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# ON THE USE OF DIRECT AND INVERSE NUMERICAL FLOW CALCULATIONS FOR SUPERSONIC TURBINE DESIGN

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## ABSTRACT

A procedure for blade design, using a time marching method to solve the Euler equations in the blade-to-blade plane is presented. This procedure uses an Office Nationale d'Etude et de Recherches Aeronautique flow solver. The classical slip conditions (no normal velocity component along the blade profile) has been replaced by another boundary conditions in such a way that the required pressure may be imposed directly. The orignal direct code was therefore transformed into an inverse solver. The unknows are calculated on the blade wall using the so-called compatibility relations. The blade geometry is then modified by resetting the wall parallel to the new flow field. The results obtained with this design process for a supersonic turbine blade of a space turbopump is presented.

## I INTRODUCTION

The design of more efficient blades is a constant challenge for turbine aerodynamic designers. It is usually done by successive improvement of the blade geometry and verification by tests or flow calculations. This process is very expensive in time and money. In the past, analytical blade design methods have been developed. The inverse method for potential flow has been solved by conformal mapping (Lightill, 1945; Schwering, 1970; Papaillou, 1968), singurality (Murugesan and Railly, 1969; Ubaldi, 1984) or characteristics methods (Goldman and Scullin, 1948). These methods are based on several assumptions and, consequently they do not give an optimal blade design for complex flow.[1] They are useful for a first guess of the geometry but this initial blade design must be modified with much more complex calculations. This is particularly true for supersonic or transonic flowfields where shock waves generate strong discontinuities. The conclusion of several authors is that the resolution of non potential flow fields requires the Euler equations (Leonard and Van den Braembussche, 1992a; Meauze and Lesain, 1980 and Meauze, 1986). Neverthless, Euler inverse design methods need realistic blade boundary conditions, compatible with an existent solution. So, these methods must be incorporated into a complete turbine design process.

This paper deals with the complete design process wich can be applied to either subsonic and supersonic turbine blades. In the first step, we use a potential method based on characteristic theory. This method gives a first "ideal" blade profile. Then, we use a direct Euler calculation to define the flow field around this blade. This

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**Nomenclature**

M1= Inlet Mach number.
P ts= Total to static pressure ratio.
P tt= Total to total pressure ratio.
b1= Inlet flow angle.
b2= Outlet flow angle.
Po= Total inlet pressure.
To= Total inlet temperature.

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step gives the complete flow conditions particularly the pressure distribution on the blade. In the final step, the initial geometry is modified: we impose a new pressure distribution on a part of the blade (pressure or suction side) to adjust the blade profile. The last two steps (direct and inverse Euler calculations) can be repeated until the optimum blade profile is reached.

## II FIRST STEP: POTENTIAL INVERSE DESIGN.

The inverse method used to define a first guess of the blade geometry is based on an approach described by Goldman and Scullin (1948). This method was developed for the design of two dimensional supersonic rotor blade sections cor-rected for boundary layer displacement thickness. The ideal rotor blade is designed by the method of characteritics to produce vortex flow within the blade passage.

## III SECOND STEP: EULER CALCULATIONS.

An advantage of the present method is that the same code is used for direct and inverse calculation. An existing analysis code was transformed into a design code by changing the blade boundary conditions. A special algorithm was added to modify the blade geometry.

A consequence is that the two methods (direct and inverse) use the same nume-rical scheme which is presented now.

## THE EULER SOLVER.

A code solving the Euler equation was developed by ONERA (Viviand and Veuillot, 1978, Cambier et al 1988). This code has been introduced in our turbine design process.

The system of Euler equations for unsteady flows is solved using a time marching procedure in a finite difference approach. This numerical scheme has the following properties: the Euler equations are discretized on the physical plane. The Mac Cormark predictor-corrector formulation is used. Artificial viscosity terms are introduced in order to avoid numerical instabilities.

The numerical domain is discretized using C grids (figure 2). This kind of grid provides a good description of the turbine rounded leading edges. The feature is of primary interest in supersonic flow to take into account the leading edge shock waves.

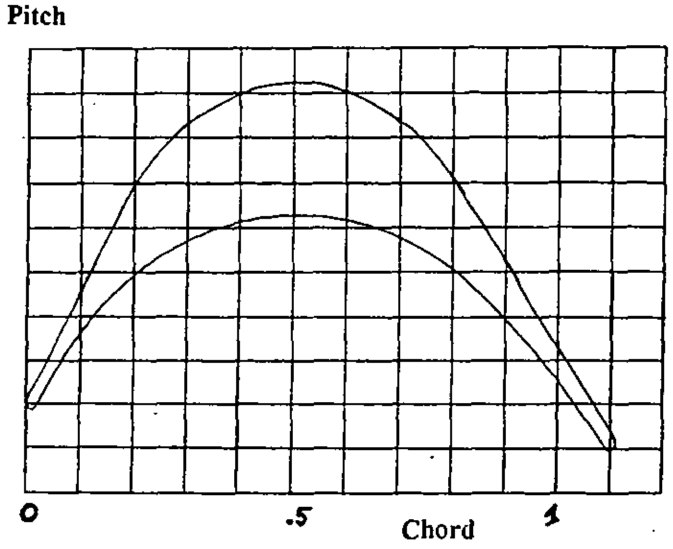

**Fig. 1 Original blade geometry.**

On the other hands, a physical rounded trailing edge creates non physical accelerations in an inviscid solution (Leonard and Van den Braembussche (1992b). So, the blade geometry is discretized in a more suitable way by a sharp trailing edge.

## BOUNDARY CONDITIONS.

Boundary conditions are treated by compatibility relations derived from the theory of characteristics. On the blade (solid walls), boundary conditions consist in imposing the normal velocity equal to zero. All the other aerodynamic values, such as Mach number, pressure and temperature, are computed.

Conversely, in the inverse mode, we assign a local static pressure and we compute all the other values by using compatibility relations. Leonard and Van den Braembusshe (1992a) show that it is possible to impose a pressure distribution only if the wall is considered as permeable. Imposing simultaneously the pressure and the slip condition on a solid boundary leads to an ill-posed problem from a mathematical point of view. This means that the blade is not modified during inverse calculation. With this technique, we obtain a large decrease of the CPU time because the grid is not modified for each iteration. Nevertheless, we do not calculate the exact solution and we need a direct calculation to validate the blade profile modification and obtain an exact flowfield solution.

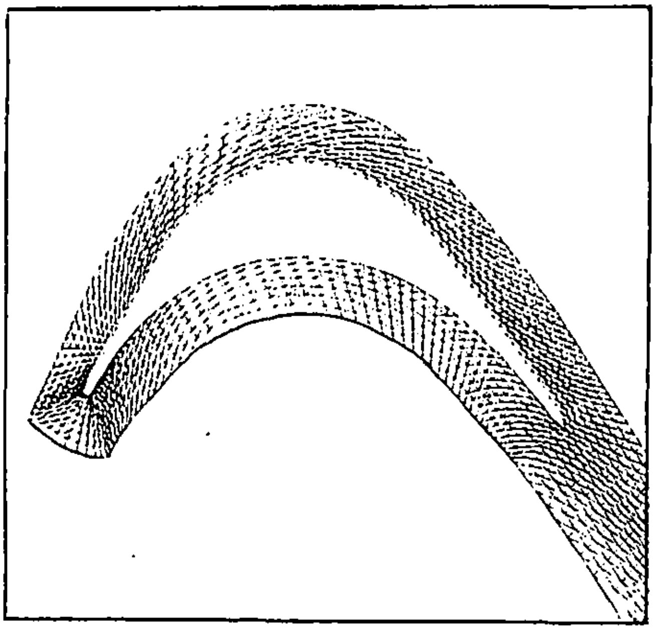

Fig. 2 - C grid discretization.

On the inlet boundary conditions, the theory of characterictics shows that (for a subsonic axial velocity) three boundary conditions must be imposed. We choose the classical inlet total conditions (Po and To) and the inlet flow angle.

Concerning the outlet conditions, we use a non reflecting condition which is, according to Viviand and Veuillot (1978), the most appropriate condition for supersonic flow with strong discontinuities like shock waves. This condition is associated (for a subsonic axial outlet velocity) to a condition concerning the mean static pressure level in the outlet boundary.

## IV THIRD STEP: OPTIMIZATION OF THE PROFILE.

As said before, the major changes in the inverse method concern the boundary condition on the blade. The new geometry is calculated using the new velocity field associated with the slip condition. The geometry modification algorithm calculates the position of the new boundary points from upstream to downstream of the cascade on the pressure or the suction side. In order to avoid discontinuity on the new profile, a smooth function was added. This smooth function is necessary in supersonic flows when shocks arise on the profile. It can create little blade modifications upstream or dowstream shock locations.

## CONSTRAINED BLADE DESIGN

The inverse design problem become even more complex if mechanical or geometrical restriction are introduced, since they may be incompatible with the aerodynamic target. Usually inverse methods impose an aerodynamic conditions (pressure or velocity distribution) on one side and the thickness distribution (Meauze, 1986; Cambier et al, 1988). But, only one side of the blade can be optimized. Leonard and Van den Braembussche (1992b) propose one another numerical procedure: the final blade design is a compromise between aerodynamic targets and mechanical restrictions. Nethertheless the required Mach number distribution must be far from being compatible with the geometrical constraints.

All these technics need an active contribution of the designer and do not allow a complete control of the blade profile. In the present method, mechanical and geometrical constraints are not introduce: only pressure distribution are imposed on the blade. The geometrical conditions (essentially leading and trailling edge thickness) are directly imposed and controlled by the designer. Meauze and Lesain (1980) demonstrated that, in most applications, the blade obtained then imposing simultaniously aerodynamic conditions on pressure and section side are not realizable. Consequently, blade modifications are limited to one side so blade thickness can be better controlled. If we want to modify the two sides, we have to execute two succesive calculations. In order to avoid no existant solution, we have to impose a physical pressure distribution on the profile. In particular, the blade surface defined by this pressure distribution must be closed.

Leonard and Van den Braembussche (1992a) show that the required pressure must also be in accordance with the aerodynamic conditions upstream and downstream of the blade. Consequently, we begin the inverse calculation from existing blades and we modify the calculated pressure distributions on these original geometries. The same numerical algorithm is used for the direct and the inverse calculation. So, the two methods are consistent.

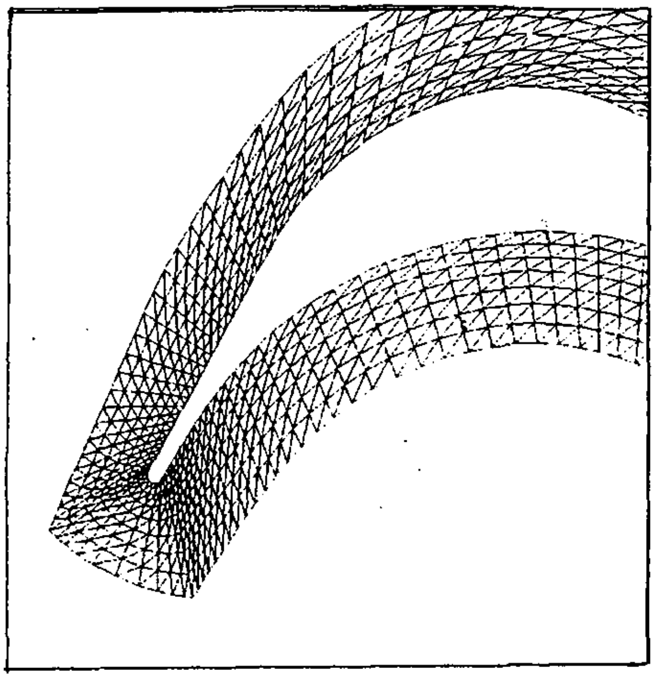

Fig 3. - Leading edge meshes.

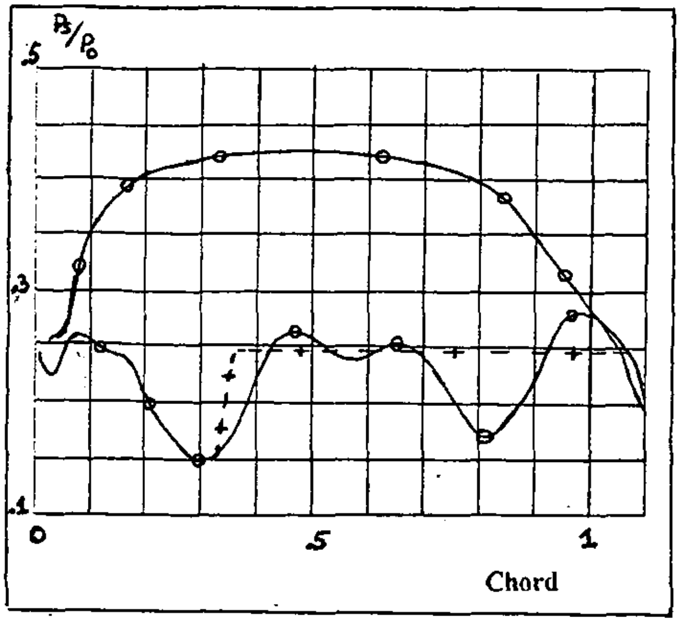

Fig 4 -Pressure distribution calculated (-o-) and prescribed ( + ) on the blade.

## V RESULTS

We will now present an application of this method on a supersonic turbine blade.

The aerodynamic conditions, in the relative frame, are:

$M_1=1.24$,
$P_{ts}=0.356$
$P_{tt}=0.71$
$b_1=64^\circ$
$b_2=53.7^\circ$

Figure 1 shows the original blade obtained with the potential method des-cribed in the first part of this paper.

A first direct calculation was made with the Euler solver. Figure 2 presents the grid mesh used for this calculation. The C grid have 1296 mesh points (8*162) around the blade. Figure 3 shows the leading edge mesh.

A convergence level of $10^{-4}$ was obtained on residuals, after 7000 iterations. This level of convergence needs 38 minutes on a SUN Sparc II station ($2.10^{-4}$ sec/iterations * mesh points).

Figure 4 shows the static pressure on the calculated blade. When we analyse these results, we note two peaks on the suction side pressure distribution on the blade. These peaks are a consequence of the acceleration on the suction side followed by shocks. They could be supressed by a new suction side design. Nevertheless, we choose first to modify only the second pressure rise because we want to test the capability of the method on a part of the blade surface and also because we can assure a better control of the blade thickness if we limit the modifications for each calculation. The first peak could be supress by a second inverse calculation.

We impose a new required pressure distribution on the suction side profile. This distribution is shown on figure 4. The objective is to reduce the suction side acceleration and in consequence suppress the region of high velocity.

The last point concerns the blade chord. We note on figure 4 that the last 10% of the blade calculated by Goldman's method does not create a pressure rise. We can expect that with a imposed pressure distribution, the calculated blade will be shorter.

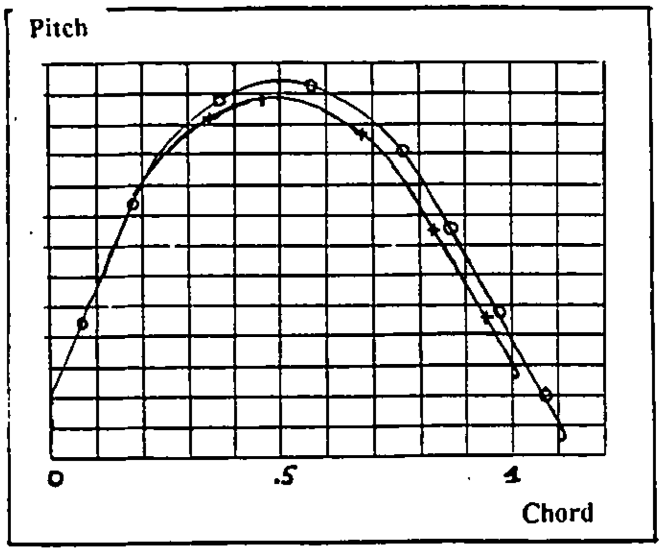

**Fig 5 Initial and corrected blade profile, suction side.**
**-o- Initial, + Corrected**

The blade profile on figure 5 has been determined by the inverse methode using the pressure distribution of figure 4. We must point out that the inverse method was applied only on the suction side. The suction side was calculated as a solid wall.

The new profile is less cambered than the older. That means a lower acceleration on the blade which is in agreement with the required pressure. We can note that the newly designed blade deviates from the original blade profile in a zone where the required pressure distribution was not modified. This phenomena is a consequence of the smooth function presented before.

As expected, the new profile is shorter than the older because the computed surface intersect the pressure surface. A new trailing edge must be construct at the pressure and suction side intersection.

A verification calculation was made on the new profile. A new blade was defined and a mesh grid construct. A direct Euler calculation was executed on this geometry. Figure 6 shows the grid used for this calculation.

The pressure distribution calculated on the blade profile is shown on figure 7. This pressure distribution is not exactly the same as the values imposed in the inverse calculation. But, this result confirms the removal of the peak velocity on the suction side.

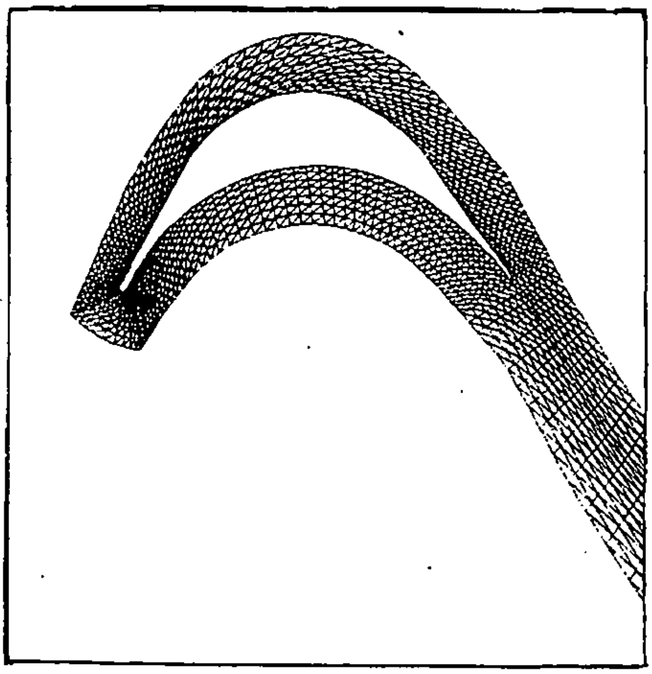

**Fig 6 - C grid on the new profile.**

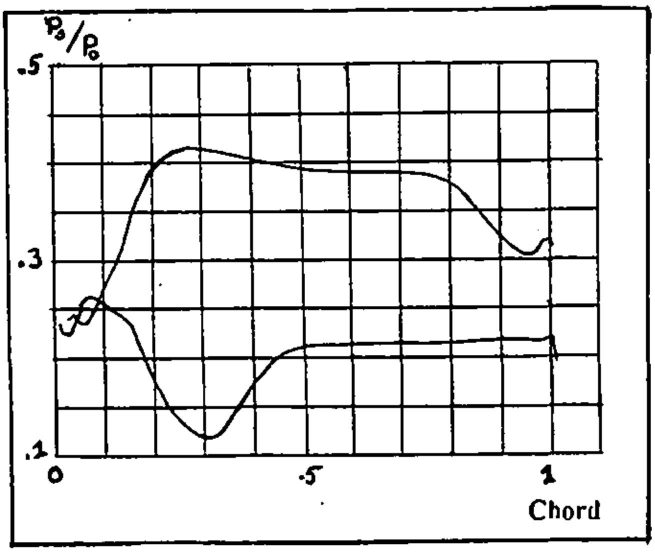

**Fig 7 - Pressure distribution calculated on the new profile.**

The differences between the prescribed and the calculated pressure distribution is a consequence of boundary conditions presented before: we do not modify the blade during the inverse calculation. So, we do not calculate exactly the same domain with the two methods.

Neverthless this validation demonstrates the profile modification required by the inverse method.

## VI CONCLUSION

A procedure for blade design has been presented. It is composed of a potential inverse method, a direct Euler solver and an inverse Euler method. An application of the procedure for the design of supersonic blade rotor has been presented. This procedure provides a simple and efficient tool for designing high-loaded blade geometry.

## AKNOWLEDGMENTS.

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