One-Dimensional, Stage-By-Stage, Axial Compressor Performance Model

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ABSTRACT

This paper presents a description of a one-dimensional, constant-radius, stage-by-stage (blade-element) axial compressor model used for compressor map generation and gas turbine off-design performance prediction. This model is designed for investigators who are without access to the proprietary compressor performance information of the gas turbine manufacturers but who are nevertheless interested in predicting the off-design performance of large utility gas turbine power systems. Model performance results (compressor maps) are reported for simulation of a nineteen-stage axial compressor designed by Allison Gas Turbine for the Electric Power Research Institute. The model is further demonstrated by simulating the NACA Eight Stage compressor. The resulting compressor maps are in good qualitative agreement with published maps and are useful for gas turbine power system performance simulation studies. This general-purpose modeling procedure can be applied to any axial compressor for which sufficient airfoil geometry and design-point performance information is known.

NOMENCLATURE

\[ A \] compressor annulus flow area
\[ C \] flow velocity relative to a stator
\[ C_o \] drag coefficient
\[ c_p \] specific heat at constant pressure
\[ g_{stg} \] constant in Newton's Second Law of Motion
\[ h \] enthalpy
\[ i \] incidence angle
\[ m \] mass flow rate
\[ m_c \] corrected flow, \( n \sqrt{\theta} / \delta \)
\[ N \] shaft speed
\[ N_c \] corrected speed, \( N / \sqrt{\theta} \)
\[ P \] pressure
\[ r \] radius
\[ R \] ideal gas constant
\[ T \] temperature
\[ U \] wheel speed

Subscripts

1 rotor inlet station of a compressor stage
2 rotor exit station of a compressor stage
\( m \) mean value
\( o \) reference value
\( amb \) ambient condition
\( std \) standard (sea level) quantity
\( stg \) compressor stage quantity
\( t \) total condition or property
\( x \) axial direction
\( \theta \) tangential direction

Superscripts

* reference, design, or critical value
airfoil inlet or exit "metal" angle

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BACKGROUND

The axial compressor performance model described in this paper was developed as a part of a larger study of the on- and off-design performance of utility gas turbines proposed for use in Gasification Combined-Cycle (GCC) power plants (Johnson, 1989). This modeling effort required compressor off-design performance information, or compressor maps, to enable gas turbine off-design performance prediction, including the prediction of performance when using fuels with heating values different from that of the "design fuels," natural gas or distillate oil. In order to investigate the off-design performance of a variety of gas turbines, one must be able to predict the off-design performance of a variety of gas turbine compressors, each characterized by a design point pressure ratio and mass flow rate. An additional requirement for the procedure is prediction of compressor performance using variable inlet guide vanes (VIGVs). These vanes can be used to control the airflow through the compressor; they are commonly used in constant speed utility gas turbines to reduce output power when following reductions in electric power demand. Also, these vanes can be used to "convert" a gas turbine from natural gas to syngas firing by helping to match the compressor and expander mass flow rates (Johnson, 1989).

A compressor modeling procedure ideally suited to these utility gas turbine performance simulations would allow the user to generate compressor performance maps for any given design compressor pressure ratio and flow rate. A one-dimensional, stage-by-stage model can provide this capability. Given such a model for a single compressor design, one can "scale" the model to a new design point pressure ratio by adding or removing the simulated compressor stages. Variable inlet guide vanes can be simulated by changing the input values for the angles describing guide vane orientation. Also, since the model is one-dimensional, the axial flow areas can be simply scaled to attain a new design point mass flow rate.

Availability of Gas Turbine Compressor Maps

The compressor map is a graphical representation of compressor performance over a range of ambient temperatures, rotational speeds, and mass flow rates. Compressor maps traditionally employ the "pseudo-nondimensional" parameters corrected flow, \( m_c \), and corrected speed, \( N_c \). These parameters are used to collapse the compressor performance characteristics to those of a machine operating under standard sea level ambient temperature and pressure conditions; they arise from the dimensional analysis of the important variables that characterize the performance of a compressor (Cohen et al. 1987). Though vital to gas turbine off-design performance analysis, compressor maps are typically not available in the open literature. Compressors are very expensive to design and test and the resulting compressor maps are held as proprietary company secrets by gas turbine manufacturers.

Other Compressor Off-Design Performance Models

Sophisticated, proprietary axial compressor off-design performance models are routinely used within the gas turbine industry; however, there are very few models published in the open literature. Three modeling techniques are described in Robbins and Dugan (1965): blade element methods, stage stacking, and a simplified method based on scaling the results of a known compressor using correlations they provide.

The simplified method would be satisfactory for many purposes if an appropriate compressor map were available for use as a starting point; the difficulty in obtaining such maps was discussed above. Also, this simplified method provides no correlations for estimating the effects of variable geometry, like variable inlet guide vanes (VIGVs).

In the stage stacking method, each compressor stage is characterized by a pair of curves for the stage work coefficient, \( \psi \), and the stage isentropic efficiency, \( \eta_{is} \), as functions of the flow coefficient, \( \phi \). These two parameters arise from the analysis of a single axial compressor or fan stage where the axial velocity through the stage is assumed constant. The resulting expression is

\[
\psi = 1 - \phi (\tan \beta_2 + \tan \alpha_i)
\]

where \( \beta_2 \) is the rotor exit flow angle measured relative to the rotor and \( \alpha_i \) is the inlet flow angle in the absolute (stationary) reference frame. The origin of this simplified nondimensional equation is presented in a later section.

When applying the stage stacking method, the user first computes \( \phi \) at the inlet to the first stage and then uses the stage performance curves to find the stage total enthalpy rise and isentropic efficiency. The inlet conditions to the next stage are then calculated and the procedure is repeated for each of the following stages, using curves characteristic of each stage. By repeating this procedure for a variety of corrected speeds and flows, a compressor map can be generated.

Howell and Calvert (1978) present a general purpose stage stacking model, but it requires cascade or compressor stage test data to be useful. For modern gas turbine compressors, this information is not available in the open literature. According to Robbins and Dugan, this is the greatest drawback to stage stacking methods. Also, because the effects of geometry are lost in a stage stacking model, predicting the effects of VIGVs requires additional test data.

In the blade element method, performance calculations for each stage are based on the actual inlet and exit flow conditions; the continuity, energy, and momentum equations are solved for every stage. The model described in this report is a blade element method applied to a single radial location for each compressor stage. This model requires two-dimensional cascade performance data to compute off-design performance, but since it is rooted in the fundamentals of thermodynamics and fluid mechanics, it is not as dependent on this information as are stage stacking methods. Since individual blade rows are simulated, this method allows the user to more easily compute the effects of variable inlet guide vanes.

SOME TURBOMACHINERY FUNDAMENTALS

The analysis of turbomachinery often begins with the Euler Turbine Equation, below:

\[
\Delta h = (U_2 C_{2s} - U_1 C_{1s}) / g_c \quad (2)
\]

Equation 2 is an expression for the increase in total enthalpy for a compressor stage as a function of inlet and outlet wheel speed and tangential flow velocities. For the analysis of axial turbomachines, it is convenient to rewrite Equation 2 in terms of axial velocities, wheel speeds, and the blade-relative exit flow angles.

\[
\Delta h = (U_{2z} - U_{1z} \tan \beta_2 - U_{1z} \tan \alpha_i) / g_c \quad (3)
\]

The angle \( \alpha_i \) is the flow angle at the rotor inlet measured relative to the upstream stator vane; \( \beta_2 \) is the rotor exit flow angle measured in the reference frame of the rotor. These angles are different from the corresponding airfoil exit "metal angles" by...
(small) deviation angles (see Figure 1). To get a general feel for compressor off-design performance, $\alpha_1$ and $\beta_2$ can be considered constant; they do not change significantly until the airfoil total pressure losses are also large (Serovy, 1985). For improved off-design predictions, the changes in deviation angles must also be accounted for; a procedure for including changes in deviation angle is described later. Note that for constant shaft speed (rpm), an increase in the axial velocity leads to a decrease in the total enthalpy rise per unit mass (and in stage pressure ratio). If an early stage in a multistage compressor fails to achieve its design pressure ratio at the design mass flow rate, the axial velocity into the next stage will be higher than design because density (pressure) is down and flow area is fixed. This stage will also fail to reach its design pressure ratio. This "snowballing" effect leads to rapid reductions in overall pressure ratio for small increases in mass flow rate; this, combined with an increase in losses as airfoil incidence angles change from the design values, leads to the near-vertical constant corrected speed lines on compressor maps.

For a one-dimensional model, $r_1 = r_2$, and Equation 3 can be simplified to

$$\Delta h_i = \frac{U(U - C_{a2}\tan \beta_2 - C_{a1}\tan \alpha_1)}{g_e}$$

If one further assumes that the axial velocity is constant ($C_{a2} = C_{a2} = C_2$) then multiplying Equation 4 by $g_e/U^2$ yields the relation between $\psi$ and $\phi$ of Equation 1. Note that Equation 1 has its roots in fan and blower analysis (Serovy, 1985) where stage density changes are small. In general, stage axial velocities cannot be considered constant.

**MODEL DESCRIPTION**

**Blade Row Total Pressure Loss**

This model uses correlations from two-dimensional cascade tests to estimate losses for flow through compressor blade rows. These losses have traditionally been quantified as losses in total pressure. The total pressure loss coefficient used in this model is defined below:

$$\bar{\omega} = \frac{P_{t2} - P_{t1}}{P_{t1} - P_1}$$

It is important to note that this is appropriate only for a constant-radius analysis and that the loss is computed in the airfoil reference frame. In this reference frame, velocities are defined relative to the airfoil; model calculations for the rotor are then similar to those for the stator.

For off-design calculations in this model, the airfoil total pressure loss is computed as a function of the incidence angle. (The correlation used here is discussed in a later section.) The incidence angle is the difference between the blade inlet "metal angle" and the inlet flow angle. A positive value of incidence is shown in Figure 1.

**Deviation Angle**

The deviation angle is the difference between the blade exit metal angle and the exit flow angle. A positive value of deviation angle is also shown in Figure 1. The deviation angle represents the "failure" of the airfoil to completely turn the flow.

The solution of Equation 4 requires the knowledge of the blade exit flow angles. The present model uses a correlation for the deviation angle as a function of incidence. The specifics of this correlation will be presented later.

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**Figure 1. Model calculations for a single blade row.**
Calculation Procedure for a Single Blade Row

The geometry of a blade row in this model is characterized by inlet and exit blade metal angles, inlet and exit axial flow areas, and the radial location of the mean streamline. In the reference frame of the airfoils, the inlet total pressure and temperature are known as well as the mass flow rate and the flow angle (it is either the inlet to the machine or the exit conditions of the previous stage have already been calculated). Three simultaneous equations must then be solved for the blade inlet static temperature, static pressure, and velocity. These equations arise from the First and Second Laws of Thermodynamics and the Conservation of Mass. The incidence angle can also be computed and with it the total pressure loss coefficient and the deviation angle.

For a constant-radius analysis in the blade-relative reference frame the total temperature is constant (since heat loss from the compressor is negligible) so the blade exit total temperature is known. The blade exit total pressure is then calculated using the loss coefficient. The blade-relative exit flow angle is computed by adding the deviation angle to the blade exit metal angle.

The exit static temperature, pressure, and velocity are computed as described above for the inlet; the total temperature, total pressure, and flow angle relative to the following airfoils are then calculated and the procedure is repeated for the next blade row. This procedure is outlined graphically in Figure 1. The actual equations solved are presented later in a section addressing the computer implementation of this model.

Two-Dimensional Cascade Loss and Deviation Data

There are few sources of two-dimensional cascade test data in the literature. Perhaps the most famous series of tests was performed by the National Advisory Committee for Aeronautics (NACA); the test results were published by Emery et al. (1958). In these experiments, a wide variety of cascade sections (characterized by airfoil geometry and blade spacing) were tested in a low speed wind tunnel. Data were collected as each cascade section was rotated relative to the incoming airflow. The results of these tests are useful for selecting a design point for a compressor stage, but they cannot be easily used to predict loss and deviation as a function of incidence.

Howell produced a correlation of British cascade test results in exactly the form needed for off-design performance prediction. Correlations for deviation and drag coefficient are replotted in Figures 2 and 3 from Howell’s plot in Horlock (1973).

The relation of the drag coefficient to the total pressure loss coefficient is shown in Equation 6 below:

\[
C_D = \frac{\bar{w}}{\sigma} \left( \cos^3 \alpha - \cos^3 \alpha * \right) \quad \text{where} \quad \tan \alpha_\ast = \frac{\tan \alpha - \tan \alpha_\ast}{2}
\]

The above relationship between the drag coefficient and the total pressure loss coefficient was derived by applying Newton’s Second Law of Motion to steady, low-speed flow through a two-dimensional cascade with uniform flow assumed at the cascade inlet and exit (see Horlock, 1973, or Gostelow, 1984).

In Figures 2 and 3, normalized values of deviation, \((\delta - \delta^*)/e^*)\), and drag coefficient, \(C_D/C_D^*\), are plotted against a normalized value of incidence, \((i - i^*)/e^*)\), where the starred quantities represent “minimum loss” of “reference” values. The dashed line in Figure 3 is an extension necessary only for predicting inlet guide vane losses; large negative values of normalized incidence arise on the inlet guide vanes as they are closed. This extension was estimated from the trend of the right-hand portion of the drag coefficient curve where large positive values of normalized incidence lead to large losses. Typical loss curves discussed by Wilson (1984) and Cohen et al. (1987) indicate that losses increase rapidly for large values of negative incidence as well.

![Figure 2. Correlation for deviation angle. (Source: Horlock, 1973.)](image)

![Figure 3. Correlation for drag coefficient. (Source: Horlock, 1973.)](image)

The correlations in Figures 2 and 3 complete the compressor off-design modeling procedure. Users can, of course, provide their own curves for loss and deviation as they implement this model. Applying the procedure to a specific compressor requires calibrating the model to the design conditions of the compressor.

Calibrating the Model to the EPRI/Allison Compressor Design Point

The Allison Gas Turbine Division of General Motors, under a contract from the Electric Power Research Institute, has designed an advanced compressor appropriate for use in utility gas turbines. This nineteen stage compressor has the design point characteristics shown in Table 1. Complete results of the compressor study, including estimated compressor maps and detailed flow path geometry, are presented in Bettner and Sehra (1986). This compressor was never built; all performance predictions are based in-house experience at Allison.
vector diagram information indicates a very small radius change
constant mean radius at about 24 inches, which corresponds to
odd numbers from one to twenty-one. The compressor has a nearly
airfoil span. These stations are identified, from tip to root, by the
the inlet and exit of every airfoil for eleven streamlines across the
computing an effective mean radius, \( r_{\text{mean}} \), using the flow angles and
each compressor stage was first "one-dimensionalized" by
axial velocities given in Bettner and Sehra and Equation 7 below.

\[
r_{\alpha} = \left( r_{\alpha} \right)^2 - r_{\alpha} \omega C_{\alpha} \tan \beta - r_{\omega} \alpha C_{\alpha} \tan \alpha
\]

This equation was derived by setting the right-hand-side of
the Euler Turbine Equation (Equation 3, where \( r_{\alpha} \) is not necessarily
the same as \( r_{\omega} \)) equal to the right-hand-side of Equation 4, which is
the Euler equation for \( r_{\alpha} \) = \( r_{\omega} \). The resulting value of \( r_{\alpha} \) yields a
one-dimensional compressor stage with same stage total enthalpy
rise per unit mass and the same axial velocities and flow angles
reported by Bettner and Sehra.

Table 2 shows all of the design point calibration information
needed for off-design performance prediction using the procedure
described above. The flow areas shown in Table 2 are
not the areas given by Bettner and Sehra. Real compressors suffer blockage of
the compressor annulus, a condition where the axial velocities at the
blade hub and tip are smaller than at the blade midspan because of
boundary layer growth on the annulus walls. To simulate blockage in this one-dimensional model, rotor inlet and exit flow areas were

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Bettner and Sehra report a design isentropic efficiency of
89.8% for the overall machine; their reported temperature and pressure values at the exit of stage 19 indicate an isentropic efficiency of 91.3% for streamline thirteen. Therefore, streamline thirteen, the basis of the model predictions, represents a more efficient streamline than the overall average for the machine. This explains the one-and-a-half point difference between the predictions for isentropic efficiency.

The current one-dimensional model predicts higher pressure ratios and efficiencies as corrected flow is reduced at constant corrected speed. This may indicate a need for an "Allison-specific"
loss curve (Figure 3) which more accurately predicts total pressure losses as incidence moves away from the design value. A two-dimensional model employing multiple spanwise streamlines may also reveal degraded off-design performance near the hub or tip of the compressor. The spacing of the constant IGV closure lines is quite good. The one-dimensional model efficiencies and pressure ratios are again higher than the Allison predictions, especially for greater than twenty degrees of IGV closure. The Allison predictions and the results of this model both show that compressor isentropic efficiency is not a strong function of guide vane angle over the first fifteen degrees of closure.

Figure 5. EPRI/Allison compressor performance with VIGVs.

Given the good qualitative agreement and acceptable (given the lack of data in the open literature) quantitative agreement with the Allison predictions, the pre-test design goals for the NACA Eight Stage compressor are listed in Table 3; the as-tested "design-point" performance of this compressor is reported in Table 4. Note that the design speed for model calibration purposes is 90%.

Table 3. Design goals for the NACA Eight Stage compressor.

| Pressure Ratio | 10.26:1 |
| Flow Rate      | 65 lbm/s (29.5 kg/s) |
| Isentropic Efficiency | 86.2% |
| Physical Speed | 13,380 rpm |
| Corrected Speed | 100% |

Table 4. Calibration point for the NACA Eight Stage model.

| Pressure Ratio | 6.45:1 |
| Flow Rate      | 59 lbm/s (26.8 kg/s) |
| Isentropic Efficiency | 87% |
| Corrected Speed | 90% |

The complete details on constructing the model of the NACA Eight Stage compressor are available in Johnson (1989). The model predictions for this compressor are shown in Figure 6. Results for efficiency and pressure ratio are quite good at 80% and 90% corrected speed. The location of the 100% corrected speed line is at a higher corrected flow than the test results, though the pressure ratio and efficiency predictions at this speed are reasonable. Because of the multiple assumptions necessary to even build this model, it is hard to identify a single "problem area" that might explain the 100% speed line. Better loss prediction for the transonic stages and/or a loss model which accounts for Mach number and choking influences may help improve this simulation. The losses across these "first generation" transonic stages at 100% corrected speed were probably higher than the designers expected. (In fact, Geye et al. claimed that

Figure 6. The NACA Eight Stage compressor map.

Geye et al. (1953) claim that this was in part due to overcompensation for boundary layer growth (blockage) by increasing the axial flow areas during the compressor design. In addition, the first two stages of the compressor were transonic stages, further complicating the application of the current model.

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improper selection of the optimum incidence angles for the transonic stages contributed to the failure to achieve the design pressure ratio at 100% speed.) Because utility gas turbine compressors are not expected to operate with transonic stages, this topic was not pursued in the GCC performance study.

COMPUTER CALCULATIONS

A FORTRAN 77 computer program was written to make the model calculations described in this paper. The next two subsections describe the calculation procedure and the importance of using accurate air properties (specific heat as a function of temperature) in this model.

Converting Between Static and Total Conditions

Since the calculation procedure outlined previously involves frequent conversion between rotor-relative and stator-relative reference frames, routines for computing static conditions given total conditions were needed. Air was modeled as an ideal gas with non-constant specific heat and care was taken to properly define and compute the total enthalpy, temperature and pressure. The relation for the specific heat as a function of temperature for air was taken from Reynolds (1979).

The total enthalpy, \( h \), is the “anchor” for defining all the other “total” quantities:

\[
h = h + \frac{c_p^2}{2g}
\]

(8)

For ideal gases, enthalpy is a function of temperature only; enthalpy and total temperature are then computed using the following equations:

\[
h(T) = \int_{T_i}^{T} c_p(T) dT
\]

(9)

\[h = h(T_i)
\]

(10)

The Gibbs Equation for a simple compressible substance

\[Tds = dh - \frac{1}{p} dP
\]

(11)

can be integrated yielding the expression for the total pressure:

\[P = P \exp \left[ \frac{\phi(T) - \phi(T_i)}{R} \right]
\]

(12)

where

\[\phi(T) = \int_{T_i}^{T} \frac{c_p(T)}{T} dT
\]

(13)

Routines were written to evaluate the above integrals using Romberg integration. Often iteration was necessary (for example, when computing static temperature given total temperature and velocity) and Newton and secant methods were used in these cases; appropriate initial guesses were made using constant specific heat equations. The importance of using the correct air properties (which is the reason behind these careful definitions and calculations) when simulating multistage compressor performance is discussed in the following section.

The Importance of Accurate Thermodynamic Properties for Air

For simple gas turbine cycle performance computations, ideal gas calculations assuming constant or average specific heats are quick and can yield sufficiently accurate results. However, when the design or analysis of turbomachinery hardware is required, significant errors can be introduced if incorrect air properties are used. This is especially true for high pressure ratio, multistage devices.

To illustrate this, the model of the EPRI/Allison compressor was run at the “design point” with two different constant specific heat models for air. The method for calculating \( c_p \) was the only change in the model for these special runs. The results are shown in Table 5 below.

<table>
<thead>
<tr>
<th>Air Specific Heat (c_p) Model</th>
<th>Compressor Pressure Ratio</th>
<th>Error (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>“exact” (Reynolds, 1979)</td>
<td>14.0:1</td>
<td>—</td>
</tr>
<tr>
<td>ambient properties (( \gamma = 1.4 ))</td>
<td>13.0:1</td>
<td>-7.1</td>
</tr>
<tr>
<td>average properties (( \gamma = 1.3865 ))</td>
<td>15.4:1</td>
<td>+10.0</td>
</tr>
</tbody>
</table>

The two constant property models yield erroneous results. The errors are about the same magnitude, but they are in opposite directions. This is particularly striking given the small (~1%) difference between the values of \( \gamma \). The reason for this is that even small deviations from the design conditions for each stage add up quickly in a multistage machine. For example, if each stage only achieved 99% of its design pressure ratio, after nineteen stages, the resulting overall pressure ratio would only be 83% of the design value. If an inaccurate model for the specific heat of air had been used (or if the important integrals above had been evaluated improperly, simultaneously matching the published EPRI/Allison design point performance and vector diagrams for each stage would have been unlikely.

SUMMARY

A one-dimensional, stage-by-stage, axial compressor performance modeling procedure has been described and demonstrated. This procedure should prove useful to gas turbine and thermal system analysts who are without access to the proprietary compressor off-design performance maps or stage performance data of the major gas turbine manufacturers. The user need only supply airfoil geometry and design-point performance information. Generic blade row loss and deviation curves have been provided; these curves can easily be replaced with the user’s own curves if so desired. This performance modeling procedure allows for easy scaling of both the pressure ratio and mass flow rate of the simulated compressor; the effects of variable geometry can also be simulated. It is further hoped that this model can serve as a foundation for improved public domain off-design performance codes incorporating more sophisticated loss and deviation correlations.

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