HYDRODYNAMIC TORQUE CONVERTERS
IN MECHANICAL TRANSMISSION SYSTEMS:
METHODS AND ANALYSIS

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ABSTRACT

Methods for analysis of general mechanical transmission systems with a hydrodynamic torque converter (HTC) are presented. The methods are adapted for computer calculations. The properties of the HTC must be known, either explicitly as speed and torque characteristics, or implicitly as internal geometry (blade angles, etc.).

Linear relationships between the torques and between the speeds of the shafts of the transmission system (except the HTC) are easily formulated. The HTC has coupled, non-linear, relationships for torques and speeds. Different ways of including these non-linear equations are presented. This can be implemented in a computer program.

Solving the equation system yields the torque and speed of each shaft of the transmission system. Power losses can be handled.

NOTATION

\(B\) Matrix for HTC internal geometry
\(c_m\) Fluid velocity in the meridional plane
\(i\) Transmission ratio
\(M\) Matrix for the linear system
\(N_h\) Number of external shafts of HTC
\(N_n, N_s\) Number of nodes and shafts in the transmission system
\(T, T\) Torque, torque vector
\(\beta, \beta\) Prescribed value, prescribed value vector
\(\eta\) Efficiency
\(\theta = T/(\rho \cdot c_m)\)
\(\lambda, \mu\) Torque level factor, torque multiplication factor
\(\nu\) Speed ratio of the HTC
\(\xi, \psi\) Internal geometry coefficients
\(\rho\) Fluid density
\(\omega, \omega\) Rotational speed, rotational speed vector

Subscripts:

\(h\) Hydrodynamic torque converter (HTC)
\(p, t\) Pump shaft, turbine shaft (of HTC)
\(T, \omega\) Torque, angular velocity

Superscripts:

\(T\) Transpose
\(-1\) Inverse

1. INTRODUCTION

Hydrodynamic torque converters (HTCs) are commonly used transmission components, especially in vehicles. The most common application is in automatic vehicle transmissions. In general, in such an application the HTC is connected in series the rest of the driveline (engine – HTC – gear transmission – load). For those cases the analysis of the behaviour of the complete transmission system is a fairly easy and straightforward task. However, in some interesting transmissions, the HTC is not connected in series. This can be referred to as parallel connection of the HTC. In these designs, the analysis is more complicated.

Some works on the analysis of transmissions with an HTC connected in parallel can be found in the literature (e.g., Förster, 1954; Diederichs, 1956; Bloch and Schneider, 1960; Helfer, 1975). Those works only deal with special cases of HTC parallel connection. Methods of analysis for general transmission systems with an HTC have been presented by Andersson (1982, 1986) and Hedman (1988, 1992). In the present paper, these methods are compiled and presented (briefly) in a uniform way.

2. TRANSMISSION SYSTEM

Here, a transmission system is regarded as composed of transmission units, e.g. gear transmissions, bearings, planetary gear trains and clutches. The HTC is also a transmission unit, albeit with special properties. The shafts of the transmission units are connected in nodes. Thus, the speeds of all shafts connected to a node are equal. All this is exemplified in Fig. 1.

![Fig. 1. Automobile driveline composed of transmission units connected in nodes (1, 2,...,7)](image-url)
The HTC has non-linear relationships between the torques and speeds of its shafts. Other types of transmission unit have (more or less) linear relationships. Therefore, in general, the analysis is facilitated by treating the HTC separately from the rest of the transmission system. This is shown schematically in Fig. 2.

![Linear transmission system](image)

**Fig. 2. Separation of HTC from remaining, linear system**

The following assumptions will be made:
1. The shafts of the transmission units are parallel. Then, one sense of rotation is chosen as positive.
2. A positive torque has the same sense as a positive speed.
3. The transmission system is consistent, i.e. no parts are over-constrained. Possible inconsistencies can be found with methods in Hedman (1988).
4. Quasi-static cases, only, are considered.
5. There is only one HTC in the transmission system.

### 2.1 Linear Transmission System

The torque and speed relationships for most transmission units (not the HTC) are linear when the losses are neglected. As an example, Fig. 3 shows a simple gear transmission.

![Gear transmission](image)

**Fig. 3. Gear transmission: left: sketch; right: schematic form with positive sense of rotation (and torque)**

Now, the speed ratio $i = -z_2/z_1$ is introduced ($z$ is the number of teeth of a gearwheel). By neglecting the losses, the relationships for speed $w$ and torque $T$ can be written:

$$w_1 - i \cdot w_2 = 0$$  \hspace{1cm} (1)
$$i \cdot T_1 + T_2 = 0$$  \hspace{1cm} (2)

When the losses are considered, the torque equation is modified with the efficiency $\eta_T$, depending on the power flow direction:

$$T_1 \cdot w_1 > 0, \quad T_1 \cdot w_1 < 0$$  \hspace{1cm} (3)

Similar linear equations can be formulated for other types of transmission unit. A large collection is listed in Hedman (1988). Additional linear torque equations are given by torque equilibrium of the nodes. Finally, prescribed values of speed and torque can be included. The speed and torque relationships for the linear transmission system can then be written in the following way (Hedman, 1988):

$$M_\omega \cdot \omega = \beta_\omega \quad \omega^T = \{\omega_1, \omega_2, ..., \omega_{N_\omega}\} \quad (4)$$
$$M_T \cdot T = \beta_T \quad T^T = \{T_1, T_2, ..., T_{N_T}\} \quad (5)$$

Here, $N_\omega$ is the number of nodes, and $N_T$ is the number of shafts in the transmission system.

### 2.2 Hydrodynamic Torque Converter

The shafts of an HTC that are connected to other shafts in the transmission system are referred to as external shafts. Those shafts are connected to turbines inside the HTC. Normally, one of the external shafts is designed for power input, the pump shaft. The other shafts are turbine shafts. Fig. 4 shows some HTC designs.

![HTCs with 2, 3 and 4 external shafts](image)

**Fig. 4. HTCs with 2, 3 and 4 external shafts; p = pump, t = turbine, \nu = uni-directional clutch, \alpha = meridional fluid velocity**

Two things distinguish the relationships for the HTC from those of other types of transmission unit. Firstly, the HTC relationships are highly non-linear. Secondly, the speeds and torques are not uncoupled, as they are in Eqs. (1) and (2). Therefore, it is convenient to study the HTC separately. Generally, this is done in either of two ways. In the first way the internal geometry of the HTC is studied, and turbomachinery equations are applied. This will be referred to as the method of blade angles. In the second way, measured (or computed) characteristics of speeds and torques are used. Accordingly, this will be referred to as the method of characteristics.

HTC description with blade angles. The application of turbomachinery equations on an HTC can be concluded in the following way (Andersson, 1982; Hedman, 1988). First the fluid velocity $c_m$ in the meridional plane (cf. Fig. 4 left) is introduced. The turbines (including the pump) are numbered consecutively in the direction of $c_m$. Thus, a turbine $i - 1$ to be interpreted as the one that precedes turbine $i$. For the case $i = 1$, turbine $N_h$ is the preceding one ("$i - 1$")

$$T_i = c_m \cdot \rho \cdot \{[v_{i1} - \psi_{i-1} - \omega_{i-1} + (\xi_i - \xi_{i-1}) \cdot c_m]; \quad i = 1, 2, ..., N_h \} \quad (6)$$

Here, $\rho$ is the fluid density and $\psi$ and $\xi$ are coefficients that depend on the turbine geometry at the outlet of the turbine blades. An additional relationship for $c_m$ can be derived from an energy balance:

$$B_{k1} \cdot c_m^2 - 2 \cdot B_{k1} \cdot c_m - B_{k0} = 0 \quad (7)$$

Here, the $B$-coefficients depend on the turbine geometry and on the rotational speeds $\omega_i$ (i = 1, 2, ..., $N_h$). Eq. (7) can be rewritten as a matrix expression:

$$\omega^T \cdot B_1 \cdot \omega = 0 \quad (8)$$

Thus, if the angular speeds $\omega_i$ are known, $c_m$ can be obtained from Eq. (7) or (8). Then, the torques are given by Eq. (6).

HTC description with characteristics. In several applications, the HTC can be regarded as just a component from a supplier. For the designer of the entire transmission system, the internal geometry of the HTC is often uninteresting or even unknown. However, there are more convenient ways to describe its speed and torque properties. This requires that the number of external shafts of the HTC is low (2 or 3). Here, the case with 2 external HTC shafts will be shown. The generalization to 3 (or more) external shafts will be outlined briefly.

The 2 external shafts will be referred to as the pump shaft and the turbine shaft, with subscripts $p$ and $t$, respectively. The speed ratio $\nu$ of the HTC is defined as follows:

$$\nu = \omega_p/\omega_t \quad (10)$$
The interaction between the torques and speeds of the HTC can be described by the following relationships:

\[ T_i = -\mu_i \cdot T_p \]  \[ \mu = \mu(\nu) \]  \[ T_p = \lambda \cdot \rho D_i^2 \cdot \omega_p^2 \]  \[ \lambda = \lambda(\nu) \]

Here, \( D_i \) is a characteristic diameter of the HTC, usually the largest diameter of the flow torus inside the HTC. Moreover, \( \mu \) is the torque multiplication factor, and \( \lambda \) is the torque level factor. They are both non-dimensional. Typical curves for \( \mu \) and \( \lambda \) is shown in Fig. 5.

![Fig. 5. Curves of torque multiplication factor \( \mu \) and and torque level factor \( \lambda \) vs. speed ratio \( \nu \) for HTC of “Triok” type](image)

Torque converters with 3 external shafts are treated thoroughly in Hedman (1992). In such designs, there are 2 turbine shafts, \( t_1 \) and \( t_2 \). That results in 2 speed ratios and torque multiplication factors:

\[ n_1 = \omega_{t1}/\omega_p \]  \[ n_2 = \omega_{t2}/\omega_p \]

Furthermore, Eq. (12) can be used for the pump torque \( T_p \) when bearing in mind that \( \lambda \) is a function of both \( n_1 \) and \( \omega_p \) here.

### 3. METHOD OF BLADE ANGLES

When obtaining the speeds and torques for the entire transmission system, the relationships for the linear system in section 2.1 are “merged” with those for the HTC. Andersson (1982) has shown how that can be done for blade angle description of the HTC. That method has been adapted for computer implementation and extended to handle uni-directional clutches in Andersson (1986). Power losses in the linear system has been included by Hedman (1988). First, the HTC torques in Eq. (6) are divided by \( c_m \):

\[ \theta_i = \frac{\mu_i}{c_m} = \psi_i - \psi_{i-1} \cdot \omega_{i-1} + \left( \xi_i - \xi_{i-1} \right) \cdot c_m \; \; i = 1,2,...,N_h \]  

This can be regarded as a set of linear equations of the variables \( \psi \), \( \omega \) and \( \theta \). Furthermore, most equations from the linear system can be used with the \( T \)-variables substituted by corresponding \( \psi \). However, here are some exceptions. In those cases the right-hand side of the linear equation is a non-zero constant. Two examples are drag torques in bearings and non-zero prescribed torques. It is shown in Hedman (1988) how that can be handled. Now, Eq. (16) can be “spliced” with Eqs. (4) and (5) to form the following matrix equation:

\[ \mathbf{M}_T \cdot \Omega_T = \beta_0 \]  

\[ \Omega_T = \{ c_m, \psi_1, \psi_2, ... , \psi_{N_h}, \omega_1, \omega_2, ... , \omega_{N_h}, \theta_1, \theta_2, ... , \theta_{N_h} \} \]

\[ \beta_0 = \{ c_m, \beta_{c1}, \beta_{c2}, ... , \beta_{cN_h}, \beta_{s1}, \beta_{s2}, ... , \beta_{sN_h} \} \]

Here, the square matrix \( \mathbf{M}_T \) is composed of the elements of \( \mathbf{M}_c \) and \( \mathbf{M}_s \), cf. Eqs. (4) and (5). The vectors \( \Omega \) and \( \beta_0 \) are composed of the speed and torque variables and the right-hand sides in Eqs. (9), (4) and (5). The first row in Eq. (17) is a simple identity: \( c_m = c_m \).

Now, in Eq. (8) the vector \( \omega_A \) can be replaced by \( \Omega \) by simply enlarging the size of the matrix \( \mathbf{B}_n \):

\[ \Omega^T \cdot \mathbf{B}_n \cdot \Omega = 0 \]  

From Eq. (17) \( \Omega \) can be expressed as:

\[ \Omega^T = \mathbf{M}_n^{-1} \cdot \beta_0 \]

Insertion in Eq. (20) gives:

\[ \beta_0^T \cdot \mathbf{M}_n \cdot \beta_0 = 0 \]

This is, in fact, a second-order algebraic equation in the variable \( c_m \). When this equation has been solved, the speeds and torques are obtained from Eq. (21).

### 4. METHOD OF CHARACTERISTICS

The HTC characteristics can be “merged” with the relationships of the linear transmission system. For the common case with 2 external shafts, a method of analysis has been presented in Hedman (1988). A more general method can be found in Hedman (1992). Those methods will now be presented briefly. However, it should first be noted that the action of possible uni-directional clutches in the HTC does not need to be handled explicitly. It is included in the characteristics.

The linear speed and torque relationships are “assembled” in Eqs. (4) and (5). The non-linear HTC relationships are given by Eqs. (11)–(15). The number of HTC relationships is equal to the number of external shafts of the HTC, \( N_h \). It can be shown that \( N_h \) linear equations for the speeds and/or torques of these external shafts can be derived, or “extracted”, from Eqs. (4) and (5). Fig. 6 shows an obvious (but not very realistic) example, where the speed ratio of the HTC is determined by the linear transmission.

![Fig. 6. HTC speed ratio implicitly determined by gear transmissions](image)

Depending on how many prescribed speeds and prescribed torques there are in the transmission system, different cases arise. Those cases, or types of governing, will now be investigated.

#### 4.1 Speed Governing

With many prescribed speeds, the number of linear speed equations can be equal to the number of speed variables. Then, Eq. (4) can be solved, yielding numerical values of all the speeds. The extracted linear relationships will be on the form \( \omega_p = 278 \; \text{rad/s} \). In this case, the HTC is said to be speed governed. When the speeds of the external HTC shafts are known, the corresponding torques can be obtained from the HTC relationships, Eqs. (11)–(15). These torque values can be used as additional “prescribed” torques in Eq. (5). The number of linear torque equations is thereby made equal to the number of torque variables. Thus, Eq. (5) can be solved. The result is the torques of the shafts of the transmission system.

#### 4.2 Torque Governing

It is possible to solve Eq. (5) if the number of linear torque equations is equal to the number of torque variables. That will give numerical values of the torques. Then, by using Eqs. (11)–(15) “backwards”, the HTC speeds can be obtained. Insertion in Eq. (4) and solving it gives the speeds of all shafts.
4.3 Mixed Types of Governing

In some cases, there is at least one speed equation and at least one torque equation among the extracted equations. Then, neither Eq. (4) nor Eq. (5) can be solved directly, independently of the HTC relationships. Special treatment is then required. For an HTC with 2 external shafts, the extracted equations are, in general form:

\[ a_{wp} \cdot \omega_p + a_{rt} \cdot \omega_t = b_0 \]  \hspace{1cm} (23)
\[ a_{T_p} \cdot T_p + a_{T_t} \cdot T_t = b_T \]  \hspace{1cm} (24)

These equations can be interpreted as straight lines in the \( \omega_p-\omega_t \) and \( T_p-T_t \)-planes. They will be referred to as the system speed line and the system torque line, respectively. Some special cases of Eq. (23) are:
- \( b_0 = 0 \): \( \nu = \omega_t/\omega_p \) is implicitly determined, cf. Fig. 6.
- \( a_{wp} = 0 \): \( \omega_t \) is implicitly determined by prescribed speeds.
- \( a_{rt} = 0 \): \( \omega_t \) is implicitly determined by prescribed speeds.

Corresponding special cases can be listed for Eq. (24).

The solution procedure is as follows. By means of the HTC relationships, the system speed line can be transformed to the \( T_p-T_t \)-plane. The intersection with the system torque line, if any, is the point of solution. This is shown in Fig. 7.

![Fig. 7. Transformation of system speed line to \( T_p-T_t \)-plane, including point of solution \( (\beta_{T_p},\beta_{T_t}) \)](image)

Now, with known values of the speeds and torques of the HTC shafts, Eqs. (4) and (5) can be “filled” and solved.

For HTCs with 3 (or more) external shafts there are similar procedures. Those are described thoroughly in Hedman (1992). For the case of 2 extracted speed equations and 1 extracted torque equation, Fig. 8 illustrates the procedure.

![Fig. 8. Solution of mixed governing for HTC with 3 external shafts](image)

5. CONCLUSIONS

Hydrodynamic torque converters (HTCs) have non-linear relationships for speeds and torques. Most other transmission units have linear speed and torque relationships. Therefore, it is of advantage to treat the HTC separately at the analysis. The HTC behaviour can be modelled in two ways. In the first way, the internal geometry is studied, and turbomachinery equations are applied. In the second way, the HTC is treated as a “black box” with known speed and torque characteristics. For both of these ways, methods have been presented that combine the HTC relationships with the linear equations of the rest of the transmission system. Thereby, it is possible to obtain the speeds and torques of all shafts in the transmission system.

REFERENCES


APPENDIX

A Numerical Example

The very simple transmission system in Fig. 6 will now be analysed with the method of characteristics. In order to simplify the analysis, the two gear transmissions will be combined into a single one. The simplified transmission system is shown in Fig. 9.

![Fig. 9. Simplified schematic form of transmission system in Fig. 6 with labelled shafts and nodes (1, 2)](image)

**Input data.** The HTC characteristics of Fig. 5 will be used along with\( D_h = 0.25 \text{ m} \) and \( \rho = 800 \text{ kg/m}^3 \). The speed ratio of the gear transmission is \( \nu = \omega_{g2}/\omega_{g1} = 2.5 \). Prescribed torques are:

\[ T_{\text{input}} = \beta_{T_{\text{input}}} = 100 \text{ Nm} \]  \hspace{1cm} (25)
\[ T_{\text{output}} = \beta_{T_{\text{output}}} = -200 \text{ Nm} \]  \hspace{1cm} (26)

**Speed relationships.** The speeds of the nodes are speed variables:

\[ \omega_1 = \omega_{\text{input}} = \omega_p = \omega_{g1} \]  \hspace{1cm} (27)
\[ \omega_2 = \omega_{\text{output}} = \omega_t = \omega_{g2} \]  \hspace{1cm} (28)

The only linear speed equation is Eq. (1) for the gear transmission. Then, the matrix equation (4) will be as follows:

\[ \begin{bmatrix} 1 & -\nu \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} = 0 \]  \hspace{1cm} (29)

**Torque relationships.** First, the losses are neglected for the gear transmission. With the notation in Fig. 9, Eq. (2) will be:

\[ \nu \cdot T_{g1} + T_{g2} = 0 \]  \hspace{1cm} (30)

**Torque equilibrium for the nodes gives:**

\[ T_{\text{input}} = T_{g1} + T_p \]  \hspace{1cm} (31)
\[ T_{\text{output}} = T_{g2} + T_t \]  \hspace{1cm} (32)
Eqs. (30)–(32) can now be arranged on matrix form, corresponding to Eq. (5):

\[
\begin{bmatrix}
0 & 0 & 0 & i_p & 1 \\
1 & 0 & -1 & 0 & 1 \\
0 & 1 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
T_p \\
T_t \\
T_{\text{input}} \\
T_{\text{output}} \\
\beta_{T_{\text{in}}} \\
\beta_{T_{\text{out}}} \\
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

(33)

**Solution.** It can be seen in Eq. (29) that there are 2 speed variables and 1 linear speed equation. Similarly, in Eq. (33) there are 6 torque variables and 5 linear torque equations. Hence, neither of the matrix equations can be solved at this stage. According to section 4.3 there is a mixed governing of the HTC. Then, one speed equation and one torque equation can be extracted as the system speed line and system torque line, respectively. For Eq. (29) this is trivial:

\[
\omega_1 - i_p \cdot \omega_2 = 0 \Rightarrow \omega_p - i_p \cdot \omega_1 = 0
\]

(34)

By means of Gaussian elimination in Eq. (33), the following linear relationship for the torques of the HTC shafts is found:

\[
i_p \cdot T_p + T_t = i_p \cdot \beta_{T_{\text{in}}} + \beta_{T_{\text{out}}}
\]

(35)

The zero right-hand side in Eq. (34) verifies the obviously constant HTC speed ratio, Eq. (10):

\[
\nu = \omega_1/\omega_p = 1/i_p = 1/2.5 = 0.4
\]

(36)

In Fig. 5 this gives:

\[
\mu \approx 1.5 \quad \lambda \approx 0.003
\]

(37)

With a constant torque multiplication factor \(\mu = -T_t/T_p\), the transformed system speed line will be a straight line in the \(T_p-T_t\)-plane. This is shown in Fig. 10.

**Fig. 10. Obtaining point of solution for transmission system in Fig. 9**

Thus, the point of solution is:

\[
T_p = \beta_{T_p} \approx 50 \text{ Nm} \quad T_t = \beta_{T_t} \approx -75 \text{ Nm}
\]

(38)

The corresponding speeds are obtained with Eqs. (12) and (36):

\[
\omega_{\text{input}} = \omega_p = 146 \text{ rad/s} \quad \omega_{\text{output}} = \omega_t \approx 58.4 \text{ rad/s}
\]

(39)

Finally, Eq. (33) can be “completed” with either of the parts of Eq. (38). Solving this matrix equation gives the other torques:

\[
T_{g1} \approx 50 \text{ Nm} \quad T_{g2} \approx -125 \text{ Nm}
\]

(40)

**Losses.** An algorithm from Hedman (1988) to include the torque losses in the gear transmission will now be used. For the efficiency, the value \(\eta_T = 0.98\) will be assumed.

Eqs. (39) and (40) give \(T_{g1} \cdot \omega_{g1} > 0\), i.e. a power flow from shaft \(g1\) to shaft \(g2\) in the gear transmission. This indicates that the left variant of Eq. (3) should be used. Then, the new version of Eq. (33) will be:

\[
\begin{bmatrix}
0 & 0 & 0 & i_p & 1 \\
1 & 0 & -1 & 0 & 1 \\
0 & 1 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
T_p \\
T_t \\
T_{\text{input}} \\
T_{\text{output}} \\
\beta_{T_{\text{in}}} \\
\beta_{T_{\text{out}}} \\
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

(41)

The system speed line is not affected by the torque losses. Eqs. (34), (36) and (37) still hold. Elimination in Eq. (41) gives the new system torque line:

\[
\eta_T \cdot i_p \cdot T_p + T_t = \eta_T \cdot i_p \cdot \beta_{T_{\text{in}}} + \beta_{T_{\text{out}}}
\]

(42)

This line intersects the transformed system speed line, cf. Fig. 10, at the following point:

\[
T_p = \beta_{T_p} \approx 47.4 \text{ Nm} \quad T_t = \beta_{T_t} \approx -71 \text{ Nm}
\]

(43)

This corresponds to:

\[
\omega_{\text{input}} = \omega_p \approx 142 \text{ rad/s} \quad \omega_{\text{output}} = \omega_t \approx 57 \text{ rad/s}
\]

(44)

Finally, the torques on the shafts of the gear transmission are:

\[
T_{g1} \approx 52.6 \text{ Nm} \quad T_{g2} \approx -129 \text{ Nm}
\]

(45)

Still, the power flows from shaft \(g1\) to shaft \(g2\) in the gear transmission; \(T_{g1} \cdot \omega_{g1} > 0\). Thus, the use of the left variant of Eq. (3) was correct.