Three-Dimensional Effects in Supersonic and Transonic Compressor Rotors—an Experimental and Computational Study

A computation method for the three-dimensional rotational flow through transonic and supersonic rotors is described and discussed by means of a comparison with experimental results. The computation treats subsonic and supersonic flows with different algorithms. The compression shocks are calculated as real three-dimensional discontinuities on the basis of the Rankine-Hugoniot-equations. The experimental data result from measurements in transonic and supersonic compressor rotors. A comparison of the data shows to what extent these three-dimensional effects are covered by the described theory.

NOMENCLATURE

- **A, a, b, r**: constants of the boundary layer
- **a**: velocity of sound
- **b(i)**: line-vector
- **D_2(s), h(s), F(s)**: dimensionsless \( \delta_2(s), H_12(s), p(s) \)
- **H_12**: shape factor
- **J**: streamline direction
- **M, Pr, Re**: Mach-, Prandtl- and Reynoldsnumber
- **r_{sh}**: surface vector of local shock-element
- **p, T, \rho**: pressure, temperature, density
- **Q**: arbitrary variable
- **R**: gas-constant
- **r, \rho_d, z**: axisymmetric coordinates
- **s**: entropy (appendix)
- **S**: dimensionless coordinate
- **u, w**: circumferential and relative velocity
- **v_0, \phi, \theta, \chi**: vector in the direction of \( \omega, \phi, \theta, \chi \)
- **x**: surface coordinate on the profile
- **a**: Mach-angle
- **\beta**: deflection angle of relative velocity
- **\beta_1, \beta_2**: circulation-, rotation-vector
- **Y, \psi, \lambda**: streamline angles
- **\delta, \eta**: angles (description of 3d shock-element)

**SUPERSCRIPTS**

- **o**: upstream of the shock
- **^\wedge**: downstream of the shock
- **(i)**: vector, components of...
INTRODUCTION

Transonic compressor stages are nowadays frequently used in stationary gas turbines as well as in aircraft engines. Generally flat profiles with slight turning are applied here for minimizing the losses. Numerous numerical procedures, e.g., (1), (2), (3), have been developed for the calculation of the flow through such bladings. The computation of viscous effects as well as the exact theoretical prediction of compression shocks is, however, problematic. This holds particularly true for transonic flows with predominantly three-dimensional character.

The present contribution covers the theoretical and experimental investigation of the three-dimensional effects in transonic and supersonic compressor rotors. The cited results mainly refer to rotors with small aspect-ratio, high turning, strong meridional hub curvature, and, moreover, with different characters of the flow. The relative flow velocity in the impulse-type rotor (\(U\)) is supersonic, whereas in the entrance region of the shock-rotor a strong compression shock is stabilized by the channel-geometry, so that the channel flow itself is mostly subsonic (\(s\)). The geometric data of the blading and the hub geometry are also given in (4), (5).

THEORETICAL APPROACH

The calculation of the rotor flow is done by a quasi-three-dimensional approximation on flow surfaces of revolution (S1) and on a S2-surface extending from tip to hub. Furthermore, the flow field is subdivided into inviscid main-flow and the boundary layers. Contrary to the time-marching methods described for example in (1), (2), the numerical calculation of the inviscid field starts from the basic equations for steady flow. Thus supersonic and subsonic regions are calculated by different procedures. The three-dimensional compression shocks are fitted by means of the Rankine-Hugoniot equations. A detailed deduction and a discussion with respect to previous published results can be found in (5), (6).

Calculation of the Inviscid Flow Field

The calculation of the rotor flow is based on the Euler-equation of motion, the energy-equation and the compatibility-equations on the S2-surface. Different algorithms were developed for regions with supersonic and subsonic flow velocities. They are briefly described in the following. For the sake of completeness the resulting finite difference equations are summed up in appendix A.

Within the supersonic regions the basic equations are solved by means of the method of characteristics in such a way that the entropy gradients are taken into consideration explicitly. Neglecting terms of higher order for the numerical procedure, the compatibility-equations on the S1-surface can be formulated as finite difference equations as given in (A1). Assuming an inviscid flow the entropy is a direct function of the stream-function. Consequently the entropy-gradient can be calculated by linear interpolation from the stream-function. The slope condition of the characteristics corresponds to that one, known for axisymmetric flow. In solving this set of finite-difference equations, a two-step-approach proved to be useful. In a predictor step the system is solved for an irrotational flow (\(\Delta s = 0\)). The solution is then improved by a corrector step considering the entropy-distribution.

The calculation of subsonic flow is based on the conservation of the circulation in the absolute system. Downstream of the compression shocks the rotation of the flow is determined by means of Crocco's law applied to the relative system (A2). The surface-integral describes the circulation behind the shock (A3). Within the supersonic region, the circulation of the finite elements of the shock- and contour-oriented grid is formulated as the line-integral of the relative velocity. This integration results in the finite difference equation (A4). From this the velocity-distribution can be determined on the assumption of a constant circulation and considering the continuity equation and the energy equation.

On the S2-surface the finite difference equations summed up in (A5 - A9) are used. The shape of the streamlines on the S2-surface is determined by integrating the mass-flow between adjacent S1-surfaces.

Compression Shocks

The transonic and supersonic flow through compressor rotors is strongly influenced by the compression shocks. The problems in the numerical treatment of these three-dimensional discontinuity surfaces are usually avoided by means of numerical or artificial viscosity. This, however, results in inaccurate shock-positions and very often in an overshoot of the changes of state across the shocks. Therefore, within the scope of the method under consideration an algorithm is established which treats the compression shocks as discontinuity surfaces on the basis of the Rankine-Hugoniot-equations.

The three-dimensional position of a local finite shock-element is defined by its surface-vector \(\mathbf{n}_{sh}\) (Fig. 1).

The shock-angle \(\phi\) is given by the dot product of this shock-surface-vector and the upstream velocity \(\mathbf{V}\):

\[
\sin \phi = \frac{\mathbf{V} \cdot \mathbf{n}_{sh}}{V_{up} \cdot \mathbf{n}_{sh}}
\]  

(1)

The deflection of the velocity-vector occurs in a plane defined by the shock-surface-vector and the velocity-vector upstream of the shock. In order to describe the three-dimensional position of the downstream velocity-vector this plane (screened in Fig. 1) may be intersected with a coaxial plane (hatched). On the intersection line these planes the vector \(\mathbf{V}_{b}\) is located, which is decisive for the veloc-
ity deflection in the shock-element. The intersection-line and thus the direction of \( \mathbf{V}_b \) can be determined by a vector in the direction of the shock-principal \( \mathbf{V}_{n-sh} \) and the upstream-velocity vector:

\[
\mathbf{v}(i) = \frac{\mathbf{V}(i) - \mathbf{V}(i')}{\mathbf{V}_{n-sh}}
\]

with \( \mathbf{v}(i) = (0; -v(y)^{i}/v(z)^{i}; -v(z)^{i}/v(z)^{i}) \) and \( \mathbf{V}_{n-sh} \), etc.

\[
\mathbf{v}(i) = (0; 1; 1) \quad \text{and} \quad \zeta_3: \text{parameter}
\]

The magnitude of the vector \( \mathbf{V}_b \) is given by geometric relations:

\[
|\mathbf{V}_b| = \frac{\sin \delta}{\sin (\delta + \phi)} (\sin^2 \phi + \tan^2 \gamma_0) 1/2
\]

The deflection angle \( \delta \) can be determined by the shock-angle \( \phi \) and the shock equations.

As the angle \( \phi \) can be established by the dot-product of the upstream-velocity and \( \mathbf{V}_b \), the downstream-velocity-vector comes from the consideration of the geometry in Fig. 1.

\[
\dot{\phi} = \phi_0 + \delta, \quad \phi = \text{arc tan} \left( \tan \gamma_0 + |\mathbf{V}_b| \cdot \sin \gamma_0 \right)
\]

\[
\sin^2 \phi = \frac{1}{2} \left( |\mathbf{V}_b| \cdot \cos \gamma_0 \sin \phi_0 - 2 \cos \phi_0 \left( |\mathbf{V}_b| \cdot \cos \gamma_0 \sin \phi_0 \right)^{-1} \right)
\]

and \( \gamma \) can be determined by the components of the equation (2).

With the aid of these relations the spatial shock can be treated as a real discontinuity. The intensity of the shock is determined by an iteration on the S1- and S2-surfaces which is described in the following. Starting with the flow properties on an arbitrary S1-surface \( \jmath \) (Fig. 2), the flow conditions on the adjacent streamsurface \( \jmath - 1 \) can be determined by means of the equations \( (A5) \), \( (A6) \), etc. (grid points \( i', k \) and \( i'' + 1, k' \)). The data downstream of the shock (grid point \( i, k \)) are calculated by an extrapolation on the stream-surface \( \jmath - 1 \). Then, the strength and the position of the shock on this S1-surface are iteratively fitted between sub- and supersonic region by means of the above mentioned shock equations. The inclination of the determined shock-element with respect to the \( r-z \)-plane also results in a weakening of the shock-wave at the interaction line with the S1-surface \( \jmath \), etc. have to be calculated again taking into account the new shock angle. By means of this iterative process the three-dimensional equations of motion can be satisfied up- and downstream of the shock-waves.

Profile-Boundary Layers and Shock/Boundary Layer Interaction

The above equations are valid only for an inviscid flow. Especially in transonic rotor flows viscous effects are of great importance. Simplifying the complex problem these effects may be considered separately for the end-wall and the profile boundary-layers. In order to minimize the calculation time the profile boundary layers are calculated by means of an integral method (7) additionally taking into account the shock/boundary layer interaction with local separation by a semi-empirical method. In the interaction regions the displacement thickness increases from \( \delta_0 \) at the beginning of the interaction to \( \delta_\text{r} \), as shown in Fig. 3 a. High flow Mach numbers and strong compression shocks may result in a local separation and consequently in a change of the direction of the shear stress on the wall. The wall pressure (Fig. 3 b) begins to increase upstream of the compression shock and reaches the pressure level of the free-stream some distance downstream of the shock. In the case of flat plates or slightly curved profiles the momentum thickness shows a nearly linear increase before and behind the interaction region (Fig. 3 c). In close proximity of the shock, however, a steep rise can be observed. Consequently the increase of the momentum thickness can be divided into one part, dependent on the freestream Mach number, and into another part induced by the shock. The shape factor \( H_2 \) (Fig. 3 d) decreases downstream in the case of a continuous freestream flow. In the interaction region with local separation the shape factor first increases considerably but further downstream, it declines to the value corresponding to continuous flow.

To calculate the flow in the interaction region a dimensionless coordinate, \( s \), is introduced. It originates at the beginning of the interaction region at the impingement point of the shock, and it is defined by:

\[
s = \frac{x - x_0}{\lambda_D}
\]

Here \( \lambda \) corresponds to the distance from the initial point of the interaction to the main shock (Fig. 3 a) and it is proportional to the intensity of the interaction. Evaluating the experimental investigations in Table 1 this dimensionless diffusion length \( \lambda_\text{D} \) can be expressed as a function of Mach and Reynolds number:

\[
\frac{\lambda_\text{D}}{\delta_0} = \frac{A}{\log \text{Re}_{\delta_0}^{1/2}}, \quad \text{with} \quad A = 232
\]

With the aid of this dimensionless coordinate, \( s \), the development of the characteristic boundary layer parameters in the interaction region, can be performed as follows:

Analogously to (8) the pressure distribution can be described, also in the case of local separation, by a transition function of a characteristic pressure coefficient. This function is defined by:

\[
P(s) = \frac{p(s) - p^0}{p_0 - p^0}
\]

and is plotted versus the flow direction in Fig. 4 a in comparison to experimental data.
Outside the interaction region near the shock a 1/7-th-power velocity distribution can be assumed. Introducing the recovery-factor, \( r \), \( (9) \) the change of the momentum-thickness in the direction of the dimensionless coordinate, \( s \), becomes:

\[
\frac{d(\delta_2^\text{cont})}{ds} = \left(1 + a \cdot r \cdot \frac{x - 1}{2} \cdot N^2\right)^{-1} \cdot \frac{b}{2} \cdot \frac{1}{\delta_2^\text{cont}} \frac{(x_k^\text{h})}{1}
\]

with \( a = 0.5; \ b = 0.045; \ r = Pr^{1/3} \)

The above-mentioned assumptions are confirmed by a comparison of the calculated gradient of the momentum-thickness near the shocks with experimental data (Fig.5).

The influence of the compression shocks on the momentum thickness can be investigated with the aid of the momentum theorem. Using the control surface in Fig. 3a, the increase of the momentum thickness caused by the influence of the shock becomes:

\[
\frac{\delta_2^\text{sh}}{\delta_2^\text{cont}} = 1 + \left( \frac{\nu}{\nu} - 1 \right) \left(1 + H_2^\text{sh}\right) + \frac{1}{\rho(\nu)^2} \frac{\delta_2^\text{cont}}{x_k^\text{h}} \frac{y}{\delta_2^\text{cont}}
\]

### Table 1: Shock/boundary layer interaction-review of cited experimental data

<table>
<thead>
<tr>
<th>( \theta^\circ )</th>
<th>( Re_{\theta^\circ} \times 10^4 )</th>
<th>( \delta_2^\circ ) [( \text{mm} )]</th>
<th>author, symbol</th>
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</tr>
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<td>1.52</td>
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<td>Leblanc (8)</td>
</tr>
<tr>
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<td>22.2</td>
<td>Mateer (15)</td>
</tr>
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<td>-2.0</td>
<td>34.6</td>
<td>Vidal (16)</td>
</tr>
<tr>
<td>1.34</td>
<td>-3.0 \times 10^{-1}</td>
<td>-</td>
<td>Ackeret (17)</td>
</tr>
<tr>
<td>1.26</td>
<td>-3.5 \times 10^{-1}</td>
<td>-</td>
<td>Delery (11)</td>
</tr>
</tbody>
</table>

Fig. 3: Shock/boundary layer interaction: distribution of characteristic boundary layer parameters

Fig. 4: Transition functions for the distribution of dimensionless pressure \( (a) \), momentum thickness \( (b) \) and shape-factor \( (c) \) in comparison with experimental results (Table 1)
Because of the deceleration of the flow the shear stress at the wall rapidly decreases. In the case of local separation the wall shear stress even changes its sign (Fig. 3a). Since, additionally, the distance 1 of the control-surfaces is small, the term including the shear stresses at the wall may be neglected. As the experimental data in Fig. 5b show, the pressure rise in the shocks results in an increase of the transition function \( \delta_2(s) = \frac{\delta(s)}{\delta_{sh}(s)} \), compared to the value given by the equation (10) within a distance of \( \delta s \). Thus considering the equations (9) and (10), the total increase of the momentum thickness in the interaction region can be computed by

\[
\frac{d(\delta_2/\delta_s)}{ds} = \frac{d\delta_2}{ds} \cdot \frac{\delta_{cont}}{\delta_{sh}} (11)
\]

The shape parameter \( H_{12} \) can also be composed by one part referring to the continuous flow and by another shock-induced part. An approximation of data taken from (10) yields:

\[
H_{12 \ cont} = 1.279 + 0.0175 \cdot M + 0.0375 \cdot M^2 (12)
\]

Since these data derive from profiles with a \( 1/7 \)-th-power velocity law, equation (11) is not valid in close proximity of the shock. If, however, the shape factor is calculated according to (11) with downstream Mach number of the shock, and the result is chosen than as a reference value, the experiments show a characteristic course of the reduced shape parameter \( h = \frac{H_{12}}{H_{12 \ cont}} \). Consequently the shock-induced shape parameter can be approximated according to Fig. 4c. The local shape parameter and the momentum thickness result in the distribution of the displacement thickness and a correction of the blade passage geometry.

**COMPARISON OF RESULTS FROM THEORY AND EXPERIMENT**

**Transonic Profile Boundary-Layers**

The presented approximation procedure for calculating transonic boundary layers with shock-induced local separation was tested, as far as flat plates and two-dimensional profiles are concerned, by means of a comparison with experimental results from different authors. As an example, Fig. 6 shows the measured and calculated distributions of Mach number, displacement and momentum thickness for a transonic circular arc profile (11). In this case, the deceleration in the compression shock results in a local separation of the boundary layer. The conformity of theory and measured data which could also be proved for other experiments (2) shows, that the computation method provides useful results for two-dimensional flows at short calculation times.
the leading edge shock waves, however, induce great pressure differences upstream of the leading edge. Though the flow conditions ahead of both rotors are equal and their profile suction side is parallel to the relative inlet flow, different isobars and leading edge shock wave structures can be observed. Thus, in the case of subsonic axial velocity, the flow field near upstream of the rotor is considerably influenced by the character of the channel flow.

The measurements in a transonic rotor performed by the Laser-two-focus method and published in (12) allow an additional comparison of theory and experiment with respect to the three-dimensional structure of the shock waves.

Fig. 8 shows the measured Mach number distributions on two cylindrical surfaces and, as dotted lines the calculated position of leading edge shock waves and compression shocks. Here again the results from computation and experiment show the three-dimensional structure of the compression shocks. Because of this three-dimensionality the real shock angles as well as the deceleration in the compression shocks are smaller than in the case of a two-dimensional approach.

**Viscous Effects in the Rotor Flow**

Within the blading, especially in the case of high turning and loading, the three-dimensional effects are still enhanced by the structure of the boundary layers and the secondary flow. This becomes evident by comparing results referring to the impulse type and the shock rotor.

In the blade channels of the impulse type rotor only weak compression shocks are generated, so that the boundary layers are lightly loaded. Taking into account only the profile boundary layers, the presented computation method already reproduces the measured distribution of the flow properties quite satisfactorily. Fig. 9 shows as an example the pressure distributions downstream of the impulse type rotor calculated under the assumption of inviscid flow and with consideration of the profile boundary layers, in comparison with the experimental data (4). It is only...
Fig. 9: Pressure distribution downstream of an impulse-type rotor: Influence of the profile boundary layers in the small regions of the hub and casing boundary layer that the results of experiment and simplified theory differ from each other.

The conditions are completely different in the rotor with a shock stabilized by the blade passage geometry. This is already shown by the isobars calculated on a S2-surface (Fig. 10). In the entrance region, the boundary layers are heavily loaded by the pressure rise in the compression shock. Near the casing, the slight acceleration of the flow within the blade channels counteracts a total separation of the boundary layers. Near the hub, however, the boundary layer is additionally loaded by the local pressure rise downstream of the shock. The visualization of the secondary flow by means of very small oil droplets showed that the hub boundary layer separates due to this heavy loading. The hub boundary layer separated downstream of the strong channel shock flows to the suction side of the profiles and is centrifuged outward in the rear part of the blading. The result can be recognized by the distribution of the flow angle behind the rotor (Fig. 11). Near the tip, the flow angle becomes considerably larger than the construction angle of the trailing edge. This reduced turning indicates a concentration of separated boundary layers at the suction side of the blading. In this case, the calculation of the flow does not reproduce correctly the distribution of the flow properties downstream of the rotor, even if the profile boundary layers are taken into account. This is shown for example by a comparison of the computed pressure distribution and the measuring results (Fig. 12).

By an adequate choice of the displacement thickness of the profile boundary layers, however, the calculation yields an outlet flow angle corresponding to the measured data (dashed line in Fig. 11). Simulating the secondary flow effects in such a way, the calculation reproduces the other flow conditions sufficiently (Fig. 12). This is confirmed by the results in Fig. 13 showing the experimental and the measured isobars in the critical casing section. Except for the rear part of the suction side of the profiles, the isobars correspond to each other in theory and experiment. As already mentioned, the theory reproduces the shock positions correctly as well as the pressure rise in the shocks.

Fig. 10: Calculated isobars on a S2-surface (shock-rotor)
CONCLUSIONS

In the present contribution computational results of a quasi-three-dimensional method, considering viscous effects by approximation, were compared with experimental data. The semi-empirical integral method for boundary layers with shock induced local separation turned out to be a useful approach for two-dimensional transonic profile flows. With regard to the rotor flow the comparison of theoretical and experimental results showed:

- The complex three-dimensional structures of front waves and compression shocks are reproduced sufficiently by the described method treating the shocks as real discontinuities.
- In the case of weak compression shocks and slight loading of the boundary layers, already the consideration of the profile boundary layers results in a more accurate agreement of calculated and experimental data.
- Also in the case of marked three-dimensional viscous effects caused by strong compression shocks the viscous effects could approximately be covered by means of the presented method and thus a better conformity with the measurements could be achieved.

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REFERENCES

3 Dodge, P.R., Lieber, L.S., "Transonic 3-D Flow Analysis of Compressor Cascades with Splitter Vanes", AFPL-TR-78-23
Supersonic Flow (S1-Surface):

\[ w = \frac{-w_s \tan \alpha + w \sin \alpha \theta}{w_s + w \alpha} \]  

Subsonic Flow (S1-Surface):

\[ \Gamma_{1,j,k} = 2 \left( d^k w \frac{d^k w}{d^k r} + \frac{\alpha_p}{\alpha_l} \frac{d^k w}{d^k r} \right) \delta^k \]  

rotation downstream of shocks

\[ \Gamma_{1,j,k} = \delta^k w \frac{d^k w}{d^k r} - u_{1,j,k} \cos \lambda \delta^k - (\delta^k u + \delta^k \omega) \eta^k \]
\[ \alpha_{ij} = \left\lfloor 1 - \left( \alpha^r_{ij} / \alpha^s_{ij} \right)^2 \right\rfloor^{1/2} \]

\[ \alpha_{ij} = \left( \frac{\alpha^s_{ij}}{\alpha^p_{ij}} \right)^{\alpha_{ij}} \left( w_{ij} + 2 w_{ij} \right) \]

with \[ Q = (w, \rho) \]

(A 5)

and

\[ \frac{\partial w_{ij}}{\partial t_{ij,k}} = \left( \frac{\partial}{\partial t} \right) \frac{\Delta_{ij,k}}{\alpha^s_{ij,k}} \left( \frac{\partial w_{ij}}{\partial t} \right) \]

(A 6)

\[ \frac{\partial w_{ij}}{\partial t_{ij,k}} = - \frac{\partial}{\partial t} \left( \frac{\partial w_{ij}}{\partial t} \right) \frac{\partial w_{ij}}{\partial t} \left( \frac{\partial w_{ij}}{\partial t} \right) \]

(A 7)

\[ \frac{\partial w_{ij}}{\partial t_{ij,k}} = \frac{\partial}{\partial t} \left( \frac{\partial w_{ij}}{\partial t} \right) \frac{\partial w_{ij}}{\partial t} \left( \frac{\partial w_{ij}}{\partial t} \right) \]

(A 8)

\[ \frac{\partial w_{ij}}{\partial t_{ij,k}} = \left( \frac{\partial}{\partial t} \right) \frac{\partial w_{ij}}{\partial t} \left( \frac{\partial w_{ij}}{\partial t} \right) \frac{\partial w_{ij}}{\partial t} \left( \frac{\partial w_{ij}}{\partial t} \right) \]

(A 9)