam turbines, but with a slightly different flow cut. The ANSYS solution used 84 linear shell elements to model the blade, and the SUPERB solution used 72 parabolic thick shell elements. The results are compared in Table 1. Based on these results and extensive experience in tuning axial turbine blades, we decided to require a ±10 percent margin on the frequencies.

<table>
<thead>
<tr>
<th>Mode</th>
<th>ANSYS Solution</th>
<th>SUPERB Solution</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2790</td>
<td>2565</td>
<td>6.3</td>
</tr>
<tr>
<td>2</td>
<td>6972</td>
<td>7438</td>
<td>6.7</td>
</tr>
<tr>
<td>3</td>
<td>8870</td>
<td>8509</td>
<td>-4.1</td>
</tr>
<tr>
<td>4</td>
<td>10741</td>
<td>10661</td>
<td>-0.7</td>
</tr>
<tr>
<td>5</td>
<td>12625</td>
<td>12759</td>
<td>1.1</td>
</tr>
<tr>
<td>6</td>
<td>17099</td>
<td>17167</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

ANSYS Solution: 84 linear shell elements
SUPERB Solution: 72 parabolic thick shell elements
greater than 7000 Hz is needed in the blade natural frequency spectrum. But with 9 nozzles, the nozzle passing frequency extends only from 4000 to 8000 Hz. The lower slope of the excitation line in the Campbell diagram thus reduces the chances of intersections with natural frequency lines. Moreover, especially after safety margins are taken into account, windows in the natural frequency spectrum of the blades are less likely at higher frequencies, where the values for higher modes tend to be closely spaced. By contrast, the window between the first and second mode of this blade tends to be a common feature of radial flow blades.

The calculated natural frequencies for the blade agree surprisingly well with the measured values. Figure 11 compares the two sets of frequencies, again ignoring centrifugal stiffening effects. (The calculated values have been adjusted slightly, corresponding to the room temperature conditions of the tests.) Impulse excitation was used in determining the first two blade frequencies, and shaker excitation was used for the next two, but in all cases the test frequencies reflect full blade-disk interaction. The worst disagreement between calculated and measured values is 5.1 percent, for the first mode. The calculated results are consistently on the low side, which is surprising since the assumption that the disk is rigid introduces an error in the opposite direction. Regardless, the blade-alone model with a rigid disk turned out to be not that bad for engineering purposes: in the case of this design.

The lowest splitter blade natural frequency is of the same order as the frequency of the second mode of the full blade. Therefore, once 9 nozzles were chosen, the splitter could be easily tuned to raise all of its natural frequencies safely above the nozzle passing frequency.
Predicted and Measured Performance

Figure 12 compares the predicted and measured performance of this steam turbine stage. The predicted efficiency at the design point was 89.2 percent. The measured efficiency was 89 percent (at a blade jet-speed ratio of 0.715). The measured mass flow rate was 4 percent higher than predicted, corresponding to a flow angle decrease of roughly 0.6 degrees at the nozzle exit.

A PROCESS TURBINE

Because the process turbine was designed as a replacement for a failed turbine, the design was done under considerable time pressure. Furthermore, the rotor was required to fit in the existing housing. Normally, we would have increased the rotor diameter beyond 560 mm in order to make the leading edge incidence angle more nearly optimal from a vibrational as well as an aerodynamic standpoint.

The prior history of fatigue failures and the less than optimal incidence angle gave us reason to be cautious. Unlike in the steam turbine, we took the second harmonic of the nozzle wake excitation into consideration. We also gave attention to potential resonances outside the specified operating range that could become hazardous during start-up of the chemical process. We introduced 30 degrees of rake at the leading edge of the blades in order to reduce the impact of the nozzle wakes on the blades by spreading it out a little. This rake is clearly visible in the photograph of the rotor (Fig 13). Finally, we resorted to some aerodynamic tricks to reduce the incidence angle and hence the blade loading near the leading edge.

Design Requirements

The principal requirement was to eliminate fatigue problems without seriously compromising aerodynamic performance. The specified operating range was 9000 to 11,500 rpm, with the flow modulated by variable nozzle vanes. A review of previous operation, however, revealed that extensive time had accumulated below design speed, anywhere from 6000 to 9000 rpm. The design point operating conditions were as follows:

- Inlet Total Temperature (K): 418.0
- Inlet Total Pressure (kPa): 727.6
- Mass Flow Rate (kg/sec): 16.445
- Rotational Speed (rpm): 11,500
- Pressure Ratio: 4.203
- Target Total-to-Static Efficiency: 0.80

The turbine also had to pass a maximum flow of 18.348 kg/sec at these conditions.

Aerodynamic Design

A flow analysis of the failed turbine revealed a high rotor inlet tangential velocity of 365.8 m/sec at the design point, and a correspondingly high incidence angle of 22 degrees. As a result, the calculated velocities along the pressure surface of the blades were negative for 40 percent of the meridional flow path, with values as high as -200 m/sec. In other words, the flow was separated over a substantial portion of the pressure surface. This is bad aerodynamically, but it is even worse for vibration. Severe separated flow at the leading edge of blades greatly amplifies the unsteady forces from wake disturbances. Accordingly, the main objective of the aerodynamic design effort was to reduce the incidence to 10 degrees or less and thus to reduce, if not eliminate, the region over which the calculated blade surface velocities are negative.

The incidence is high in the failed turbine because the rotor is undersized for its design conditions. Since the diameter of the rotor could not be increased, however, the problem had to be dealt with in other ways. Two moves were made to reduce the incidence. First, the rotor inlet width was reduced. This increases the inlet meridional velocity, but it also lowers the incidence angle. Second, negative swirl - i.e. negative tangential velocity - was introduced at the rotor exit. This shifts more of the total work done toward the trailing edge. The following comparison of two of the preliminary designs for the replacement turbine shows the efficacy of these two moves:

<table>
<thead>
<tr>
<th>Design</th>
<th>Initial Prelim.</th>
<th>Final Prelim.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor Tip Width (mm)</td>
<td>32.0</td>
<td>27.0</td>
</tr>
<tr>
<td>Rotor Inlet Tangential Velocity (m/sec)</td>
<td>365.0</td>
<td>349.0</td>
</tr>
<tr>
<td>Rotor Inlet Relative Flow Angle (deg)</td>
<td>18.7</td>
<td>7.1</td>
</tr>
<tr>
<td>Rotor Exit Swirl Angle (deg)</td>
<td>0.0</td>
<td>15.2</td>
</tr>
<tr>
<td>Total-to-Static Efficiency</td>
<td>0.853</td>
<td>0.845</td>
</tr>
</tbody>
</table>

The final preliminary design satisfied the objectives.
FIG. 14 - PROCESS TURBINE, MERIDIONAL FLOW PATH

Rotor Design. Figure 14 shows the final meridional flow path. Initially, the blade thickness distribution was the same, save for scale, as in the steam turbine. In order to tune the blade for vibration, however, this thickness distribution had to be changed, as indicated in Figure 15. Figure 16 shows the blade surface velocities for the two different blade thickness schedules. The change in the thickness distribution had only a small effect, confirming that this is a good variable to use in tuning. Moreover, in both cases the region of negative velocity on the pressure surface has been greatly reduced. Splitter blades would have eliminated these regions completely, but the further complications they would have introduced were inappropriate. The diffusion parameters exceed the design criteria at the hub, but they are adequate at the mean and shroud:

<table>
<thead>
<tr>
<th>Hub</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diffusion, Dp</td>
<td>0.6</td>
</tr>
<tr>
<td>Diffusion Rate, Dr</td>
<td>0.8</td>
</tr>
</tbody>
</table>

The design was accepted on the basis of the surface velocities for the mean streamtube.

Nozzle Design. Initially, 12 nozzles were selected and designed following the established design procedure. Subsequently, however, when the number of nozzles was reduced to 9, the design procedure could no longer be used. Obtaining acceptable blade shapes for solidity ratios below 1.2 within the radius ratio constraint of 1.4 proved difficult.

Therefore, a geometric construction method was used to define the nozzle vanes shown in Figure 17. Points A and B were defined by using a log spiral with an angle of about 78 degrees. The throat opening at the design operating condition was calculated to be 50.8 mm, and a smooth curve was drawn from B to C to obtain this opening. On the pressure surface, the curve from D to E was defined to have a converging channel passage from the inlet to the throat location. This design maintains the converging passage when the vanes are restaggered about point F, open or closed through a total of about 10 degrees of rotation.

No flow analysis was performed on this vane design. The log spiral surface minimizes the diffusion from the throat to the trailing edge on the suction surface, which is the main design requirement.
Also, the trailing edge blockage is only about 1 percent, compared to 15 percent for the vane design in the failed turbine.

**Vibrational Design**

Figure 18 presents the calculated blade natural frequencies for the rotor design, prior to any blade tuning, and Figures 19 through 22 display the corresponding mode shapes. The calculations were made using SUPERB, with the blade and a sector of the disk modeled by thick shell elements, as shown in the mode shape figures. The first mode is the classical first mode of radial flow blades, in which the trailing edge of the blade is vibrating much like an axial blade in a first bending mode. The second mode has a nodal point along the shroud line, so that the trailing edge is vibrating 180 degrees out of phase with the rest of the blade. In the third mode this nodal point is further upstream, and there is also a nodal point along the trailing edge, corresponding to a second bending mode of an axial blade. The fourth mode has two nodal points along the shroud, and both the leading edge and the corner of the trailing edge are vibrating significantly.
Including the disk sector in the finite element model of the blade had only a small effect on the calculated frequencies. The frequencies of these four modes are between 1 and 2 percent higher with a rigid disk. This is consistent with the comparatively low vibration levels in the disk in the mode shapes.

When the natural frequency spectrum in Figure 18 is viewed in the light of the steam turbine discussed above, two approaches to tuning the blades appear possible. On the one hand, the first two modes will safely straddle the nozzle passing frequency, just as in the steam turbine, if 12 nozzles are used. The only problem on this approach is to raise the frequency of the third mode so that it is no longer in resonance with the second harmonic of the nozzle wake excitation. This must be done without raising the first two frequencies into resonance. On the other hand, 9 nozzles can be used, as in the steam turbine, if the first two frequencies can somehow be raised substantially. (Not enough space is available in the existing housing to allow fewer than 9 nozzles.) The main problem on this approach is to be able to predict the natural frequencies with the required accuracy. Almost certainly the smallest safety margin will be for the most highly responsive first mode at maximum steady state stress conditions.

We ended up pursuing both of these approaches, one following the other. First, we tuned the blades for 12 nozzles by modifying their thickness distribution. We then manufactured the rotor and tested its natural frequencies. Once we found we could predict the blade natural frequencies with enough accuracy, we shifted to 9 nozzles, raising the blade frequencies by cutting back the trailing edge.

Figure 15 contrasts the original blade thickness distribution and the modified thickness distribution required to tune the blades for 12 nozzles. The modified thickness distribution was arrived at in a series of trials, constrained on one side by manufacturing considerations and on the other by the maximum flow the turbine must pass. The thickness change indicated in the figure raises the third natural frequency 9 percent, the first two frequencies 7 percent, and the fourth frequency 2 percent. Figure 23 shows the Campbell diagram for the tuned blades. The one obvious point of concern is that the margin between twice the nozzle passing frequency and the third natural frequency is only 6.2 percent at the maximum operating speed. We nevertheless accepted this design partly because centrifugal stiffening at maximum speed will tend to increase the margin, partly because the potential resonance in question involves the second harmonic of the excitation, and partly because raising this frequency further created worse problems elsewhere.

Table II compares the calculated and measured natural frequencies of the tuned blades. The agreement is comparable to what we found for the steam turbine, with a maximum discrepancy slightly below 5 percent. The tests revealed a large number of other natural frequencies, which modal analysis showed were disk modes, although they consistently involved blade-disk interaction. So long as the number of nodal diameters in a disk mode does not coincide with the number of excitation peaks around the circumference at the same frequency, the disk mode is not excitable. The only disk modes of concern here were one-nodal-circle modes at 4150 and 4600 Hz. Failure analyses had indicated that one-nodal-circle modes, excited by nozzle wakes acting on the blades, contributed to the failures of the original turbine.

Three considerations led us to pursue the alternative of a 9 nozzle design. First, the one-nodal-circle mode at 4150 Hz is in resonance with the second harmonic of the nozzle wake excitation in the middle of the operating range. Second, at maximum speed, the margin between this harmonic and the measured value of the third frequency is only 2.6 percent. Although centrifugal stiffening will increase it, this margin is smaller than we would prefer. Third, the first mode is in resonance with the nozzle passing frequency at 7200 rpm. Although this is below the specified operating range, 7000 to 7500 rpm is nevertheless a preferred dwell point during start-up of the chemical process.

The question was whether cutting back the trailing edge could raise the blade natural frequencies enough to allow 9 nozzles to be used. Figure 24 indicates how the calculated frequencies vary as a function of the angle at which the trailing edge is cut back. Based on this curve, we elected to cut the trailing edge back 5.5 degrees. Cutting it back more increased the incidence angle at the leading edge too much. After the cut-back, the measured first

<table>
<thead>
<tr>
<th>Mode</th>
<th>Calculated Frequency</th>
<th>Measured Frequency</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1575</td>
<td>1600</td>
<td>-1.6</td>
</tr>
<tr>
<td>2</td>
<td>3467</td>
<td>3325</td>
<td>4.3</td>
</tr>
<tr>
<td>3</td>
<td>4903</td>
<td>4725</td>
<td>3.8</td>
</tr>
<tr>
<td>4</td>
<td>5566</td>
<td>5325</td>
<td>4.5</td>
</tr>
</tbody>
</table>
frequency was 1850 Hz, as predicted, and the second frequency was 3982 Hz, 5.6 percent below the calculated value, but still safely above twice the nozzle passing frequency. Figure 25 shows the final Campbell diagram for the turbine, using the measured values for the first two frequencies. The smallest margin of safety for resonance is 7 percent, viz. for the first mode at maximum speed. This margin is adequate since it is based on a measured value and centrifugal stiffening will tend to increase it. As the Campbell diagram makes clear, the major advantage of using only 9 nozzles is to eliminate all nozzle passing resonances not merely within the operating range, but below it too.

Even though the final design was arrived at in two steps, only 14 weeks elapsed from the time work began on the design until the turbine was fully operating. It has performed continuously for more than 10,000 hours since then. One reason for the short design and production period was the use of an integrated, interactive master computer program in the design. The blade geometry could be modified, the new natural frequencies could be determined, and the effects on aerodynamic performance could be assessed within 2 hours. But the primary reason was the use of numerically controlled machining in the production of the impeller. The integrated master computer program automatically generates the tape for machining.

**Predicted and Measured Performances**

The cut-back trailing edge reduced the negative swirl at the rotor exit. Consequently, the incidence angle was increased to 10 degrees - slightly greater than we would have preferred, but of somewhat less significance for a design with no nozzle passing resonances in or below the operating range. The cut-back trailing edge had no effect on the predicted efficiency.

Figure 26 compares the predicted and measured performance of the turbine. The predicted total-to-static efficiency of 84.5 percent was only slightly above the efficiency measured, which was not at design operating conditions.

**ROTOR MANUFACTURE**

The blade thickness distribution and trailing edge contour are good variables to use in tuning precisely because aerodynamic performance is on the
whole less sensitive to them. The obvious question, however, is whether the same thing is true of these two variables with respect to manufacturability. In particular, how do comparatively complicated blade thickness schedules affect the manufacturability of the wheels? The answer depends entirely on how the blade surfaces are defined. On our approach, the blade surfaces are always defined by straight-line elements, regardless of the thickness schedule specified along the hub and shroud. Consequently, the blades are always easy to manufacture, using numerically controlled 5-axis milling machines. The straight-line element construction allows the blades to be milled with the cutter flanks, as shown in Figure 27. Flank milling reduces the machining time by an order of magnitude compared to point milling. For example, a 65 mm diameter wheel with 15 blades required only two and a half hours to be machined in aluminum.

NREC adopted the straight-line element definition of blades many years ago, largely out of the need to have some standardized convention. But, with the advent of numerically controlled machining, it has turned out to offer major advantages in manufacture.

CONCLUDING REMARKS

The central point of this paper is that it is now feasible to design radial inflow turbines for nozzle wake induced vibration in the same way as axial turbines. Current finite element programs can predict blade natural frequencies reasonably well. The frequencies can be changed significantly by changing the blade thickness distribution and trailing edge contour. Because aerodynamic performance is comparatively insensitive to these variables, tuning does not in general require extensive aerodynamic redesign. Furthermore, state-of-the-art methods permit the aerodynamic effects of tuning to be assessed, so that aerodynamic aspects can be traded off against vibrational aspects, as we did in the process turbine. Finally, so long as the blade surfaces are defined in an appropriate way, using blade thickness distribution and trailing edge contour to tune has no adverse effect on manufacturability.

Using a small number of nozzle vanes is especially helpful when the turbine operates over a wide speed range. It reduces the number of modes that have to be tuned, and it focuses concern on modes that are easiest to tune. Also, windows in the natural frequency spectrum tend to be largest at the low end of the spectrum.

Nevertheless, tuning radial inflow turbines for nozzle wake vibration is still far short of the level of sophistication of axial blade tuning. The next step is to use more powerful finite element programs. Based on experience with axial turbines, three refinements are especially needed. First, centrifugal stiffening of both the blade and the disk should be taken into account, if necessary by including special elements for modeling radial inflow turbine disks. Most designs will require extra safety margins if centrifugal stiffening is treated in the qualitative way we have followed here. Second, the tangential boundary conditions for a sector of a bladed disk should allow arbitrary phase lags in the excitation acting on, and hence in the vibration of, adjacent blades. Otherwise, blade-disk interaction is mis-represented unless the entire bladed disk is modeled. Finally, the finite element programs should permit a forced response analysis for an arbitrarily phased excitation acting along the blade. Even though excitation and damping cannot yet be calculated with great accuracy, they can be estimated well enough for purposes of determining whether a particular blade resonance is hazardous—e.g., whether a twice nozzle passing resonance of a higher mode is enough of a hazard to require further tuning.

REFERENCES