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ACTUATOR PLACEMENT FOR ACTIVE SURGE CONTROL IN A MULTI-STAGE AXIAL COMPRESSOR

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ABSTRACT. This paper describes an actuator placement methodology for the active control of purely one-dimensional instabilities of a seven-stage axial compressor using an air bleeding strategy. In this theoretical study, using stage-by-stage non-linear modelling based on the conservation equations of mass, momentum, and energy, a scheduling LQR (Linear Quadratic Regulator) controller is designed for several actuator locations in a compressor from the first stage to the plenum. In this controller design, the LQR weighting matrices are selected so that the associated cost function includes only air bleeding mass flow leading to the minimisation of the air bleed. The LQR cost function represents a measure of the consumption of air bleeding and can be calculated analytically using the solution of an Algebraic Riccati Equation. From analysis of the cost at different compressor stages, the location of an air bleeding actuator is selected at the stage with the minimum cost. Finally, using an ACSL simulation program, the scheduling controller has been integrated with a non-linear stage-by-stage model and the time response of the air bleeding mass flow at different locations has been obtained to confirm the results from the analytical approach. Results are presented to show actively stabilised compressor flow beyond the surge point where the air bleed is minimised. These results also indicate the preferred location of the actuator at the compressor downstream stages for both low and high compressor speeds.

NOMENCLATURE

ACSL	:	Advanced Continuous Simulation Language
ARE	:	Algebraic Riccati Equation
A	:	Flow area and system matrix
avpn	:	Actuator plane number
B	:	Control matrix
C_v	:	Specific heat at constant pressure
E_{net}	:	Net energy input to the flow
F_{net}	:	Net force input to the flow
h	:	Enthalpy
K	:	Feedback coefficients matrix
LP	:	Lumped Parameter
LQR	:	Linear Quadratic Regulator
MCE	:	Momentum, Continuity and Energy
MS	:	Mach number at the steady state value
P_S	:	Static pressure
Q, R	:	LQR weighting matrices
SP	:	Static pressure
T_S	:	Static Temperature
t	:	Time co-ordinate
u	:	Control signal
V_a	:	Axial velocity of air
W	:	Mass flow
W_b	:	Bleed mass flow
x	:	Axial co-ordinate
X	:	State variables vector
ρ	:	Density

1. INTRODUCTION

In an axial flow compressor, as the mass flow is reduced the pressure rise increases. Generally a point is reached at which further reduction in mass flow leads to non-linear flow instabilities in the compressor. The line marking the locus of these points for different rotational speeds is known as surge line. Beyond the surge line, the flow instabilities can develop into rotating stall or surge. Rotating stall is a local instability to the compressor and a two-dimensional model is required to represent its characteristics. However, surge is a system instability and can be treated as a one-dimensional flow which is influenced by both the overall annalus averaged compressor characteristics and the system geometry.

Active control of compressor instabilities has received attention recently (Epstein et al. 1989). Such an approach allows the compressor to operate beyond the surge line and to suppress the flow instabilities using a feedback control system so that the whole system including the compression system, actuator, sensor and controller become stabilised as shown in fig 1. Experimental and theoretical studies have been undertaken for both rotating stall (Paduano et. al., 1991) and surge suppression (Ffowcs Williams and Huang 1989, Pinsley et. al. 1990). However, there have been few studies which have considered the effect of control configuration on an active control strategy.

Simon et. al. (1992) evaluated the effect of an actuator-sensor pair on the active control of surge using a global model and a single-sensor control configuration. However, this type of modelling is limited to centrifugal and single-stage axial compressors. For multi-stage axial compressors, active surge control is more complicated due to the role of mismatching between non-linear stage characteristics as reported by Esscuret and Elder (1993) and Hosny (1991).

One of the features of a multi-stage axial compressor that influences the performance of an active surge control system is the location of the actuator. This problem becomes more critical when an air bleeding strategy is used for suppression of instabilities because the resultant large amplitude bleed mass flow may disrupt the steady-state characteristics.

This paper presents a methodology for actuator placement in the active control of surge in multi-stage axial compressors using a bleed strategy. This method uses an analytical optimisation approach that minimises the consumption of air bleeding.

In this study, a scheduling optimal controller is designed so that it guarantees the minimisation of the consumption of bleed and also the robustness of the control system at the steady-state operating points beyond the surge line. By selecting different locations of the air bleeding actuator along the compressor, a cost function is used to measure the consumption of bleed mass flow during the stabilisation process. From analysis of this cost function at different locations, the actuator is located at the axial plane with the minimum cost. Finally, using an ACSL simulation program, time responses of bleed mass flow during the stabilisation process of a seven-stage axial compressor are provided to confirm the analytical approach.

2. ONE-DIMENSIONAL STAGE-BY-STAGE MODELLING

The physical problem to be modelled is the time varying airflow through the compressor. The complex flow is simplified by considering only the annulus averaged, or one-dimensional flow. The compressor is divided into axial elements. In general, an element in the compressor corresponds to a stage and is composed of a rotor row, its downstream inter-row volume, a stator row and another inter-row volume. The physical compression system to be modelled in this study is a test bed installation where the compressor takes in ambient air and exhausts into a downstream volume terminated by a throttle valve.

The basic equations used in the model are the conservation equations of mass, momentum and energy. The equations can be written in a general one-dimensional conservation form as follows:

Continuity:

$$\int_{x_r}^{x_{r+1}} \left(\frac{\partial \rho}{\partial t}\right) A(x)dx = W_r - W_{r+1} \qquad (1)$$

Momentum:

$$\int_{x_r}^{x_{r+1}} \left(\frac{\partial W}{\partial t}\right)dx = (WV_a + P_S A)_r - (WV_a + P_S A)_{r+1} + F_{net,r} \qquad (2)$$

Energy:

$$\int_{x_r}^{x_{r+1}} A\frac{\partial}{\partial t}\left(\rho(C_V T_S + \frac{V_a^2}{2})\right)dx = (Wh_o)_r - (Wh_o)_{r+1} + E_{net,r} \qquad (3)$$

where the subscripts r and $r+1$ denote the flow conditions at the inlet and outlet of the r^{th} element respectively. $F_{net,r}$ represents the net force input to the flow in the axial direction resulting from the action of the blading, the pressure on the walls and the frictional losses. $E_{net,r}$ accounts for the net energy transfer to the airflow through shaft work and heat transfer.

These equations can be simplified to manipulate them more easily. A non-linear LP-MCE-MS model has been selected for the control system study. In this model, the Lumped Parameter (LP) representation is obtained from the equations by assuming that the axial rate of change of properties remain constant throughout the control volume. A further distinction is made as the model considers all Momentum, Continuity and Energy equations (MCE). It is also assumed that the Mach number is constant at steady state points during

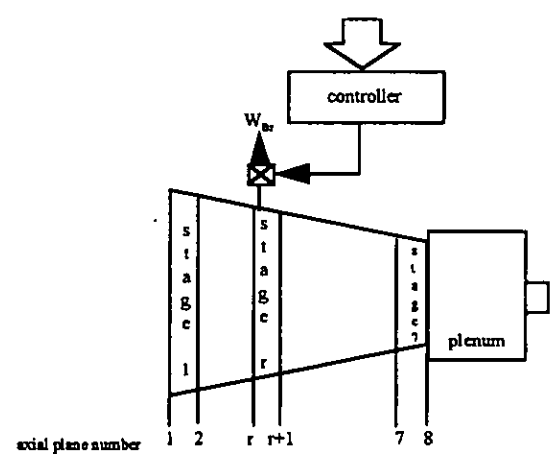

Fig 1 Active surge control using a single-actuator

transient behaviour (MS models). In addition, considering a quasi steady-state approximation, the net momentum and energy terms are obtained from the steady-state equations and experimental steady-state compressor characteristic.

A seven-stage axial compressor was modelled using an ACSL (Advanced Continuous Simulation Language, 1992) program which predicted the surge points within 6% of the experimental data (Montazeri et. al., 1994). In addition, the simulation results revealed that the origin of the purely one-dimensional instabilities is affected by the steady-state stage characteristics. One-dimensional instabilities originate from the front stages at low speed and from the last stage at high speed.

3. SCHEDULING LQR CONTROLLER DESIGN

Using air bleeding as an actuation strategy for active stabilisation of the purely one-dimensional instabilities, the selected mathematical model is a set of unstable 24th order differential equations, which models the non-linear compressor characteristics.

Linearising the compressor flow and actuation model at each steady state point, the equations are as follows:

$$\dot{X} = AX + BW_b \qquad (4)$$

where, X denotes the vector of 24 state variables (i.e. independent flow parameters including static pressure, mass flow and density at each axial plane) and W_b is the bleed mass flow (i.e. the control signal). Also, A and B are the small perturbation system and control matrices, respectively.

A linear quadratic regulator (Anderson and Moore, 1971) full-state feedback controller has been used as follows:

$$W_b = -KX \qquad (5)$$

where K is the feedback gain vector and is determined from the following equation:

$$K = R^{-1}B^T P \qquad (6)$$

where P is a semi-definite positive matrix from the solution of the following Algebraic Riccati Equation (ARE):

$$PA^T + AP - PBR^{-1}B^T P + Q = 0 \qquad (7)$$

With an LQR approach, the feedback controller coefficients are designed so that the following cost function is minimised:.

$$J = \int_0^\infty (X^T QX + W_b^T RW_b)dt \qquad (8)$$

where the matrices Q and R are positive semi-definite and positive definite respectively.

The performance of a control system designed using LQR method depends on the weighting matrices Q and R (Kalman 1964, Safanov 1977). The weighting matrices Q and R are a compromise between the control activity and the stability robustness of a control system. As Q increases in comparison with R, feedback gain coefficients increase providing more stability robustness for the feedback control system but the magnitude of the control signal also increases. However, as R increases in comparison with Q, the feedback gain coefficients decrease resulting in less energy in the control signals but also reducing the stability robustness.

For an unstable system, as Q approaches zero, the closed-loop poles move toward the position of the stable poles providing the minimum control activity. This is termed the *minimum energy control law.*

For the active stabilisation of compressor surge using a bleed strategy, zero (or a very small value) Q and identity R weighting matrices can be used to yield a cost function with the following form:

$$J = \int_0^\infty (W_{b,r})^2 dt \qquad (9)$$

where r is the axial location of the bleed actuator. In this case, where the cost function is 'the Integral of the square of the bleed mass flow', the LQR method minimises the consumption of air bleeding during the stabilisation process.

On the other hand, the selected weighting matrices may not provide sufficient stability robustness for the active control system due to nonlinearities when the compressor is operating beyond the surge line. Therefore, a gain-scheduling approach (Astrom, 1989) has been used to cope with these non-linear effects. In other words, the controller design approach is based on linearising the selected non-linear model about a set of closely-spaced steady-state operating points beyond the surge line, and applying the linear quadratic regulator technique to design the corresponding state-feedback controllers in order to minimise the air bleeding mass flow at each design point. In this way, the non-linear control problem is reduced to a series of linear control problems and a non-linear feedback controller where the feedback gains vary with the compressor steady-state conditions.

4. ACTUATOR PLACEMENT APPROACH

As shown in fig 1, in a single-actuator configuration, the actuator may be located at any of the compressor stages or the compressor plenum. Therefore, it is necessary to determine if the location of air bleeding influences the amplitude of bleed mass flow in an active surge system. To select the optional location, a performance index is defined which is an index of the performance of the active control system for different actuator locations along the compressor.

The cost function J (Eq. 9) defines a measure of the consumption of air bleeding when the active control system is subject to an initial condition x(0). In other words, calculating cost J gives an index that measures the required bleed to force the compressor flow from a perturbed condition to zero perturbation (i.e. steady-state condition). Therefore, analysis of the cost J at different locations along the compressor is used to determine the axial plane with the minimum cost for the bleed actuator location.

Appendix A shows that the cost J can be calculated analytically for an active surge control that is subject to an initial condition x(0). The following expression is then satisfied:

$$J = \int_0^\infty W_{B,r}^2 dt = x^T(0)Px(0) \qquad (10)$$

where P is the solution of the ARE (7), Q is zero and R is unity.

5. ANALYSIS OF THE RESULTS

Using ACSL (1992), the non-linear process model (integrated with the air bleeding model) is linearised at a steady-state design point. Using Matlab (Grace, 1990), the matrix P (i.e. the solution of the ARE) and the feedback gain coefficients, K, are then calculated for different locations of the air bleeding actuator along the compression system. Using equation (10), the cost function J is finally computed for an initial state condition x(o). This procedure has been applied to a low pressure seven-stage axial flow compressor and repeated for several steady-state points and several initial conditions.

5.1 Low speed (70% design speed) operation

Fig 2 shows that the cost J varies as the actuator location (avpn) changes from the compressor first stage (avpn=1) to the compressor plenum (avpn=8). In this figure, the cost J is calculated for 24 series of initial conditions at the steady-state point Wss=97-kg/sec (slightly beyond the surge point). Each set corresponds to an initial condition equal to 1% of the steady-state value at one of the state variables. For example, the last set corresponds to the initial condition x(0)=1%Wss_8, that is 1% of the steady-state mass flow at plane number 8. In addition, fig 3 depicts the cost J at several steady-state points

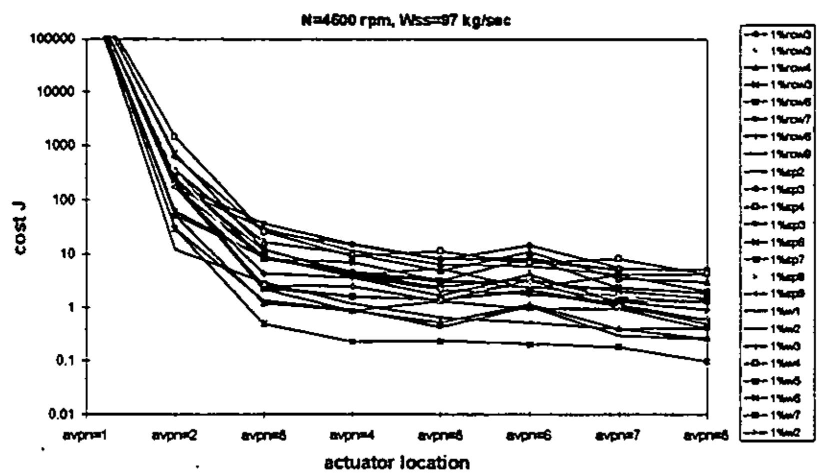

Fig 2 Cost function J at Wss=97kg/sec in the presence of different initial conditiond

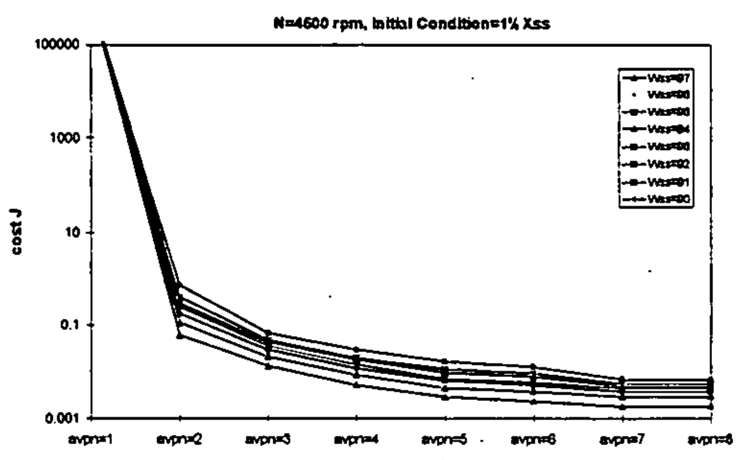

Fig 2 Cost function J versus actuator location at several steady-state points at low speed

beyond the surge line for values of avpn (actuator plane number) from 1 to 8 in the presence of 1% initial perturbation at all 24 state variables i.e. $x(o)=1\%X_{ss}$. A common feature in these figures is that the cost J reduces for the location of the air bleeding actuator toward the compressor outlet (i.e. high pressure flow). In other words, the cost J is minimum when the actuator is located at the last stage of the compressor.

This result can be justified by considering the input matrix B, shown in Table 1. The B matrix coefficients indicate the sensitivities of the flow parameters to the bleed mass flow. For example, bleed mass flow from the 4th stage (avpn=4) influences the 3rd and 11th state variables, ρ_4 and SP_4, respectively. Comparing these coefficients for different avpn values, it can be seen that the B coefficients increase as avpn increases from 1 to 7. This implies that the compressor flow dynamics is more sensitive to the air bleeding from the compressor downstream stages than the upstream stages, even though the instabilities originate from the compressor front stages at low speed.

In addition, figure 4 shows that the cost J increases as the steady-state mass flow is reduced beyond the surge point. This means that the consumption of air bleeding is higher when the compressor is operating at a point farther from the surge point than a point closer to the surge point. However, the difference reduces as the actuator location is placed towards the compressor downstream (i.e. avpn=7).

5.2 High speed (design speed) operation

Fig 4 shows the cost J versus avpn at the steady-state point Wss=184kg/sec close to the surge point where initial conditions are changed through 24 state variables. In addition, fig 5 shows the cost J versus actuator location (avpn) at several steady-

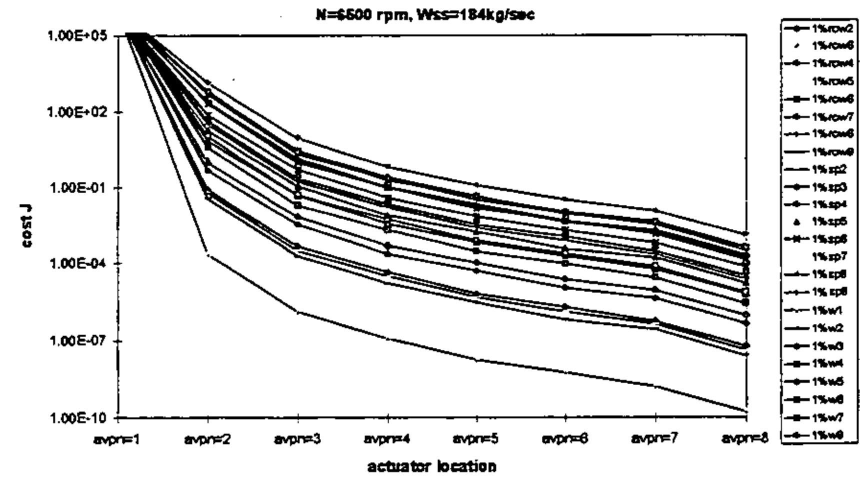

Fig 4 Cost function J at Wss=184kg/sec in the presence of different initial conditions

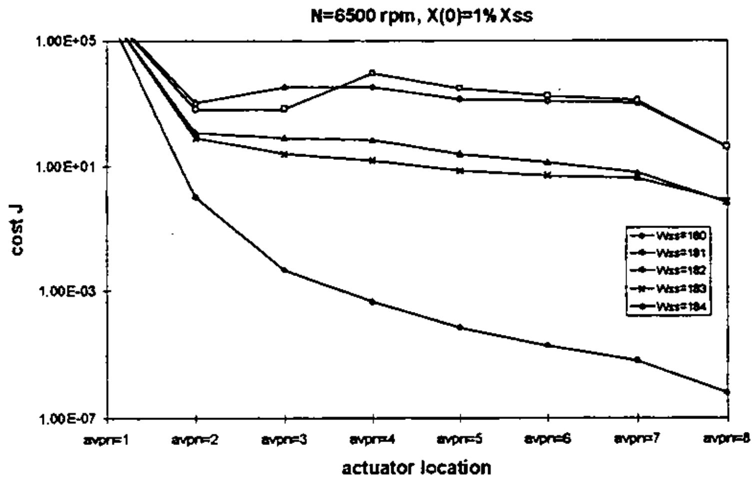

Fig 5 Cost function J versus actuator location at several steady-state points at high speed

avp n=1	avpn=2	avpn=3	avpn=4	avpn=5	avpn=6	avpn=7	avpn=8
0	-3.03E+00	0	0	0	0	0	0
0	0	-7.83E+00	0	0	0	0	0
0	0	0	-1.11E+01	0	0	0	0
0	0	0	0	-1.36E+01	0	0	0
0	0	0	0	0	-1.64E+01	0	0
0	0	0	0	0	0	-1.89E+01	0
0	0	0	0	0	0	0	-2.51ED+01
0	0	0	0	0	0	0	0
0	-3.61E+05	0	0	0	0	0	0
0	0	-9.75E+05	0	0	0	0	0
0	0	0	-1.44E+06	0	0	0	0
0	0	0	0	-1.83E+06	0	0	0
0	0	0	0	0	-2.29E+06	0	0
0	0	0	0	0	0	-2.71E+06	0
0	0	0	0	0	0	0	-3.79E+06
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Table 1 Control matrix B at Wss=97kg/sec at low speed

avp n=1	avpn=2	avpn=3	avpn=4	avpn=5	avpn=6	avpn=7	avpn=8
0	-3.03E+00	0	0	0	0	0	0
0	0	-7.83E+00	0	0	0	0	0
0	0	0	-1.11E+01	0	0	0	0
0	0	0	0	-1.36E+01	0	0	0
0	0	0	0	0	-1.64E+01	0	0
0	0	0	0	0	0	-1.89E+01	0
0	0	0	0	0	0	0	-2.51E0+01
0	0	0	0	0	0	0	0
0	-3.44E+05	0	0	0	0	0	0
0	0	-9.57E+05	0	0	0	0	0
0	0	0	-1.49E+06	0	0	0	0
0	0	0	0	-2.00E+06	0	0	0
0	0	0	0	0	-2.62E+06	0	0
0	0	0	0	0	0	-3.22E+06	0
0	0	0	0	0	0	0	-4.51E+06
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Table 2 Control matrix B at Wss=184kg/sec at high speed

state points beyond the surge point at high speed. These figures also reveal that the cost J is minimum where the actuator is located at the plane 8 (i.e. the compressor plenum). As for the low speed case, the control matrix B at high speed, shown in Table 2, indicates that the compressor flow is more sensitive to the air bleeding from downstream stages than upstream stages in an active surge control system.

5.3 Simulation study

The LQR feedback gain coefficients are assigned in the non-linear simulation program of the active surge control system where the non-linear process model of the compressor is combined with the linear feedback of the compressor flow perturbations relative to their steady-state values.

The results obtained from non-linear simulation of the active control system for different locations of air bleeding at low rotational speed are shown in fig 6. This figure shows the bleed mass flow during stabilisation of the compression system at the steady-state point Wss=97kg/sec in the presence of the initial condition 0.1%SP2 where the actuator location is changed from plane 2 to 8 (i.e. from the second stage to the plenum). As shown in this figure, the amplitude of the bleed mass flow is reduced as the actuator is located towards the last stages confirming the results from the analytical approach to the actuator placement at low speed.

In addition, fig 7 shows similar results at the steady state point Wss=184 kg/sec at high speed. As shown in this figure, the amplitude of the bleed mass flow is minimum when the actuator is located at the compressor plenum (avpn=8). Therefore, the simulation results at high speed also confirm the results from the analytical method.

6. CONCLUSIONS

A methodology is proposed in this paper for actuator placement in the active control of surge in multi-stage axial compressors using an air bleeding strategy. ACSL simulation of a seven-stage axial compressor has then been used to confirm the analytical approach. For the low pressure seven-stage compressor, both schemes show that an actuator placement at the compressor downstream stages consumes the minimum air bleeding for stabilisation of the compression system at both low and high speed.

Using this approach, simulation results show that the bleed mass flow for stabilisation of the compressor surge using the single-actuator active control system is sufficiently small in comparison with the compressor steady-state mass flow in the presence of small perturbations and also for large perturbations. In addition, although it is appreciated that this paper has assumed the existence of in-bleed (injection), the simulation study has been extended to restrict actuation to model outward bleed alone (no injection) and similar results have been obtained.

APPENDIX A: *Analytical approach to compute the cost function J*

A Lyapunov function for a dynamical system is a semi-definite positive function in terms of the system state, x(t). For a feedback control system with an LQR controller, the Lyapunov function can be defined as follows: -

$$V(t)=x(t)^T Px(t) \tag{11}$$

where P is a semi-definite positive matrix from the solution of the ARE. The time derivative of V(t) can then be written as follows:

$$\dot{V} = \dot{x}^T Px + x^T P\dot{x} \tag{12}$$

where x and $\dot{x}$ are the short form of x(t) and $\dot{x}(t)$.

Considering the closed-loop state equation of the feedback control system:

$$\dot{x} = (A - BK)x \tag{13}$$

Equation (12) can be written as follows:

$$\dot{V}(t) = x^T (A^T P + PA - K^T B^T P - PBK)x \tag{14}$$

where $K = R^{-1}B^T P$ is the LQR feedback gain matrix.

Furthermore, considering the ARE (7), equation (14) yields the following form:

$$\dot{V}(t) = -(x^T Qx + W_b^T RW_b) \tag{15}$$

Substituting (11) in (8), the cost function J can be written as follows:

$$J = \int_0^\infty (x^T Qx + W_b^T RW_b)dt = \int_0^\infty -\dot{V}(t)dt = -V(t) \tag{16}$$

Finally, considering the equations (11) and (16), the cost function J for an LQR control system where the target state is zero, i.e. $x(\infty)=0$, will have the following form:

$$J = \int_0^\infty (x^T Qx + W_b^T RW_b)dt = x^T(0)Px(0) \tag{17}$$

For a single-actuator system with zero Q and unity R weighting matrices, equation (17) yields:

$$J = \int_0^\infty W_{B,r}^2 dt = x^T(0)Px(0) \tag{18}$$

Equation (18) gives an analytical expression for computing the cost function J of an active control system where the control system is subject to an initial deviation x(0).

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