INVARIANT COORDINATE SYSTEMS FOR COMPRESSOR CONTROL

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Abstract

To protect a compressor from surge, it is necessary to accurately calculate the location of the interface between stable operation and surge. Describing the Surge Limit Interface in certain coordinate systems results in a surface which is invariant to compressor suction conditions such as temperature and molecular weight. We refer to these coordinates as invariant coordinates. We explore these invariant coordinate systems and some nearly invariant systems useful for antisurge control. Some of them are commonly used in the industry, others are quite novel. This work serves to point out the unifying basis of them all.

The applications for these methods are mainly industrial compressors. Varying molecular weight represents the main challenge since a real-time measurement for this parameter is unavailable.

We present compressor maps constructed from test data in these coordinates. The validity of this approach is well supported by these data.

NOMENCLATURE

Roman symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$A$</td>
<td>orifice coefficient</td>
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<tr>
<td>$a$</td>
<td>local acoustic velocity</td>
</tr>
<tr>
<td>$d$</td>
<td>characteristic length</td>
</tr>
<tr>
<td>$H_p$</td>
<td>polytropic head, $ZRT_e (R_e^c - 1) / \sigma$</td>
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<tr>
<td>$h_r$</td>
<td>reduced polytropic head</td>
</tr>
<tr>
<td>$k$</td>
<td>isentropic exponent</td>
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<tr>
<td>$M$</td>
<td>slope of the surge limit line</td>
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<tr>
<td>$MW$</td>
<td>molecular weight</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>mass flow rate</td>
</tr>
<tr>
<td>$N$</td>
<td>rotational speed (rpm)</td>
</tr>
<tr>
<td>$N_e$</td>
<td>equivalent speed, $N / \sqrt{(ZRT_e)}$</td>
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<tr>
<td>$n$</td>
<td>polytropic exponent</td>
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<tr>
<td>$P$</td>
<td>power</td>
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<td>$R_T$</td>
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<tr>
<td>$S$</td>
<td>surge parameter with safety margin factored in</td>
</tr>
<tr>
<td>$S_s$</td>
<td>surge parameter</td>
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<tr>
<td>$T$</td>
<td>temperature</td>
</tr>
<tr>
<td>$V$</td>
<td>velocity</td>
</tr>
<tr>
<td>$Z$</td>
<td>compressibility</td>
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Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>inlet guide vane angle</td>
</tr>
<tr>
<td>$\delta$</td>
<td>distance between operating point and surge control line</td>
</tr>
<tr>
<td>$\eta_p$</td>
<td>polytropic efficiency</td>
</tr>
<tr>
<td>$\mu$</td>
<td>viscosity</td>
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Presented at the International Gas Turbine and Aeroengine Congress & Exhibition
Birmingham, UK — June 10-13, 1996
In this article we will present the theoretical foundation on which invariant coordinates are based. The invariant sets of coordinates are actually five-dimensional spaces. By assuming the Reynolds number is of negligible effect, these spaces can be reduced to four dimensions. Characteristics for compressors of fixed geometry (for example, compressors with no variable inlet guide vanes or adjustable stators) can be represented in three dimensions. From the actual invariant coordinates we will suggest coordinates which are not truly invariant to inlet conditions, but are more easily implemented in real-life control situations.

Antisurge control can be effected in a space which is, dimensionally, one less than the space required to describe the operating region of the compressor. Therefore, when inlet guide vanes are used, in general, we need three coordinates and only two when there are none.

The various coordinate systems yield several possibilities for control. Accurate control can be accomplished when the installation lacks certain transducers such as flow rate measurement, temperatures, or downstream pressure. Besides providing flexibility for the primary control strategy for a given installation, the alternatives provide avenues for fall-back strategies and fault tolerance.

Common Coordinates

Many coordinate combinations have been suggested for use in antisurge control. Many of these were not, in general, invariant to changes in inlet temperature or molecular weight. A small number have been presented and used which are invariant to the bulk of the variations in inlet conditions. Most of these are not invariant to changes in the isentropic exponent, $k$. Such coordinates are referred to, in this work, as nearly invariant since the variation in $k$ is often small.

White (1972) and Staroselsky and Ladin (1979) reported a method of calculating the distance between the surge line and the operating point using two differential pressure transducers. One is a flow rate transmitter signal, $\Delta p_f$. The other, the difference between the suction and discharge pressures, $\Delta p_g$. This was a simple solution to the problem which could be implemented using control hardware of the day and is nearly invariant.

Another coordinate system has been utilized for some time for compressor antisurge control. That
two dimensional space is \((q^2, R_e)\). The square of the reduced flow rate can be calculated by the control system as

\[
q^2 = \frac{\Delta p_o}{p}
\]

and, as outlined below, represents a quantity closely related to a Mach number associated with the compressor. The pressure ratio is simply

\[
R_e = \frac{p_d}{p_s}
\]

Only three transducers are required to implement this control strategy. Because two nearly invariant coordinates are used, a general function of one can be set up to fit any shape curve desired.

This combination of coordinates was reported by Nõgel et al. (1973) who calculated the reduced flow rate based on a differential flow measurement device in discharge. Other disclosures of this approach have been presented over the years. Some of them are by Kolnsberg (1979), Agar (1977), Campos (1993), and Gaston (1992).

**THEORETICAL BASIS**

The method of analysis known as *dimensional analysis or similitude* is used in this section to construct three systems of coordinates which are invariant to inlet conditions. The operating point and the Surge Limit Interface of the compressor are mapped into these coordinates.

**Invariant Parameters**

For a system of coordinates to be invariant to inlet conditions, it must satisfy *dynamic similarity*. This requirement implies three things:

- Geometric similarity
- Kinematic similarity
- Pertinent forces must be considered in a set of valid dimensionless parameters.

Geometric similarity is given because we are considering different inlet conditions in the same compressor. We show below that kinetic similarity is assured by number three — the dimensionless parameters.

The process is explained as follows. First we set up a functional relationship for the variables (dependent and independent) that apply to the situation at hand (see, for example, Dixon 1978). In our case we will choose polytropic head and shaft power for the dependent variables due to their popularity in the industry. Volumetric flow rate was chosen as an independent variable — also due to its popularity in the industry. An example of a compressor map is shown in Figure 1. The list of chosen variables are as follows:

\[
H_p = f_0(Q, \omega, \mu, \rho, a, d, \alpha)
\]

\[
P = f_1(Q, \omega, \mu, \rho, a, d, \alpha).
\]

These relationships each consist of eight dimensional parameters. This number can be reduced to five using the Buckingham Pi Theorem (see, for example, White 1979 or Kline 1965). It is beyond the scope of this paper to go through that procedure, but a result is

\[
\frac{H_p}{a^2} = f_2\left(\frac{Q}{a_d^2}, \frac{\omega d}{a}, \frac{\rho a d^2}{\mu}, \alpha\right)
\]

\[
\frac{P}{\rho a^2 d^2 \omega} = f_3\left(\frac{Q}{a_d^2}, \frac{\omega d}{a}, \frac{\rho a d^2}{\mu}, \alpha\right).
\]

The dimensionless parameters resulting from this procedure are not unique. In fact, an infinite number of dimensionless parameters can be constructed from those shown above by constructing linear and nonlinear combinations of them. The
ones shown above were chosen for their usefulness for control.

The first term appearing in the independent variable list on the right-hand side of each of the above expressions is proportional to the Mach number based on the bulk (one dimensional) velocity at the suction of the compressor. The second term is proportional to the Mach number based on the tangential speed of the rotor. The third term is proportional to the Reynolds number based on the same wheel speed. The last term takes into account variations in the compressor geometry such as variable inlet guide vanes.

When a compressor is designed, the characteristics of that compressor are constructed for a given set of inlet conditions and gas properties (reference conditions). For those characteristics to apply under other conditions they must be presented in (or transformed into) the above coordinates — or equivalent ones. In other words, the compressor map needs to be mapped into the five dimensional spaces 

\[
\left( \frac{H_p}{a^2}, \frac{Q}{ad^2}, \frac{\omega d}{a}, \frac{\omega p a d^2}{\mu}, \alpha \right)
\]

and

\[
\left( \frac{P}{\rho a^2 d^3 \omega}, \frac{Q}{ad^2}, \frac{\omega d}{a}, \frac{\omega p a d^2}{\mu}, \alpha \right).
\]

As mentioned above, along with the requirements of Equations 1, the new conditions must satisfy kinematic similarity.

Kinematic similarity Kinematic similarity states that the velocities for the two conditions at any point must be related by a constant scaling factor. Therefore,

\[
\frac{V_1}{V_2} = \left( \frac{V_1}{V_2} \right)_{ref}
\]

where the subscripts 1 and 2 imply velocities from two different locations (such as the suction and discharge of the compressor).

We can show that kinematic similarity is assured if we satisfy Equations 1 and geometric similarity. In theory, when Equations 1 are satisfied at one point, the flows “match” — thus the dimensionless parameters are equal at all points. Therefore, at the locations 1 and 2 used above, the flow coefficient (inlet flow Mach number) and wheel Mach number must both satisfy

\[
\left( \frac{Q}{ad^2} \right)_1 = \left[ \left( \frac{Q}{ad^2} \right)_1 \right]_{ref}
\]

(2)

\[
\left( \frac{\omega d}{a} \right)_1 = \left[ \left( \frac{\omega d}{a} \right)_1 \right]_{ref}
\]

(3)

Now dividing Equation 2 by Equation 3 we get

\[
\left( \frac{Q}{ad^2} \right)_1 = \left( \frac{Q}{ad^2} \right)_2_{ref}
\]

(4)

Note that \( Q = VA = \pi V d^2 / 4 \). Remembering that geometric similarity is satisfied, and \( \omega_1 = \omega_2 \), we see that the characteristic length, \( d \), and the rotational speeds divide out. Equation 4 can be rewritten as

\[
\frac{V_1}{V_2} = \left( \frac{V_1}{V_2} \right)_{ref},
\]

which says that kinematic similarity is satisfied by virtue of Equations 1 and need not be specified separately.

Once geometric and kinematic similarity are assured and the dimensionless parameters of Equations 1 are matched, dynamic similarity is said to be satisfied. Therefore, the set of dimensionless parameters in Equations 1 represent coordinates which are invariant to inlet conditions.

Simplifications

We can reduce Equations 1 further using common sense and experience. When the Reynolds number is high the flow is turbulent and the frictional effects (due to the viscosity) tend to be approximately constant. In the normal operating range of compressors the Reynolds number is high. Thus, its influence can be neglected — and this is supported by experience. (For analyses of the effect of the Reynolds number, see Casey 1985, Strub et al. 1987, Wiesner 1979.) We are left with

\[
\frac{H_p}{a^2} = f_4 \left( \frac{Q}{ad^2}, \frac{\omega d}{a}, \alpha \right)
\]

(5)

\[
\frac{P}{\rho a^2 d^3 \omega} = f_5 \left( \frac{Q}{ad^2}, \frac{\omega d}{a}, \alpha \right).
\]
Polytropic head is represented as

\[ H_p = \frac{n}{n-1} (ZRT) \left[ \left( \frac{p_d}{p_s} \right)^{\frac{n-1}{n}} - 1 \right]; \]

where \( n \) is the polytropic exponent. The volumetric flow rate is related to the differential-pressure flow measurement device as

\[ Q = A \sqrt{\frac{(ZRT) \Delta p_o}{p}}. \]

The local acoustic velocity is

\[ a = \sqrt{k ZRT}. \quad (6) \]

So, using the suction values of the local acoustic velocity, \( a_s = \sqrt{(k ZRT) s} \), the density, \( \rho_s \), and the volumetric flow, we can reduce some of the terms of Equations 5:

\[ \frac{H_p}{a^2} = \frac{1}{k_s} \left[ \left( \frac{p_d}{p_s} \right)^{\sigma} - 1 \right] = \frac{1}{k_s} h_r \]

\[ \frac{Q_s}{a_s d^2} \frac{P}{\rho_s d^2 d^3 \omega} = \frac{P}{\omega d^2 \rho_s k_s} = \frac{30 P}{\pi N d^3 \rho_s k_s} = \frac{30}{\pi d^3 k_s} P_r \]

\[ \frac{k d}{a} = \frac{d \pi N}{30} = \frac{d \pi}{30 \sqrt{k_s}}. \]

where

\[ \frac{h_r}{k_s} = \frac{1}{\sigma \left[ \left( \frac{p_d}{p_s} \right)^{\sigma} - 1 \right]} \]

\[ q_s = \sqrt{\frac{\Delta p_{o,s}}{p_s}} \]

\[ P_r = \frac{P}{\rho_s k_s} \]

\[ N_e = \frac{N}{\sqrt{(ZRT) s}}. \]

So Equations 5 can be written

\[ \frac{1}{h_r} = \frac{f_4 \left( A_q, \frac{d \pi}{30 \sqrt{k_s}} N_e, \alpha \right)}{30 \sqrt{k_s} P_r} = \frac{f_5 \left( A_q, \frac{d \pi}{30 \sqrt{k_s}} N_e, \alpha \right)}{\pi d^3 k_s}. \quad (7) \]

The physical dimensions of the compressor are constant, therefore \( d \) can be ignored in all of the terms in Equations 7. The orifice coefficient, \( A \), is also constant and can thus be eliminated. It should be noted, however, that some of the resulting terms are not dimensionless. To convert from the rotational speed, \( \omega \), in radians per second to \( N \) in rpm, we used the constant \( 2\pi / 60 = \pi / 30 \). This constant can be ignored as well, remembering that units have been changed. The final set of invariant coordinates (with the flow rate and rotational speed terms squared for convenience) are

\[ \frac{h_r}{k_s} = f_9 \left( \frac{q^2}{k_s}, \frac{N^2}{k_s}, \alpha \right) \]

and

\[ P_r = f_7 \left( \frac{q^2}{k_s}, \frac{N^2}{k_s}, \alpha \right). \quad (8) \]

**Isentropic exponent** The isentropic exponent, \( k \), appears in each of the invariant parameters appearing in Equations 8. However, \( k \) is not directly measurable for use in antisurge control. Two approaches to this problem are:

- Assume it is constant (when \( k \) does not vary considerably).
- Approximate \( k \) as a function of \( \sigma \) assuming the efficiency, \( \eta_p \), varies insignificantly along the Surge Limit Interface (usually a valid assumption).

**Nearly Constant \( k \)**

The results of ignoring the variation in \( k \) are the coordinate systems

\[ h_r = f_9 \left( q^2, N^2, \alpha \right) \]

\[ P_r = f_7 \left( q^2, N^2, \alpha \right). \]

**Calculation of \( k \)**

In some cases, gases of very different \( k \)'s are compressed with the same compressor. A common example of this is the use of nitrogen to purge a system normally used for hydrocarbons. It is possible to characterize the isentropic exponent as a function of \( \sigma \) (which is calculable using the pressure and temperature ratios).

For these situations the fundamental coordinate systems are

\[ \frac{h_r}{f_k(\sigma)} = f_{10} \left( \frac{q^2}{f_k(\sigma)}, \frac{N^2}{f_k(\sigma)}, \alpha \right). \]
We can multiply both sides of Equation 11 by \( \dot{m}_d / \dot{m}_s = 1 \). Using \( \dot{m} = \rho V A \) we get

\[
\frac{(\rho V A k Z RT)_d}{(\rho V A k Z RT)_s} = \left[ \frac{(\rho V A k Z RT)_d}{(\rho V A k Z RT)_s} \right]_{ref}
\]

but, again, we are assured of geometric similarity, so the \( A \)'s cancel out; and using kinematic similarity, the velocities will also divide out. Now using \( p = p Z RT \) we get

\[
\frac{(kp)_d}{(kp)_s} = \left[ \frac{(kp)_d}{(kp)_s} \right]_{ref}
\]

which says that a coordinate system in which the pressure ratio, \( R_c = p_d / p_s \), is one of the coordinates is not invariant to changes in the isentropic exponent, \( k \). As long as the ratio \( k_4 / k_s \) does not vary "considerably" this approach is quite satisfactory for antisurge control.

In practice, pressure ratio can be used as a substitute for \( \dot{m}_d / \dot{m}_s \). This implies that we have three four-dimensional coordinate systems to choose from. They are:

\[
\begin{align*}
\frac{h_r}{k_s} & = f_6 \left( \frac{q_2}{k_s}, N_c^2, \alpha \right) \\
\frac{R_c k_d}{k_s} & = f_{12} \left( \frac{q_2}{k_s}, N_s^2, \alpha \right) \\
\frac{P_r}{k_s} & = f_7 \left( \frac{q_2}{k_s}, N_s^2, \alpha \right)
\end{align*}
\]

By solving the first two of Equations 12 for \( q_2 / k_s \) and substituting the result for \( q_2 / k_s \) in the last equation, the result is reduced power being a function of reduced head or pressure ratio. These combinations can be of use when flow measurements are not available, are unreliable, or fail.

**Parameters using \( \Delta p_c \) and \( \Delta p_o \)** In the introduction, a common parameter was described which was made up of two differential pressure signals:

- the differential pressure across the compressor, \( \Delta p_c = p_d - p_s \), and
- the differential pressure across a flow measurement device, \( \Delta p_o \), in suction or discharge.

We can show that these two signals can be combined into a single nearly invariant parameter for antisurge control. An important concept is that
any combination of invariant coordinates (linear or nonlinear) is still invariant. So we can combine a linear function of $R_c$ with $q_s^2$ as such (it is assumed that $k$ does not vary significantly):

$$\frac{R_c - 1}{q_s^2} = \frac{\frac{p_d}{p_s} - 1}{\Delta p_c/\Delta p_o},$$

then simplify:

$$\frac{p_s(\frac{p_d}{p_s} - 1)}{\Delta p_o} = \frac{p_d - p_s}{\Delta p_o} = \frac{\Delta p_c}{\Delta p_o}. \quad (13)$$

Therefore, $\Delta p_c/\Delta p_o$ is as invariant as the parameters used in its construction ($R_c$ and $q_s$). And it requires only two transmitters.

Although it may not be obvious that the Surge Limit Line can be described by a single invariant parameter, it is true. However, when $\Delta p_c/\Delta p_o$ is the sole coordinate for antisurge control, the shape and location of the Surge Control Line are somewhat limiting. Available turndown will be reduced for many compressors. An example of a Surge Limit Line and lines of constant $\Delta p_c/\Delta p_o$ are shown in Figure 2. The discrepancy between the actual surge line and the line of constant $\Delta p_c/\Delta p_o$ for this compressor is readily apparent.

**Temperature Ratio**

The temperature ratio appears in Equation 11. The gas constant, $R$, divides out (being the same at the suction and discharge). Thus, Equation 11 can be rewritten as

$$\frac{(kZ)_d}{(kZ)_s} R_T = \left[ \frac{(kZ)_d}{(kR)_s} R_T \right]_\text{ref}.$$

So, insofar as the ratio, $(kZ)_d/(kZ)_s$, does not change significantly, the temperature ratio can be assumed invariant.

**SURGE LIMIT INTERFACE**

We now return to the sets of invariant coordinates given in Equations 12. These equations describe the surface on which the operating point lies at steady state. This surface is of dimension one less than the space in which it resides. The number of coordinates required to fix the location on the surface is equal to its dimension. This is a useful concept because, for control, no more than three (and much of the time only two) dimensions need to be calculated to fix the location on that surface.

The Surge Limit Interface is a surface of dimension one less than the complete surface or two less than the space it resides in. Thus for a compressor with variable equivalent speed and inlet guide vane angle, two dimensions are necessary to fix a point on the surge limit surface. Only one dimension is required when just one of these parameters is variable.

Some of the dimensions required to fix a point on the steady state operating surface can be used to locate a point on the Surge Limit Interface. For controls applications, where we need a comparison of the operating point location to a point on the Surge Limit Interface, we need to calculate no more than three of the coordinates — and often, only two.
Reduction of a Surge Limit Surface

A compressor with variable inlet guide vanes as well as equivalent-speed \((N_e)\) changes will have a two-dimensional Surge Limit Surface. By slicing this surface with planes of constant guide vane angle, \(\alpha\), this surface can be thought of as a family of Surge Limit Lines. Therefore, the Surge Limit Line can be constructed for the value of \(\alpha\) in the two-dimensional coordinate system chosen for control of that compressor [for instance \((q_s, R_c)\)]. These two dimensions are referred to as the primary coordinates, and the guide vane angle, \(\alpha\), is referred to as the secondary coordinate.

Antisurge control would be carried out under such circumstances as though no inlet guide vanes were present — except that the location of the Surge Limit Line (and thus the Surge Control Line) varies with \(\alpha\).

When the molecular weight of the gas is known at all times, the Surge Limit Surface can be sliced with planes of constant equivalent speed instead of \(\alpha\). So \(N_e\) becomes the secondary coordinate in the above discussion. This is useful when the guide vane position signal is unreliable. Surge Limit Lines at constant equivalent speed are shown in Figure 3.

The choice of which coordinate system to use is driven by two things:

- the information available in the specific installation (that is, what measurements are available?), and

\[\Delta - \alpha = 20.70\]
\[\vartriangleright - \alpha = 21.09\]
\[\vartriangledown - \alpha = 22.14\]
\[\bigcirc - \alpha = 22.72\]

![Figure 3: A Family of surge limit lines, each at a constant equivalent speed.](attachment:image)

**Table 1:** Various coordinate systems for control of compressors without inlet guide vanes.

<table>
<thead>
<tr>
<th>(P_e)</th>
<th>(h)</th>
<th>(R_c)</th>
<th>(N_e^2)</th>
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**Table 2:** Various coordinate systems for control of compressors with inlet guide vanes.

<table>
<thead>
<tr>
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</table>

INARIANT SYSTEMS

Using the coordinates in Equations 12, we can calculate the distance between the operating point and a point on the Surge Limit Interface. As mentioned above, we need only two or, at most, three of the coordinates. Therefore, we can choose any two or three depending on the information available. The combinations are listed in Table 1 for a compressor which has no inlet guide vanes. Table 2 describes the options when guide vanes must be taken into consideration.
• the coordinate system that provides the most accurate control.

Each of the above parameters are invariant to inlet conditions. Therefore, antisurge control algorithms can be constructed using a single parameter instead of a pair or even three. An example of this is a minimum flow rate algorithm. A Surge Limit Line for control can be described by \( q^2/k_s = \text{constant} \). This describes a line which is invariant to inlet conditions, so is equally valid for all gases. It may not, however, represent an accurate depiction of the true Surge Limit Line.

Fall-back and fault-tolerant strategies In many applications, more than one set of coordinates are viable. Thus, one or more of the alternatives can be used as fall-back strategies in the event of a transducer failure or similar event.

Fault-tolerance is also a consideration. Any of the methods could be checked against any other method (or methods) to improve the integrity of the control.

Discharge Properties

Flow measurement devices are often located in the discharge of compressors. This presents no problem in constructing a (nearly) invariant flow rate parameter.

We can define four dimensionless flow rate parameters as

\[
q^2_{s,s} = \frac{\Delta p_{o,s}}{p_s}, \quad q^2_{s,d} = \frac{\Delta p_{o,s}}{p_d}, \\
q^2_{d,d} = \frac{\Delta p_{o,d}}{p_s}, \quad q^2_{d,d} = \frac{\Delta p_{o,d}}{p_d},
\]

utilizing all of the possible measurement locations. Each of the last three \((q^2_{s,d}, q^2_{s,s}, \text{and } q^2_{d,d})\) can be derived from the first \((q^2_{s,s})\) using functions of \(R_c\) and \(R_T\). Therefore, they are sufficiently invariant for most antisurge control applications.

Using similar reasoning as above, the reduced power can be defined using discharge pressure:

\[
P_{r,d} = \frac{P}{p_d N}.
\]

Also, the equivalent speed, \(N_e\), can be constructed using the discharge acoustic velocity:

\[
N_{e,d} = \frac{N}{\sqrt{(ZRT)_d}}.
\]

Limitations

Some of the invariant coordinate systems suggested above are not suitable for all applications. For instance, the use of equivalent speed paired with pressure ratio may require high precision in calculating the pressure ratio to maintain a "small" safety margin. The reason for this is that lines of constant equivalent speed tend to nearly coincide with lines of constant pressure ratio near the Surge Limit Line — especially for centrifugal compressors. Therefore, the antisurge control system must detect a very small increase in pressure ratio as surge is approached from the right along a constant equivalent speed curve. A larger safety margin can be applied — moving the Surge Control Line to a region of greater slope on the equivalent speed curves. The tradeoff is the cost of recycling more often versus the cost of installing a flow measurement device.

DISTANCE TO SURGE

We have, up to this point, derived coordinate systems which are invariant (or nearly invariant) to inlet conditions (see Equations 12). We noted that the difficulty of measuring \(k\) could be overcome by assuming it was constant, or calculating it as a function of \(a\) which can be computed using measured quantities.

When there are no inlet guide vanes, we need only two coordinates to calculate the relative positions of the operating point and the Surge Limit Line in any of the above coordinate systems. The presence of inlet guide vanes increases the dimensionality of the problem to three and the Surge Limit Line becomes a Surge Limit Surface. The question remaining is how we use this information in antisurge control of compressors.

As described above, the Surge Limit Surface in existence for compressors with inlet guide vanes is considered a family of Surge Limit Lines by looking at the intersection of the Surface with a constant \(\alpha\) or \(N_e\) plane. The following description deals only with Surge Limit Lines. Thus, for the case where variable inlet guide vanes exist, the Surge Limit Surface has already been reduced to a Surge Limit Line for a given \(\alpha\) or \(N_e\).

The coordinate system (primary coordinates if inlet guide vanes are present) considered here is generic. Let it be described simply as \((X, Y)\). Each \((X, Y)\) can be any pair of the coordinates
from Table 1 [for instance, \((q_s^2, R_c)\)].

The Surge Control Line is defined as a line removed from the Surge Limit Line a “safe distance” into the stable operating region. The distance between these two lines is the safety margin. The goal of antisurge control is to provide safe operation with the minimum safety margin — which translates into the maximum operating envelope. The antisurge controller will attempt to maintain the operating point no closer to the Surge Limit Line than the Surge Control Line. For this reason, the error used in the closed loop control is a measurement of the distance between the operating point and the Surge Control Line.

There are several possible approaches to defining this distance. One method is outlined here. The basis will be the distance between the operating point and the Surge Limit Line (rather than to the Surge Control Line). Including a safety margin with this distance defines the Surge Control Line. The topic of the safety margin is covered in a later section.

We can describe the operating point location in any of the suggested coordinates to be the slope of a line passing through the origin and the operating point. The Surge Limit Line is described by the slope of a line passing through the origin and the Surge Limit Line at the point the operating point is compared to. Either of these slopes can be written as

\[
M = \frac{Y}{X}
\]

A comparison between the location of the operating point and the Surge Limit Line is required. One way to compare quantities is to take their ratio. Here we divide the slope corresponding to the operating point by that of the Surge Limit Line:

\[
S = \frac{M_{OP}}{M_{SLL}} = \frac{(Y/X)_{OP}}{(Y/X)_{SLL}}
\]

This ratio is then used to calculate the deviation, \(\delta_s\), between operating point and the Surge Limit Line. The ratio of the slopes is unity when the operating point is on the Surge Limit Line, so the deviation (“distance to surge”) is

\[
\delta_s = 1 - \frac{(Y/X)_{OP}}{(Y/X)_{SLL}}.
\]  

(14)

Thus, the deviation is positive when the compressor is operating in the stable region.

There are an infinite number of points on the Surge Limit Line which we could compare to the operating point. One possibility is to use the intersection of the Surge Limit Line and a horizontal line passing through the operating point. Therefore, \(Y_{SLL} = Y_{OP}\), which simplifies Equation 14 to

\[
\delta_s = 1 - \frac{X_{SLL}}{X_{OP}}
\]

(15)

which describes a normalized distance to the Surge Limit Line along that horizontal line.

As an example of this approach, let the primary coordinates be \((q_s^2, h_r)\). Then the slope of the line passing through the origin and the operating point is \((h_r/q_s^2)_{OP}\). The slope of the line passing through the point of comparison on the Surge Limit Line is \((h_r)_{OP}/(q_s^2)_{SLL}\), where \((q_s^2)_{SLL}\) is the reduced flow rate at the intersection of the horizontal line defined as \(h_r = (h_r)_{OP}\) and the Surge Limit Line. The deviation is

\[
\delta_s = 1 - \frac{(q_s^2(h_r)_{OP})^2}{(q_s^2)_{SLL}}.
\]

This is illustrated in Figure 9 where the operating point is at \((92.79, 2.31)\). The value of the reduced flow rate at surge, \((q_s^2)_{SLL}\), is 60.29. Thus,

\[
\delta_s = 1 - \frac{60.29}{92.79} = 1 - 0.6497 = 0.3503 > 0
\]

which shows that the compressor is operating in the safe region.

**Implementation**

Using Equation 15 we characterize the Surge Limit Line as a function of \(Y\):

\[
f_\delta(Y) = X_{SLL}.
\]

So the point on this line to which we compare the operating point is

\[
(f_\delta(Y_{OP}), Y_{OP}) = (X_{SLL}, Y_{OP}).
\]

The deviation can now be computed as

\[
\delta_s = 1 - \frac{f_\delta(Y_{OP})}{X_{OP}}.
\]  

(16)

Besides calculating the values of \(X\) and \(Y\), the control system must be able to compute the function, \(f_\delta(Y)\). This function is usually determined...
empirically through surge testing the compressor at a few points and constructing a curve through these points.

No control system is perfect. A safety margin must be included to keep the operation of the compressor away from surge under transient conditions. One approach to implement a safety margin is to modify Equation 16 as follows:

\[ \delta = 1 - \left( \frac{f(X_{OP})}{X_{OP}} + b \right). \]

This produces a Surge Control Line which intersects the Surge Limit Line at the origin and diverges from it as it goes to the right and up. Of course, the Surge Control Line is always to the right of the Surge Limit Line.

The deviation, \( \delta \), is the "error" used in a proportional-integral (PI) control loop. The output of the PI loop is a signal to the antisurge (recycle or blow-off) valve to keep the operating point on or to the right of the Surge Control Line.

**EXAMPLES**

Some graphical examples of the coordinate systems described in the preceding sections are presented here. Data for two different compressors are shown. Neither compressor was fitted with adjustable inlet guide vanes. Different hydrocarbon gas mixtures are represented with varying molecular weights for each compressor. Surge Limit Lines have been plotted in each coordinate system. Lines of constant equivalent speed are shown in those plots not having \( N_e \) as a primary coordinate.

Some of the data have been scaled by a constant scaling factor. As mentioned above, constants can be dropped in constructing coordinate spaces invariant to suction conditions.

**Compressor I**

The first set of plots is a complete set of coordinates for a low-pressure stage, gas injection compressor. They are calculated from characteristic maps, labeled "based on test," obtained from the manufacturer. This compressor is rated at, nominally, 870 kW (at the guarantee point).

The characteristic speed curves are shown in Figure 4 for a single gas composition. Surge Limit Lines for several molecular weights are displayed on the same plot. Since these coordinate systems
are not invariant to inlet conditions, the various curves do not coincide. Power and volumetric flow have been normalized using design conditions.

The remaining figures for this compressor are coordinates invariant to suction conditions. Some of the scatter seen in the data is due to human error in converting the manufacturer’s maps to numerical data for reduction. Ignoring the slight scatter, note that the curves for all molecular weights represent a single, overall curve (not necessarily a straight line) in the various coordinate spaces — this is what is meant by invariant coordinates.

Figure 6: When a power measurement is available, this coordinate system can be used to construct a no-flow algorithm.

The coordinate spaces \((P_r, h_r)\) and \((P_r, R_e)\) depicted in Figures 5 and 6 require neither a flow measurement device, nor knowledge of the molecular weight.

The invariant surge limit line shown in Figure 7 is constructed in the coordinate space \((q_r^2, P_r)\). Neither of these parameters require any transmitters to be located downstream of the compressor.

The last coordinate system involving the reduced power, \(P_r\), is seen in Figure 8. The control method utilizing this space would be another “no-flow” algorithm — not requiring a flow measurement device. However, to calculate the equivalent speed, \(N_e\), necessitates the continuous knowledge of the molecular weight. Given the technology today, practicality demands that this approach be used for constant molecular weight applications only. However, the invariance of the equivalent speed parameter is demonstrated by this and all of the plots in which it appears.

Depicted in Figures 9 and 10 are more common coordinate spaces for antisurge control. They both require pressure measurements upstream and downstream of the compressor, and a differential-pressure flow measurement device. In addition, the control utilizing the parameters \((q_r^2, h_r)\) would require that temperature transmitters be installed both in suction and discharge.

Figures 11 and 12 are similar to the previous two, except that the reduced flow rate is replaced by the equivalent speed. As mentioned above, these are only practical for constant molecular weight applications.

The final figure for Compressor I is Figure 13. This is another space in which information in the discharge of the compressor is unnecessary.

**Compressor II**

The second set of plots represent data acquired in the field during normal surge testing. This compressor is the second stage of a two stage MTBE process machine. Its design power is nominally 700 kW.

Two molecular weights are shown on the plots.
Figure 8: Another coordinate space which leads to a no-flow control algorithm. It requires that molecular weight be known.

Figure 9: A coordinate space for antisurge control using reduced head and reduced flow rate.

Figure 10: One of the most commonly implemented antisurge control approaches uses these coordinates.

Figure 11: Coordinates in which flow rate does not appear. Molecular weight must be known.
These should be considered approximate. The suction temperature and pressure varied throughout the test — even when the molecular weight was fairly constant. No power indication was available for this particular compressor, so no plots of reduced power are shown.

The Surge Limit Line is represented in reduced head versus reduced flow rate rate coordinates in Figure 14. The lowest flow rate point of the $MW = 32$ curve shows some error in this plot (as with all of the plots showing reduced flow rate for this particular compressor). The signals were recorded using a digital recorder. Due to the discrete nature of these data, some uncertainty is associated with determining the exact point at which the compressor surged. There is also some measure of subjectivity involved in this determination.

Figure 12: A no-flow control algorithm could be constructed around these coordinates if molecular weight is known at all times.

Figure 13: This surge limit line is plotted in the space $(q_s^2, N_s^2)$. Knowledge of molecular weight is required.

Figure 14: Surge limit line in reduced head versus reduced flow rate squared. Empirical data.

Very similar results are plotted in Figure 15 in pressure ratio versus reduced flow rate coordinates. Comparing this graph to that of Figure 14, there would not appear to be an advantage to control using reduced head over control using pressure ratio for this compressor.

The Surge Limit Line is depicted for no-flow control algorithms in Figures 16 and 17. Although these plots are for gases of variable molecular weight, as before, the control approaches corresponding to them are only viable for constant molecular weight applications.

The final plot is in $(q_s^2, N_s^2)$ coordinates (Fig-
ure 18). Once again, the associated antisurge control system would not require information in the discharge of the compressor.

**Suction or Discharge Conditions**

As mentioned in a previous section, an option is to utilize discharge properties rather than those in suction. The parameters for which information is used on one side of the compressor only are reduced power, equivalent speed, and reduced flow rate. These coordinates, therefore, could be altered such that they make use of information either in the suction or discharge. Comments were made in the previous sections that some coordinate spaces do not require discharge information (transmitters located there). It is equally true to say that control algorithms can be designed such that suction transmitters are unnecessary.

**CONCLUSION**

Using similitude, coordinate systems were constructed which are invariant to inlet conditions. Some simplifications have been suggested when necessary and possible. The results were three general four-dimensional coordinate systems.

At most, only three of these dimensions are required for control purposes — for compressors with adjustable inlet guide vanes. Therefore, a variety of possible combinations may be available for any given installation. Tables 1 and 2 summarize various coordinate pairs and triples, respectively. More than one of these combinations may be applicable to any one compressor. Thus, the coordinates systems not chosen may be useful for fallback and/or fault-tolerance strategies. It was noted that antisurge control could be implemented using only one of these invariant parameters with the potential for an inaccurate representation of the Surge Limit Line.

A practical definition of a space which is invariant to suction conditions is when the Surge Limit Line (Surface) is represented as a single curve (or Surface) in that space. Actual data presented showed that the Surge Limit Lines for varying inlet conditions (most notably molecular weight) did, in fact, collapse into one curve. This is the desired result, and allows the implementation of antisurge control for compressors handling a variety of gases.

There are many other aspects to good antisurge control which have not been covered here. However, before even adequate control can be performed, the distance between the operating point and the Surge Control Line must be known. Due to variations in suction conditions, this distance measurement must be transparent to these variations.
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ACKNOWLEDGMENTS

I would like to thank Krishnan Narayanan of Compressor Controls Corporation for reducing the data for Compressor I for this report; Roman Bershader for making the data for Compressor II available to me; and Compressor Controls Corporation for their support as I wrote this paper. Special thanks goes to my reviewers whose valuable comments made this a much better paper.

Figure 17: No flow measurement device required. However, molecular weight must be known.

Figure 18: Surge limit line based on equivalent speed and reduced flow rate.