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THE PATH TO PREDICTING BYPASS TRANSITION

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ABSTRACT

A theory is presented for calculating the fluctuations in a laminar boundary layer when the free stream is turbulent. The kinetic energy equation for these fluctuations is derived and a new mechanism is revealed for their production. A methodology is presented for solving the equation using standard boundary layer computer codes. Solutions of the equation show that the fluctuations grow at first almost linearly with distance and then more slowly as viscous dissipation becomes important. Comparisons of calculated growth rates and kinetic energy profiles with data show good agreement.

In addition, a hypothesis is advanced for the effective forcing frequency and free-stream turbulence level which produce these fluctuations. Finally, a method to calculate the onset of transition is examined and the results compared to data.

NOMENCLATURE

c_f skin friction coefficient
k kinetic energy of the laminar fluctuations, $[m^2/s^2]$
p static pressure, $[N/m^2]$
Re Reynolds number
t time, [s]
Tu free-stream turbulence level
u velocity component in the x-direction, [m/s]
u^* friction velocity, [m/s]
U free-stream velocity, [m/s]
U_∞ free-stream velocity for unaccelerated flow, [m/s]
v velocity component in the y-direction, [m/s]
x coordinate in the free-stream direction, [m]
y coordinate normal to the surface, [m]

Greek
δ boundary layer thickness, [m]
ε dissipation of kinetic energy, $[m^2/s^3]$
η Kolmogorov's length scale, [m]
λ viscous dissipation length scale, [m]
Λ integral length scale of turbulence [m]
ν kinematic viscosity, $[m^2/s]$
ρ density, $[kg/m^3]$
υ Kolmogorov's velocity scale, [m/s]
ω angular frequency, [1/s]

Additional Marks
$O(q)$ of order of magnitude q
$\bar{q}$ time-averaged component of q
q' fluctuating component of q, $\overline{q'} = 0$

INTRODUCTION

One of the remaining difficulties in calculating laminar-to-turbulent transition in boundary layers is predicting its onset. For natural transition, onset is usually determined using the "e^n" method developed by Smith (1956) and others. This method, which is widely used in the aircraft industry, uses the amplification rate of the most unstable Tollmien-Schlichting wave at each stream wise position to determine a disturbance-amplitude ratio. Onset is then presumed to occur at the position where this ratio attains an experimentally determined critical value related to the free-stream turbulence level (Mack, 1977). For bypass transition, which is the usual mode of transition in gas turbine engines, onset is usually determined without too much regard concerning the physics involved. In this case, empirical correlations providing the best fit to transition data are used, and these are applied either directly to the mean flow (Abu-Ghannam and Shaw, 1980, and Mayle, 1991), or indirectly to the production of turbulent-kinetic-energy (from the many examples see the earliest, McDonald and Fish, 1973; one of the latest, Schmidt and Patankar, 1991; and a comparison of several, Sieger et al., 1993). In spite of these methods,

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however, predicting the onset of either natural or bypass transition is still more of an art than a science (see Savill, 1991, or Sieger et al. for examples).

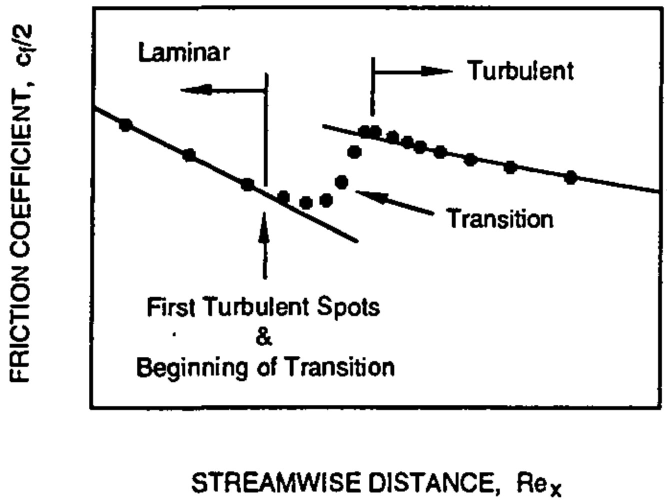

Figure 1. Transition from laminar to turbulent flow in a boundary layer.

As pictured in Fig. 1, transition from a practical standpoint may be considered to begin where a quantity such as the surface shear stress first deviates from its laminar value. In 1951, Emmons showed that this corresponds to the first position along the surface where isolated spots of turbulence within the boundary layer are formed. Clearly then, everything before the spots are formed happens in a completety laminar boundary layer. Measurements show, however, that this pre-transition flow is not steady (see Fig. 2).

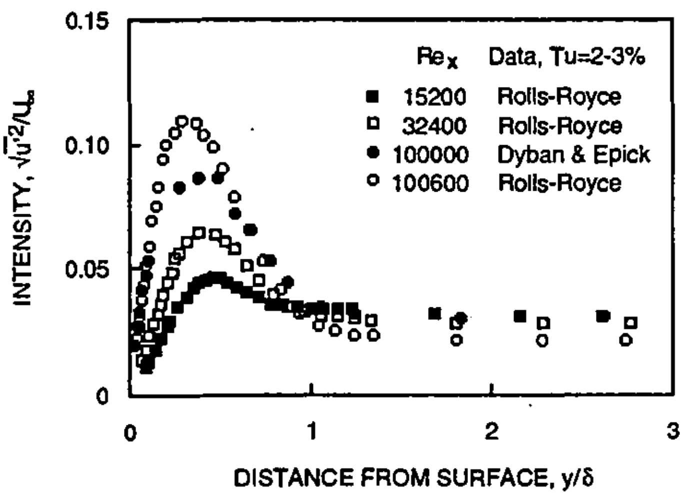

Figure 2. Fluctuations in a laminar boundary layer before the onset of transition.

For natural transition, which occurs when the free-stream turbulence level is zero or nearly so, Tollmien-Schlichting waves can be found. For bypass transition, which occurs at high free-stream turbulence levels, "turbulent-looking" fluctuations mimicking those in the free stream are found, and similar to the Tollmien-Schlichting waves these fluctuations also amplify eventually forming turbulent spots. Their growth in an unaccelerated flow is shown in Fig. 3.

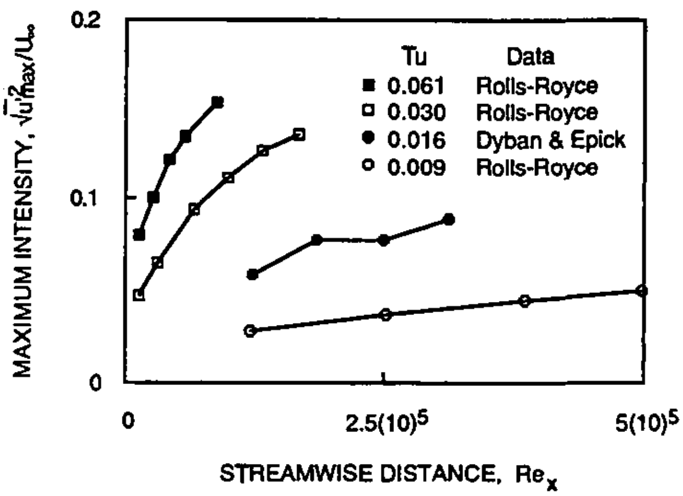

Figure 3. The growth of fluctuations in a laminar boundary layer before transition.

Although our eventual goal is to predict the onset of transition, our aim in this paper is to calculate the pre-transition laminar fluctuations. As will be seen, this is done by developing and solving a laminar-kinetic-energy equation for the fluctuations, but more importantly it is accomplished by recognizing a new mechanism and determining the effective frequency and turbulence level in the free stream which is responsible for both producing and amplifying them. The approach is new. It is based on Lin's (1957) analysis for unsteady laminar boundary layers and Dullenkopf and Mayle's (1995) concept of an effective frequency and turbulence level for laminar boundary layers in a turbulent free stream. Because it is easy to incorporate in any modern boundary-layer computer code, the approach is also practical.

The paper is divided into several sections. In the first we briefly review Lin's analysis and high-frequency results. In the next, we develop the laminar-kinetic-energy (LKE) equation, introduce the new mechanism, and propose models for the production and dissipation terms in the equation. After briefly describing how to include the new kinetic energy equation in k-ε type computer codes, we compare the results of our calculations with experimental data. Then we propose a method to determine the effective forcing frequency for the fluctuations and the effective free-stream turbulence level. Finally we examine a criterion for the onset of transition and conclude with suggestions of work yet to be done.

C.C. LIN'S ANALYSIS

Lin (1957) examined the effect of frae-stream fluctuations on laminar boundary layers by decomposing the velocities and pressura into time-averaged and time-dependent components similar to Reynolds' analysis for turbulent flow, i.e., $u(x,y,t) = \bar{u}(x,y) + u'(x,y,t)$, $v(x,y,t) = \bar{v}(x,y) + v'(x,y,t)$ and $p(x,t) = \bar{p}(x) + p'(x,t)$. In contrast to the usual turbulent approach, however, both the pressure and its fluctuation may be eliminated from the equations of motion by using the unsteady Euler equation, namely,

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} \qquad (1)$$

where ρ is the density and U is the free-stream velocity given by $U(x,t) = \bar{U}(x) + U'(x,t)$. The time-averaged boundary-layer equations for mass and momentum are idantical to those for turbulent flow except that the momentum equation contains the addition term $\overline{U'(\partial U'/\partial x)}$ on the right hand side[2], and except that now they apply to a time-averaged laminar flow. The equations for the fluctuating components of velocity can be obtained by subtracting the time-averaged equations from the equations for the instantaneous motion and, contrary to the situation for turbulent flow, can be solved once the unsteady free-stream velocity distribution is given.

Solutions are easily obtained when the frequency of the free-stream fluctuations is high enough such that $\omega >> \nu/\delta^2$, where ω is the frequency, ν is the kinematic viscosity, and δ is the boundary layer thickness. In this case, the fluctuations occur mostly adjacent to the wall within a thickness $\delta_\omega = \sqrt{2\nu/\omega} << \delta$ independent of the mean flow. Considering the temporal and spatial varying free-stream velocity $U(x,t) = U_\infty(x) + U'_\infty(x)\cdot sin\,\omega t$, the solution for the u' component is

$$u'(x,y,t) = U'_\infty\left[sin\,\omega t - e^{-(y/\delta_\omega)} sin\left(\omega t - \frac{y}{\delta_\omega}\right)\right];\ \left(\omega >> \nu/\delta^2\right) \qquad (2)$$

which can also be obtained from Stokes solution for a plate oscillating parallel to itself in a quiescent fluid (Schlichting, 1979). Since the equation for u' is linear, a solution for an arbitrary free-stream fluctuation may be obtained by superposition. If $U'_\infty \neq fnc(x)$, the normal component of the fluctuating velocity v', which is proportional to dU'_∞/dx, is zero, the apparent shear stress $\left(\overline{u'v'}\right)$ is zero, and the time-averaged velocities $\bar{u}$ and $\bar{v}$ are exactly those given by the laminar solution. Thus, for unaccelerated flow over a surface with slowly decaying free-stream turbulence, we should expect the mean flow velocity profiles to be those given by Blasius. This result Is well documented by Dyban et al. (1976), Sohn and Reshotko (1991), Zhou and Wang (1993), and others.

The kinetic energy for the u' component given in eq. (2) is

$$k = \frac{1}{2}\overline{u'^2} = k_\infty\left[1 - 2e^{-(y/\delta_\omega)} cos\left(\frac{y}{\delta_\omega}\right) + e^{-2(y/\delta_\omega)}\right] \qquad (3)$$

where, since there is no other fluctuating component in the free stream for this case, $k_\infty = \overline{U'^2}/2 = U'^2_\infty/4$. This distribution has a maximum of roughly $1.1k_\infty$ near $y = 2\delta_\omega$.

[2] See Lin's original paper or Schlichting (1979).

For details regarding higher order approximations, the reader is referred to Lin's original paper. For other approaches to solving laminar-boundary-layer flows with an unsteady free stream, the reader should see Moora (1951), Lighthill (1954), and Ackerberg and Phillips (1972).

THEORY

The Laminar-Kinetic-Energy Equation

To the authors' knowledge, the energy equation for laminar fluctuations, henceforth called the "LKE" equation, has never been presented before. It can be obtained in the same way as the turbulent-kinetic-energy equation (see Bradshaw, 1971), except that the pressuras $\bar{p}$ and p' can be eliminated by using the time-averaged and instantaneous form of eq. (1). For boundary layer flows, one obtains

$$\bar{u}\frac{\partial k}{\partial x} + \bar{v}\frac{\partial k}{\partial y} = -\left(\overline{u'v'}\right)\frac{\partial \bar{u}}{\partial y} - \frac{\partial}{\partial y}\left[\overline{v'k} - \nu\frac{\partial k}{\partial y}\right] - \varepsilon + \left\{\overline{u'\frac{\partial U'}{\partial t}}\right\} \qquad (4)$$

where k is the kinetic energy of the laminar fluctuations, and ε is the viscous dissipation of kinetic energy defined by $\varepsilon \equiv \nu\overline{(\partial u'/\partial y)^2}$. All of the terms but the last are similar to those in the TKE equation and represent the convection of laminar kinetic energy (on the left), and the production, diffusion and dissipation of laminar kinetic energy respectively (on the right). The term in curly brackets arises from taking the average of $u'(dp'/dx)$ and represents the production of laminar kinetic energy by the work of the imposed fluctuating pressure forces. This term is new. It is also the only term in the equation providing a direct link between the fluctuations in the free stream and boundary layer, and as will now be shown, it is the most important production term.

If the kinetic energy is to increase as shown in Fig. 3, then at least one of the production terms must be larger than the dissipation term. Supposing the orders of magnitude for each quantity in eq. (4) are $u' \approx O\left(\sqrt{k}\right)$, $v' \approx O\left(\delta\sqrt{k}/L\right)$ where L is a distance along the surface and $\delta \approx O\left(\sqrt{\nu L/U_\infty}\right)$, $\partial/\partial y \approx O(1/\delta)$, $\bar{u} \approx O(U_\infty)$ where U_∞ is the free-stream velocity, $\partial/\partial t \approx O(\omega)$, and $U' \approx O\left(\sqrt{k_\infty}\right)$ where k_∞ is the free-stream kinetic energy, one obtains

$$\overline{u'v'}\frac{\partial \bar{u}}{\partial y}\Big/\varepsilon \approx O(1) \quad \text{and} \quad \overline{u'\frac{\partial U'}{\partial t}}\Big/\varepsilon \approx O\left(\frac{\omega\delta^2}{\nu}\sqrt{k_\infty/k}\right)$$

This implies that only the new production term has a chance of overwhelming the dissipation term, and that this occurs when Lin's high frequency criterion is met, namely, when $\omega >> \nu/\delta^2$. If the free-stream fluctuations result from turbulence having a broad spectrum of frequencies, this criterion will always be met. Therefora, the main effect of free-stream turbulence on a laminar boundary layer is similar to the high frequency response examined by Lin.

Using the same order of magnitude analysis, it can also be shown that the diffusion of kinetic energy by the v' component of the fluctuations can be neglected comparad to the viscous diffusion. Hence, the relevant LKE equation for a laminar boundary layer with a turbulent free stream becomes

$$\bar{u}\frac{\partial k}{\partial x} + \bar{v}\frac{\partial k}{\partial y} = \overline{u'\frac{\partial U'}{\partial t}} + \nu\frac{\partial^2 k}{\partial y^2} - \varepsilon \qquad (5)$$

which reveals that the mechanism for producing and amplifying laminar fluctuations in a boundary layer has the form of a forcing function.

The impact of this mechanism can be seen immediately by considering only the first terms on each side of eq. (5) and setting the production term equal to $\omega\sqrt{k\cdot k_\infty}$. Considering $\omega \neq$ fnc(x) and $\bar{u} \approx U_\infty/2 \neq$ fnc(x), the equation may be integrated immediately to yield $\sqrt{k} \approx \sqrt{k_\infty}(\omega x/U_\infty)$. This is a fundamental result which shows that any free-stream fluctuation produces a fluctuation in the boundary layer directly proportional to itself that initially grows linearly with distance.

Modeling the Kinetic Energy Equation

Measurements by Liepmann (1943), Dyban and Epik (1985), and Rolls-Royce (1993) show that the major contribution of kinetic energy in a laminar boundary layer comes from the u' component of the fluctuations. Hence, to a good approximation, we have $k \approx \overline{u'^2}/2$. (Indeed, for boundary layers, eq. (4) is the transport equation for $\overline{u'^2}/2$.) In the free stream, $\overline{u'^2} \to \overline{U'^2}$ which for isotropic turbulence requires that $k \to k_\infty/3$.

Turning attention first to the production term in eq. (5), we propose that $\overline{u'\partial U'/\partial t}$ may be modeled by the expression $\omega_{eff}\sqrt{k\cdot k_{eff}}$ where, following Dullenkopf and Mayle (1995), ω_{eff} is an effective frequency of turbulence and k_{eff} is an effective free-stream turbulent kinetic energy for forcing the fluctuations. This provides $\overline{u'\partial U'/\partial t} \propto \omega_{eff}(Tu_{eff}/Tu_\infty)\sqrt{k\cdot k_\infty}$ where Tu denotes a turbulence level and k_∞ is the measured free-stream kinetic energy. Since any fluctuation in the free stream and its temporal derivative is ninety degrees out of phase, the production of kinetic energy must decay as the free stream is approached. The "damping" factor for Lin's problem is e^{-y/δ_ω}. Sharma et al. (1982) noted that the fluctuation intensity profiles always seem to have a maximum near $y^+ \equiv u^* y/\nu \approx 25$, where u^* is the friction velocity. Supposing δ_ω is proportional to the position of maximum intensity provides $\delta_\omega u^*/\nu =$ constant, say C^+, and the damping factor becomes $\exp(-y^+/C^+)$. Therefore a suitable expression for the production term might be

$$\overline{u'\frac{\partial U'}{\partial t}} = C_\omega \frac{U_\infty^2}{\nu}\sqrt{k\cdot k_\infty}\,e^{-y^+/C^+}$$

where C_ω = constant x $(\omega_{eff}\nu/U_\infty^2)(Tu_{eff}/Tu_\infty)$, and where both C_ω and C^+ remain to be determined. Although the damping factor in this expression appears similar to Van Driest's (1956) for turbulent boundary layers, and is arrived at through similar considerations, his refers to the viscous damping of fluctuations as the surface is approached while the present factor refers to a decreased interaction caused by a phase shift as the free stream is approached.

Turning attention now to the dissipation term, we have $\varepsilon = \nu\overline{(\partial u'/\partial y)^2} \propto \nu k/\lambda^2$ where λ is a viscous dissipation length scale. Knowing that the dissipation remains finite near the surface, indeed a maximum, and that velocity fluctuations near the surface must be proportional to y, the length scale near the surface must be proportional to y. Since the dissipation and diffusion of kinetic energy are equal at the surface, the coefficient of proportionality must equal two. Hence, a suitable expression for the dissipation term might be

$$\varepsilon = 2\nu\frac{k}{y^2}$$

This expression is identical to that proposed by Chien (1982) for the viscous dissipation of turbulent kinetic energy.

Substituting these expressions into eq. (5), the LKE equation becomes

$$\bar{u}\frac{\partial k}{\partial x} + \bar{v}\frac{\partial k}{\partial y} = C_\omega\frac{U_\infty^2}{\nu}\sqrt{k\cdot k_\infty}\,e^{-y^+/C^+} + \nu\frac{\partial^2 k}{\partial y^2} - 2\nu k/y^2 \quad (6)$$

The boundary conditions are k = 0 at y = 0 and, for isotropic free-stream turbulence, $k \to k_\infty/3$ as $y \to \infty$. In addition, an initial kinetic energy profile must be provided.

Solution Methodology using *k,ε*-Codes

Eq. (6) can easily be solved by most modern boundary-layer computer codes. For codes based on the Spalding-Patankar algorithms, incorporating an additional transport equation is quite straightforward. Caution is needed only to ensure that the diffusivity of kinetic energy, which is usually calculated from an effective diffusivity using molecular and turbulent Prandtl numbers, is set equal to ν. It is also necessary to provide for the possibility that k diffuses beyond the viscous boundary layer. In addition, all turbulent quantities in the time-averaged equations of motion must be set equal to zero and any transition model must be deactivated such that the flow remains completely laminar. This can usually be accomplished by setting the production of turbulent kinetic energy equal to zero.

For the present calculations, eq. (3) with $\delta_\omega = \delta/4$ was used as the initial profile. Calculations with other values of δ_ω had virtually no effect on the solutions. In addition, calculations were begun at $Re_x = 1000$ and no modifications to the equations of motion for the mean flow were made. Since the stream wise distribution of maximum fluctuation intensity is used for predicting the onset of transition, provisions for obtaining it should also be included in the code.

COMPARISONS WITH EXPERIMENTS

Calculations for this paper were performed using the computer code called "ALFA" (Sieger et al., 1993). This is a standard boundary-layer code of the k-ε variety which was modified as just described. Three of the following comparisons are made with data obtained by Rolls-Royce (1993) for free-stream turbulence levels of about 1, 3, and 6 percent. Their experiments are well documented and all necessary turbulence data are available (Roach, 1987, Roach and Brierley, 1990). In addition, these data have become standard test cases for transitional flow modeling (Savill, 1991). A comparison with data from Dyban and Epik (1985) is also made for a free-stream turbulence level of about 2 percent.

Preliminary calculations indicated that reasonably good results could be obtained by considering both C_ω and C^+ independent of x. The "best" values for C_ω and C^+ were then obtained by fitting the data by eye realizing that C_ω has the greatest effect on the growth of kinetic energy, while C^+ has the greatest effect on its growth rate (see eq. 6). When no combination of C_ω and C^+ could be found to fit the kinetic energy distributions in both the x and y directions, a compromise was made to fit the stream wise distribution of maximum kinetic energy. With this, the best value for C^+ turned out to be virtually the same for all of the data and was consequently set equal to a constant, namely, $C^+ = 13$.

The calculated and measured maximum intensity distributions are shown in Fig. 4. Agreement is excellent, and although not shown, a good fit for the 1% data is found all the way out to $Re_x \approx 1.3(10)^6$. The values of C_ω which provide these results are presented in Table 1. Their variation will be discussed in the next section.

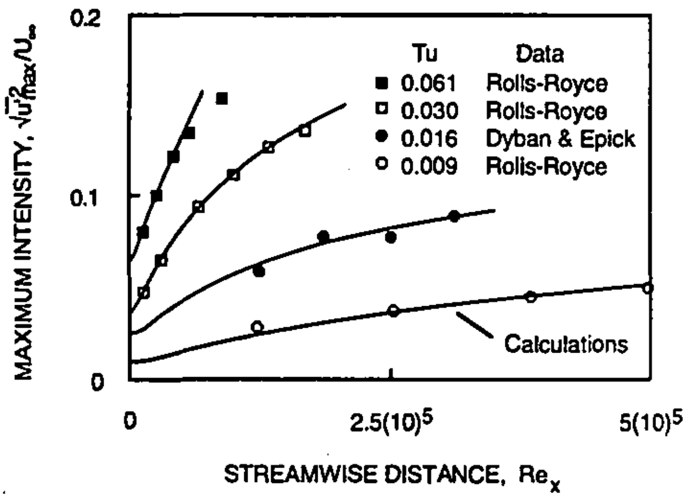

Figure 4. A comparison of the calculations with measurements.

Cross-sections of calculated and measured intensity profiles are shown in Figs. 5a and 5b. While on the average reasonable, the calculated profiles are not quite right. In general, the peaks of the intensity profiles are calculated closer to the wall than measured. This is particularly true for the $Tu_\infty = 1\%$ test case where just before transition (comparison not shown), the peak is predicted to be one-third of the measured distance from the wall. In this case, however, fluctuations at the Tollmein-Schlichting frequency were detected and transition is suspected to be caused by a natural instability. In spite of these discrepancies, the agreement between the calculated and measured intensity profiles is remarkable considering the simplicity of the model and the fact that no transitional boundary-layer code using low-Reynolds-number turbulence modeling has yet been able to calculate these data.

Table 1

Test Case	Tu_∞ [%]	C_ω
Rolls-Royce (1993)	0.9	0.00010
Dyban &Epik (1985)	1.6	0.00014
Rolls-Royce (1993)	3.0	0.00021
Rolls-Royce (1993)	6.0	0.00017

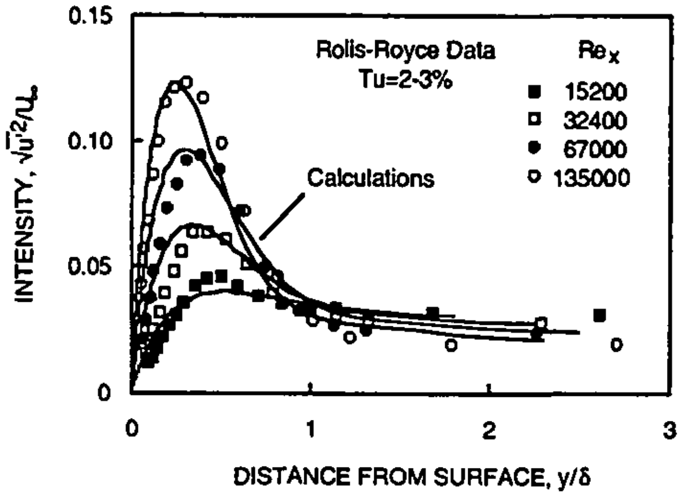

Figure 5a. Calculated and measured intensity profiles for one set of free-stream conditions.

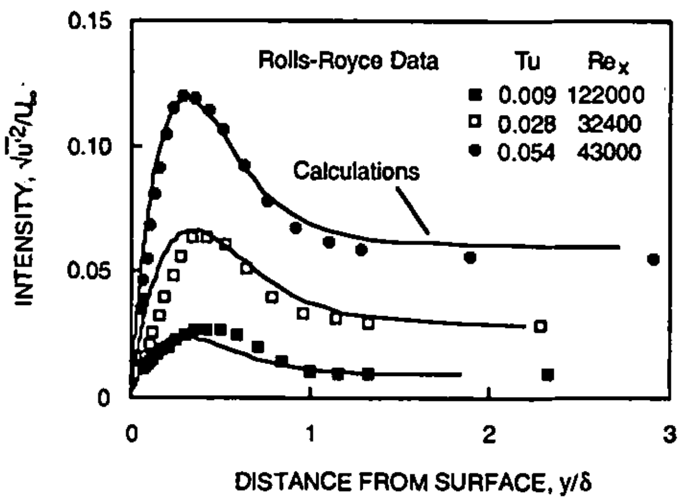

Figure 5b. Calculated and measured intensity profiles for three different sets of free-stream conditions.

EFFECTIVE FREQUENCY AND TURBULENCE LEVEL

By definition, C_ω depends on both an effective frequency and turbulence level. Therefore, one may expect that the variation of C_ω with Tu_∞ as shown in Table 1 is not in general valid. In fact we believe that C_ω, and consequently the production of LKE, is intimately connected to the turbulence spectrum characterized at the very least by the turbulence level, integral length scale, and the dissipation length scale.

Since the boundary layer is thinnest at the front of a surface, the first fluctuations in the layer will be produced by turbulence in the free stream having the highest frequencies. The "highest" frequencies are not necessarily "effective", however, since fluctuations at these frequencies will be viscously dissipated. Considering free-stream dissipation is most intense at the frequency ω_d, say, any fluctuations with frequencies near or higher than ω_d will certainly be dissipated in the boundary layer. Thus, the first fluctuations to amplify in the boundary layer will have frequencies less than ω_d, and from Lin's criterion this will occur once $Re_x >> U_\infty^2/\omega_d\nu$.

The frequency ω_d can be determined from the second moment of the turbulence spectrum (Hinze, 1975) and corresponds roughly to the frequency where the energy begins to fall off from Kolmogorov's equilibrium spectral distribution. An estimate for it, however, can be obtained by dividing the free-stream velocity by the eddy size in the free stream most affected by viscous dissipation. Introducing Kolmogorov's length scale η, one obtains (see Hinze) $\omega_d \approx 0.1U_\infty/\eta = 0.1U_\infty/(\nu^3/\varepsilon_\infty)^{1/4}$ where ε_∞ is the free-stream dissipation of turbulence. Assuming that the effective frequency for fluctuations in a laminar boundary layer is some fraction of this, one obtains

$$\frac{\omega_{eff}\nu}{U_\infty^2} \propto \frac{\omega_d\nu}{U_\infty^2} \propto \frac{(\varepsilon_\infty\nu)^{1/4}}{U_\infty} = \frac{\upsilon}{U_\infty}$$

where υ is Kolmogorov's velocity scale.

Following Dullenkopf and Mayle (1995), fluctuations in a laminar boundary layer will only respond to the energy contained within a relatively small band of frequencies near the effective frequency. If the energy spectral distribution near ω_{eff} is that given by Kolmogorov, and only the energy contained within a band near this frequency is important, an estimate for the effective turbulence level can be obtained. According to Dullenkopf and Mayle, the effective turbulence level is

$$\frac{Tu_{eff}}{Tu_\infty} \propto \left(\frac{\omega_{eff}\Lambda}{U_\infty}\right)^{-1/3} \propto \left(\frac{\omega_d\Lambda}{U_\infty}\right)^{-1/3} = \left(\frac{\omega_d\nu}{U_\infty^2}\right)^{-1/3}(Re_\Lambda)^{-1/3}$$

where Λ is the integral length scale of turbulence and $Re_\Lambda = U_\infty\Lambda/\nu$ is the integral-length-scale Reynolds number.

Substituting these expressions into the definition of C_ω yields

$$C_\omega = C\left(\frac{\upsilon}{U_\infty}\right)^{2/3} Re_\Lambda^{-1/3} \qquad (7)$$

where C is an accumulation of proportionality factors which is expected to be a constant. The quantities υ/U_∞ and Re_Λ can be determined directly from the turbulence energy density spectrum. Since υ is related to the dissipation of turbulence, it can also be determined from the decay of turbulence, namely $(\upsilon/U_\infty)^4 = -(3/2)[d(Tu_\infty^2)/dRe_x]$. Estimating Re_Λ from the dissipation length scale is not recommended, however, since comparisons between these estimates and measurements using the data of Roach (1987) show poor agreement.

The 1, 3, and 6% data of Rolls-Royce were obtained using three very different turbulence grids. Since the turbulence field generated by these grids is well documented by Roach (1987), however, one can easily determine the values of υ/U_∞ and Re_Λ. These values are listed in Table 2. The values for υ/U_∞ were obtained using the equation above together with Roach's correlations for the decay of Tu_∞. The correlation for the 1 and 3% test cases, which used grids with round bars, is found in Roach's Fig. 3. The correlation for the 6% case, which used a grid with square bars, is found in Roach's Fig. 4. The grid for this test, as for the 3% test case, had bars placed in a parallel array. The grid for the 1% case had bars placed in a square array. Unlike the correlations given for Tu_∞, the correlation for Λ does not fit the data very well. Therefore the values for Re_Λ in Table 2 were obtained directly from the data plotted in Fig. 9 of Roach's paper, not from his correlation. For Dyban and Epik's test case, υ/U_∞ was evaluated from the measured decay of turbulence. No data on length scale, however, are reported.

Table 2

Test Case	υ/U_∞	Re_Λ	C
Rolls-Royce (1%)	0.0035	4740	0.073
Dyban &Epik (2%)	0.0080	-	-
Rolls-Royce (3%)	0.0098	3590	0.070
Rolls-Royce (6%)	0.0117	9830	0.071

The values for C in the last column of Table 2 were obtained using eq. (7) and the values for C_ω given in Table 1. The fact that C is virtually identical for all three cases, in spite of the large variations in Tu_∞, υ/U_∞, and Re_Λ, is truly remarkable, and nicely supports our hypotheses for an effective frequency and turbulence level. Making our best estimate for Dyban and Epik's test, however, we obtain $Re_\Lambda \approx 3000$ which in turn provides $C \approx 0.05$. This difference we believe is in the uncertainty of Re_Λ and perhaps even in the value given for υ/U_∞ which had to be extrapolated back to the beginning of the plate. Considering this, we presently suggest using C = 0.07 in eq. (7) and strongly urge using measurements for obtaining both υ/U_∞ and Re_Λ. To the experimenter, of course, we strongly encourage measuring the complete spectrum of turbulence in the free stream, paying particular attention to frequency spikes, and ask for stream wise distributions of turbulence intensity, integral- and micro- length scales and dissipation. In addition, all experimenters should measure the intensity and spectral energy distribution of the stream wise component of vibration of the test surface which, of course, contributes to the "real" turbulence "sensed" by the flow over the surface.

THE ONSET OF TRANSITION

In 1943, Liepmann proposed that transition begins where the maximum Reynolds stress in the boundary layer equals the wall shear stress. According to Sharma et al. (1982) this may be expressed as

$$\sqrt{(\overline{u'^2})_{max,t}} \approx 3(u^*)_t \qquad (8)$$

where the subscript "t" denotes the condition at the onset of transition, and u^* is the friction velocity.

The calculated values of $\sqrt{(\overline{u'^2})_{max}}/U_\infty$ from Fig. 4 have been replotted in Fig. 6 together with the quantity $3u^*/U_\infty$. For unaccelerated flow, $u^*/U_\infty = 0.576/Re_x^{1/4}$. The criterion expressed in eq. (8) is met where the curves cross. These locations are where transition is "predicted" to begin. The observed locations are marked with arrows. In general, agreement is best for the higher turbulence levels. The discrepancy at the lowest turbulence level is discussed below. Nevertheless, since the calculations of $\sqrt{(\overline{u'^2})_{max}}$ were correlated to give the best fit to the data, any disagreement here is a result of the criterion.

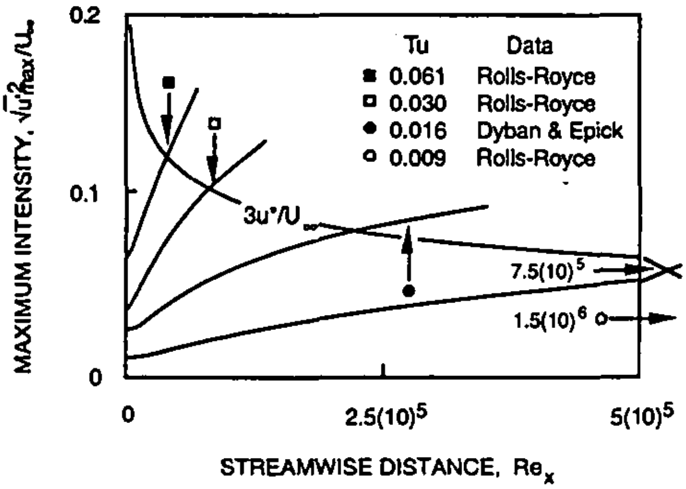

Figure 6. Calculated and measured locations for the onset of transition.

The $Tu_\infty = 1\%$ test case is interesting and impossible for most transitional boundary layer codes to predict. As reported by Roach and Brierley (1990), the fluctuations far downstream have a strong component at the Tollmien-Schlichting frequency. While fluctuations of this sort first appear contrary to the concept of an effective frequency and therefore impossible to predict using the present theory, we believe this test is an excellent and perhaps the only documented example of the subtle transformation between bypass and natural transition. Our interpretation of this test is that the initial fluctuations developed according to the present theory for bypass transition, but the growth rate was too low. Thus once the critical Reynolds number was reached, the Tollmien-Schlichting instabilities, which amplify exponentially compared to the linear amplification for bypass transition, dominated, and transition finally occurred "naturally." Since the average growth of the fluctuations, whether forced or natural, was reasonably calculated by the present theory, the effective and Tollmien-Schlichting frequency must be nearly identical for this case. This suggests that once the effective frequency nears the Tollmien-Schlichting frequency, transition is more likely to occur naturally.

CONCLUSIONS

The main idea proposed in this paper is that the laminar fluctuations preceding transition are primarily caused by the work of the imposed fluctuating free-stream pressure forces on the flow in the boundary layer. Based on this thought, we presented a theory for calculating these fluctuations using the laminar-kinetic-energy equation which, after some modeling, assumes the form

$$\bar{u}\frac{\partial k}{\partial x} + \bar{v}\frac{\partial k}{\partial y} = C_\infty \frac{U_\infty^2}{\nu}\sqrt{k\cdot k_\infty}\,e^{-y^+/C^+} + \nu\frac{\partial^2 k}{\partial y^2} - 2\nu k/y^2$$

where $C^+ \approx 13$.

Additional ideas concerning the frequency which drives the fluctuations were also proposed. These ideas permitted us to relate the coefficient C_∞ in the above equation to the free-stream turbulence-energy-density spectrum according to

$$C_\infty = C\left(\frac{\upsilon}{U_\infty}\right)^{2/3} Re_\lambda^{-1/3}$$

where $C \approx 0.07$.

These ideas are new and, we believe, clear the path to predicting the onset of transition. But before onset can reliably be predicted, much more work remains to be done. Some of our suggestions for work are:

- Obtain more data in the pre-transitional flow, including the quantities v' and $\overline{u'v'}$, as well as u', comprehensive free-stream turbulence measurements, and surface vibration measurements,
- Develop more sophisticated production and dissipation models, a more universal criterion for turbulent spot formation, and hence the onset of transition, and models for the second order effects of the fluctuations on the time-averaged velocity and temperature profiles, and
- Investigate the effects of free-stream acceleration, and the transformation between bypass and natural transition,

This list is neither all inclusive nor in an order of priority except, perhaps to say, that more data is clearly needed.

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