GENETIC ALGORITHM BASED STRATEGIES FOR RADIAL FLOW IMPELLER DESIGN

Raffaele Tuccillo  Adolfo Senatore

Dipartimento di Ingegneria Meccanica per l'Energetica (D.I.M.E.)
Università di Napoli "Federico II"
Napoli (Italy)

ABSTRACT

The authors present a method for radial flow impeller design which pays attention to the set-up of optimal design strategies, the latter based upon the employment of a genetic algorithm which controls a Quasi-3D flow simulation of a radial flow impeller, with integral boundary layer calculation.

The design procedure develops through a number of steps, which are characterized by a progressive reduction in the search interval of each independent variable. In addition, a more accurate definition of the blade geometry and a more reliable flow analysis is obtained through a mesh refinement in the flow model, when the whole optimization process is close to convergence.

The results demonstrate the possibility of achieving an optimal choice of the impeller geometrical parameters by approaching, in a first phase, the design data, and then by proceeding with a number of refinement steps, as outlined above. In this way, the accuracy of the whole procedure is strongly increased, so that the typical effectiveness of the genetic algorithms in the preliminary selection steps is combined with a detailed investigation of the impeller blade features in the final steps of the optimized design. The effectiveness becomes, therefore, comparable with that of classical gradient based methods of optimization. At the same time, the genetic algorithm is capable of handling discontinuities in the objective function, thus overcoming the typical limits of numerical methods.

In order to emphasize the effectiveness of the proposed procedure, the results refer to a wide variety of design data. Both the impeller configurations and the flow distributions found demonstrate that the final solution is in agreement with the design target and constraints.

NOMENCLATURE

\(\mathbf{\mathcal{H}}\)  
Boundary layer form factor

\(\mathcal{H}_R\)  
Rothalpy

\(\mathcal{H}_T\)  
Total-to-total adiabatic head

\(k_1, k_2\)  
Coefficients for blade angle law

\(L_q\)  
"Quasi-Orthogonal" length

\(m\)  
Meridional coordinate

\(m\)  
Mass flow rate

\(M\)  
Mach Number

\(n\)  
Rotational speed

\(N_s\)  
Specific Speed

\(p\)  
Pressure

\(P\)  
Penalty factor

\(q\)  
"Quasi-Orthogonal" coordinate

\(r\)  
Radial coordinate

\(R\)  
Reaction degree

\(T\)  
Temperature

\(V\)  
Peripheral velocity

\(V\)  
Absolute velocity

\(V, V_r\)  
Volume flow rate

\(V_x, V_r\)  
Variation in axial or radial coordinate

\(W\)  
Relative Velocity

\(x\)  
Axial coordinate

\(\mathcal{x}\)  
Vector of independent variables

\(y\)  
Boundary layer coordinate

\(Z\)  
Number of blades

Greeks

\(\beta_0\)  
Blade angle referred to axial and radial directions

\(\beta_{\Pi}\)  
"Total-to-total" pressure ratio

\(\delta^*\)  
Displacement thickness

\(\eta_{\Pi}\)  
"Total-to-total" efficiency

\(\delta\)  
Peripheral coordinate

\(\delta^*\)  
Momentum thickness

\(\lambda\)  
Lagrange multiplier

\(\xi_{split}\)  
Non-dimensional coordinate of splitter blade

\(\rho\)  
Density

\(\omega\)  
Angular velocity

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based on the solution of the relative motion equation:

\[ \mathbf{W} \times (\nabla \times \mathbf{W}) - 2\mathbf{\omega} \times \mathbf{W} = \nabla \mathbf{H_R} - \nabla \mathbf{s} \]  

(1)

together with those for mass and rothalpy conservation:

\[ \nabla \cdot \mathbf{H_R} = 0 \]

\[ \nabla \cdot (\rho \mathbf{W}) = 0 \]  

(2)

The solution is iteratively performed on through-flow and blade-to-blade surfaces, following the quasi-orthogonal approach (Katsanis, 1966, 1977) and the Stanton approximation for potential flow.

The dissipative effects along both end-wall and blade surfaces evaluated through the integral boundary layer equations:

\[ \frac{d\mathbf{H}^*}{dm} = \frac{1}{\delta^*} \mathbf{F}(\mathbf{H}^*) - \frac{1}{\delta^*} \mathbf{H}^* \frac{d\mathbf{W}_e}{\mathbf{W}_e} - \frac{d\delta^*}{dm} \]  

(3)

The velocity profile inside the boundary layer is assumed to be of the type:

\[ \frac{\mathbf{W}}{\mathbf{W}_0} = \left(\frac{\gamma}{\delta} \right)^{\frac{1}{2}} ; \quad \mathbf{H} = \frac{\delta^*}{\delta} \]  

(4)

so yielding:

\[ \mathbf{H}^* = \frac{\delta - \delta^*}{\delta} = 2\mathbf{H} \]  

(5)

In order to describe the \( \mathbf{F}(\mathbf{H}^*) \) function, the Lieblein (1965) approach is followed for the blade surface boundary layer, while the end-wall boundary layer is modelled by the De Ruick and Hirsch (1981) relationships. The friction coefficient, \( c_f \), is evaluated either by the original Ludwieg and Tillman equation or by that modified by Annand and Lakshminarayana (1975).

An iterative solution scheme is adopted in order to account for the interactions between the potential and the viscous flow regions along both through-flow and blade-to-blade surfaces. In fact, decelerations in the potential flow field affect the boundary layer growth according to eqs. (3 - 5), while the boundary layer blockage produces alteration in the velocity level throughout the inviscid region.

The flow model above summarized, although being of a simplified type if compared with a fully 3D viscous simulation, allows a satisfactory evaluation of the local flow and blade loading conditions and is characterized by a good sensitivity to changes in impeller geometry. The Eckardt’s definition (1976) of load coefficient was assumed:

\[ C_L = \frac{W_\text{tot} - W_\text{ps}}{W} \]  

(6)

The mass, momentum and energy balances at the impeller discharge, as a result of both the outflow distribution and the boundary layer thickness, provide average values for absolute velocity and flow angle, \( V_2, \alpha_2 \) and static pressure and temperature, \( \bar{p}_2, \bar{T}_2 \).

Therefore the impeller total-to-total pressure ratio and adiabatic efficiency may be evaluated (Senatore and Tuccillo, 1990):

\[ \beta_n = \frac{\bar{p}_{2t}}{\bar{p}_{01}} ; \quad \eta_{\text{ad}} = \frac{\bar{T}_{2t}}{\bar{T}_{01}} - 1 \]  

(7)

Hence the impeller total-to-total pressure ratio and adiabatic efficiency may be evaluated (Senatore and Tuccillo, 1990):

\[ \beta_n = \frac{\bar{p}_{2t}}{\bar{p}_{01}} ; \quad \eta_{\text{ad}} = \frac{\bar{T}_{2t}}{\bar{T}_{01}} - 1 \]  

(7)
Total performance parameters in eqs. (7) result from both the mechanical energy transfer and total pressure losses. The latter are due to both the boundary layer momentum thickness and to incidence losses which are estimated as a function of the difference between flow and blade angles at compressor inlet.

In addition, the isentropic reaction degree is assumed as a criterion to evaluate the increase in static pressure, with respect to the total mechanical energy transfer through the impeller:

\[ R = \frac{\left( \frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1}{\beta_{in}^{\gamma} - 1} \]  

(8)

**IMPELLER GEOMETRY DEFINITION**

The impeller geometry is defined by the parameters listed in table I which represent, together with the rotational speed, the independent variables of the optimizing process. Some of these parameters affect more significantly the performance level and the mass flow rate range, while others do influence the flow and load distribution.

For instance, the dimensional factor, \( F_{dim} \) allows the scaling of all the main impeller coordinates \( \left( x_1, \ldots, x_9 \right) \) in fig. 1), to be scaled in order to obtain geometrically similar impellers of different sizes. A proper selection of this parameter allows the impeller size to meet the mass flow delivery requirement. The rotational speed \( n \) itself, whose admissible range is strongly influenced by the dimensional factor, affects the amount of mechanical energy transfer and, therefore, the pressure ratio level. In this paper, the rotational speed was included in the independent variable list, while in previous authors' papers (1994) examples were presented by referring to given values of \( n \).

More detailed variations in the impeller geometry are produced by varying the impeller leading or trailing edge coordinates, according to the \( \left( V_{XC}, \ldots, V_{rD} \right) \) parameters in table I. A variation in a single coordinate produces, of course, non-similar impellers and allows a more appropriate choice of a number of impeller geometrical features, like the blade height at leading and trailing edges, tip diameter, etc. A better control can therefore be exerted on constrained parameters like relative and absolute Mach numbers and tip speed.

The impeller shroud and hub profiles are defined through the \( \left( c_{h}, d_{h} \right) \) and \( \left( c_{b}, d_{b} \right) \) coefficients which affect the length of the elliptic and straight lines, so defining the slope and curvature of the meridional streamlines and affecting the related flow field (Senatore and Tuccillo, 1991, 1994, 1995).

Similarly, the inlet tip blade angle \( \beta_{1bs} \) and the blade exit angle \( \beta_{2b} \) affect both the operating range of volume flow rate and the energy transfer level. The blade definition is completed by the \( k_1 \) and \( k_2 \) coefficients which determine the minimum value of the blade angle and its location in the meridional path \( (h_{min} = \beta_{2bs}/k_2 \) at \( \theta = (\pi/2)/k_1 \) in fig. 1), respectively. A quadratic law is assumed for the blade angle development from the leading to trailing edge. The hub-to-tip blade angle distribution at blade inlet follows a forced vortex law.

The number of full blades, \( Z \), is included in the independent variable list and affects, in particular, the aerodynamic load conditions. A further contribution to the search of the optimal impeller configuration is given by the \( \xi_{split} \) variable which represents the non-dimensional coordinate of the splitter blade leading edge along the meridional path. It is self evident that a \( \xi_{split} \) value greater than 1 indicates the absence of splitter blades, while a value close to zero suggests the necessity of doubling the number of blades.

The authors' choice for the impeller geometrical parameters was determined by the need of introducing a limited number of independent variables into the optimization and, on the other hand, by the requirement of a good sensitivity to the features of the flow model. Really, the geometrical variables listed above seem to affect both the impeller performance level and the local flow distribution as well. In this way a reasonable evaluation of the design target and constraints may be performed.

The initial values of the parameters in table 1 refer to a reference configuration which allowed an experimental validation of the flow model accuracy (Senatore and Tuccillo, 1990). In particular, the evaluation of the interaction between the inviscid and the viscous flow region leads to a correct estimation of the static pressure increase along the flow path.

**OBJECTIVE FUNCTION FORMULATION**

The purpose of the optimizing process is the achievement of the maximum in total-to-total adiabatic efficiency, \( \eta_{tt} \), for given inlet flow conditions and pressure ratio level \( \beta_{in}^{\gamma} \). The objective function is therefore defined by:

\[ f(X) = 1 - \eta_{tt} \]  

(9)

\( X \) being the independent variable vector as listed in table I. In addition a number of equality and inequality constraints must be simultaneously satisfied. The first one is given by the prescribed pressure ratio level:

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**Table I. Independent Variables of the Optimization**

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Genetic Optimization Notation</th>
<th>Initial Value</th>
<th>Initial Search Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{dim} )</td>
<td>81</td>
<td>1</td>
<td>1/16 + 16</td>
</tr>
<tr>
<td>( V_{XC}, % )</td>
<td>82</td>
<td>0</td>
<td>Init. value ± 17.5</td>
</tr>
<tr>
<td>( V_{XD}, % )</td>
<td>83</td>
<td>0</td>
<td>Init. Value ± 17.5</td>
</tr>
<tr>
<td>( V_{rA}, % )</td>
<td>84</td>
<td>0</td>
<td>Init. Value ± 10</td>
</tr>
<tr>
<td>( V_{rB}, % )</td>
<td>85</td>
<td>0</td>
<td>Init. Value ± 15</td>
</tr>
<tr>
<td>( V_{rC}, V_{rD}, % )</td>
<td>86</td>
<td>0</td>
<td>Init. Value ± 20</td>
</tr>
<tr>
<td>( n, \text{rpm} )</td>
<td>87</td>
<td>38000</td>
<td>2375 + 60800</td>
</tr>
<tr>
<td>( Z )</td>
<td>88</td>
<td>13</td>
<td>9 + 17</td>
</tr>
<tr>
<td>( \xi_{split} )</td>
<td>89</td>
<td>0.4</td>
<td>0.1 + 0.8</td>
</tr>
<tr>
<td>( c_{h} )</td>
<td>90</td>
<td>-1</td>
<td>Init. Value ± 0.25</td>
</tr>
<tr>
<td>( d_{h} )</td>
<td>91</td>
<td>.35</td>
<td>Init. Value ± 0.05</td>
</tr>
<tr>
<td>( c_{s} )</td>
<td>92</td>
<td>.42</td>
<td>Init. Value ± 0.1</td>
</tr>
<tr>
<td>( \beta_{1bs}, \text{deg} )</td>
<td>93</td>
<td>-62.92</td>
<td>Init. Value ± 10</td>
</tr>
<tr>
<td>( \beta_{2b}, \text{deg} )</td>
<td>94</td>
<td>-34.78</td>
<td>Init. Value ± 10</td>
</tr>
<tr>
<td>( k_1 )</td>
<td>95</td>
<td>2.60</td>
<td>Init. Value ± 1.5</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>96</td>
<td>1.77</td>
<td>Init. Value ± 4.1</td>
</tr>
</tbody>
</table>

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**Fig. I. Meridional Impeller Profile.**
\[ c(\mathcal{D}) = 1 - \frac{\beta^*}{\beta_k} = 0 \]  

(10)

Inequality constraints are imposed in order to avoid critical conditions in terms of both flow pattern and blade mechanical stresses. In particular such constraints exert a control on the inlet relative Mach number, the outlet absolute Mach number, the blade load coefficient and the peripheral velocity, the impeller reaction degree. The formal relationship is:

\[ d_j(\mathcal{D}) \leq 0 \quad (j = 1, \ldots, \text{no. of constraints}) \]  

(11)

Such constraints also allow the flow model limitations to be taken into account. An excess in relative Mach number would result not only in increased shock losses but in a non proper employment of the flow model, when dealing with transonic flows. Similarly an excess in blade loading would emphasize some three-dimensional effects, like flow separation, secondary and leakage flows which are not presently included in the model. As it is self-evident, some constraints may be relaxed when adopting a more accurate 3-D approach and further constraints may be added, according to the designer's experience. The base methodology does not undergo significant variations when a different choice for the design constraint is adopted.

As a consequence, the overall objective function is built as to take account both the main objective of the design and the constraints imposed. A classical Augmented Langrangian Function formulation was therefore adopted:

\[ F(\mathcal{D}) = f(\mathcal{D}) + \lambda \cdot c(\mathcal{D}) + P \left[c(\mathcal{D})\right]^2 \]  

(12)

\[ F(\mathcal{D}) = f(\mathcal{D}) + \lambda \cdot c(\mathcal{D}) + P \left[c(\mathcal{D})\right]^2 + P \sum_{ij} \left[ d_j(\mathcal{D}) \right]^2 \quad (d_j > 0) \]

THE GENETIC OPTIMIZATION ALGORITHM

The algorithm employed by the authors can be considered as belonging to the genetic algorithm class (Goldberg, 1989) and it is characterized by an high effectiveness, mainly in overcoming the typical difficulties which are encountered by other classical optimization methods. Such difficulties arise when discontinuities in the objective function are present because of non-linear constraint violations or unreliable flow conditions. This suggested the authors to exploit the attractive feature of the genetic algorithm approach, which proceeds through a discrete-step process, combined with a progressive refinement strategy which can lead to a more accurate search of the final solution. In the following, a brief outline is given of the algorithm which has been widely discussed in previous authors' papers.

The search of the optimal impeller geometry is performed throughout a population, whose individuals are represented by a genetic code of the type:

\[ \mathbf{g} = (g_1, g_2, \ldots, g_{16}) \]  

(13)

Each gene, \( g_i \), may vary within the \((1 - 9)\) integer interval, so subdividing into eight parts the range of variation of the independent variables in table I. The generally linear relation holds between the gene and the corresponding \( X_i \) variable:

\[ X_i = X_{i_{\text{min}}} + \frac{g_i - 1}{8} (X_{i_{\text{max}}} - X_{i_{\text{min}}}) \phi \quad (g_i = 1, \ldots, 16) \]  

(14)

As it will be better explained in the following, the search interval \((X_{i_{\text{min}}}, X_{i_{\text{max}}})\) can be shifted and its extent can be reduced by a proper value of the \( \phi \) factor, if a more refined search is required. When the optimizing procedure starts a \( \phi = 1 \) value is assumed and the variable ranges are those reported in table I.

A different relationship is established for those variables which more dramatically affect the impeller size and its operating range, say the dimensional factor and the rotational speed. The changes in these parameters take place according to a geometric progression:

\[ X = X_{i_{\text{init}}} 2^{(6r-1)\phi} \]  

(15)

The latter may therefore vary within an interval of \((1/16 \text{ to } 16)\) times their initial value. A more refined search is obtained by varying the reduction factor, \( \phi \), during the following phases of the optimization process.

The operative structure of the algorithm set up by the authors follows those which can be found in the classical scientific literature (Goldberg, 1989). Each cycle of genetic search is arranged through a number of fundamental steps:

1) A random generation of an initial population of individuals.
2) A mating-pool phase which matches the best individuals for a following reproduction step. The choice criterion is the roulette-wheel one, which assigns a higher probability level to those individuals which are characterized by the best values in the objective function.
3) A cross-over process which allows a gene exchange between the individuals mated and therefore the generation of new individuals, to the aim of achieving a species evolution in the impeller population.
4) Genetic Mutations in the best individuals, according to random \((\pm 1)\) changes in a single gene within their genetic string.
5) Selection of individuals which either will survive or must be killed.
6) The cycle of genetic search is repeated from the step (1), but the random generation is allowed for a reduced number of new individuals, in order to take into account those survived after the selection at the step (5).

The process is considered to be concluded if the same optimal genetic code has been found after five consecutive cycles.

The procedure described above has been already introduced by the authors in their previous works (1994, 1995) but some innovative solution have been adopted, as explained below:

- The initial size of the population at the step (1) was, in previous authors' experiences, of \( 3 \times N \), \( N \) being the number of genes. Although this choice ensures, in some cases, a satisfactory speed of the optimizing process, it does not necessarily involve a good effectiveness, mainly if the final objective of the design is very distant from the original configuration data. For this reason, the examples in this work started from an initial population of \( N \times N \) individuals.
- Regarding the mating-pool phase (2), an increased effectiveness has been found by assuming the best four individuals as the basis for the matching with the other individuals. In previous applications of this method, the basis of the mating pool just consisted of the best individual.
- The selection among the individuals to be killed or survived is operated through a smoother criterion, i.e., those individuals whose objective function level exceeds by more than 20 % the minimum level at previous cycles are killed. This allows an increased number of information about the optimal trend to be followed.
OPTIMAL DESIGN STRATEGIES AND RELATED EXAMPLES

The examples of optimized design in this paper have been conceived to the aim of checking the effectiveness of the method described in previous sections, with special reference to the changes which have been introduced into both the genetic search cycles and the design strategy. The design data of the radial flow impellers in table 2 are strongly different one from each other, in terms of both mass flow rate and pressure ratio, as to highlight the capability of the algorithm to perform a search in a wide range of geometric and operating parameters.

All examples have been developed starting from a reference impeller with a mass flow rate of about 3 kg/s and a design pressure ratio of 7. Geometric and operating data of this impeller are reported in table 1. Table 2 gives the final non-dimensional specific speed of the optimized impellers for inlet ISO standard conditions:

\[ N_s = \frac{2\pi n v}{60 H_2} \]  

(16)

The final \( N_s \) values, compared with that of the reference compressor (\( N_s = 0.9961 \)), confirm that the design method is capable of finding solutions with non-similar geometrical and fluid-dynamic characteristics. The second and third optimization present quite the same \( N_s \) level but with strongly different scale factors, due to the different values of both mass flow rate and pressure ratio.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Gen. Value</th>
<th>Gene Value</th>
<th>Gene Value</th>
<th>Gene Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_{\text{in}} )</td>
<td>4.984</td>
<td>4.788</td>
<td>4.979</td>
<td>4.930</td>
<td>(5.029)</td>
</tr>
<tr>
<td>( \eta_{\text{in}} )</td>
<td>0.923</td>
<td>0.853</td>
<td>0.890</td>
<td>0.906</td>
<td>(0.896)</td>
</tr>
<tr>
<td>( R )</td>
<td>0.170</td>
<td>0.454</td>
<td>0.510</td>
<td>0.502</td>
<td>(0.543)</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>1.483</td>
<td>1.107</td>
<td>1.049</td>
<td>1.056</td>
<td>(1.008)</td>
</tr>
<tr>
<td>Func. Eval.</td>
<td>498/4982</td>
<td>1610/9973</td>
<td>2524/9608</td>
<td>524/1724</td>
<td>(1235/3258)</td>
</tr>
<tr>
<td>Actual / total</td>
<td>524/1724</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. RESULTS OF OPTIMIZATION CASE #1.

As stated above, the design problem must satisfy, from a mathematical point of view, constrained optimization conditions according to the following limits:

- \( \phi \leq 0.25 \)
- \( \beta_{\text{in}} \geq 2 \)
- \( \eta_{\text{in}} \geq 0.853 \)
- \( R \geq 0.170 \)

The limit in the reaction degree is assigned in order to avoid that the total-to-total pressure ratio is reached with an excess in kinetic energy level and a low increase in static pressure. In order to allow an easier search during the first phase, this constraint was initially imposed at a very low level (\( R \approx 0.1 \)) and became more severe during the following phases (\( R \approx 0,5 \)).

Only in the first example, a fourth phase was added. Such a phase starts from the results of the third one but the flow calculation is performed with a more refined computational mesh (i.e. a 40x25x20 grid instead of a 20x11x10 one in the [\( \mu, q, \beta \)] reference). This case was also examined with a reaction degree limit increased up to 0.75. The related results are reported as 4.1st phase and 4.2nd phase and are reported within brackets in table 3. Both 4th and 4.1st phase started from the results of the third one and were developed with the purpose of simulating a typical multi-step process of optimized design. The higher order steps could be replaced, in practice, by procedures involving more accurate approaches for the prediction of both flow field and impeller performance. The duplicate development of the fourth phase was made with the purpose of investigating the effect of different limits imposed to the constrained variable values.
Tables 3 to 5 show the results of the application during the progress of the optimized design procedures, while figures 2 and 3 put into evidence their overall development or, in major detail, that of a single phase, so allowing the following considerations:

- The organization in subsequent phases of increased accuracy and with a progressive constraint sharpness has confirmed its effectiveness. Really, it can be noted that, mainly in the first and third example, the first phase allowed establishment of the range of values for both dimensional factor and rotational speed in relation to the required mass flow and pressure ratio levels. In both cases, the impeller reaction degree is quite low at the end of the first phase. As a result, a high level of absolute velocity (or of the corresponding Mach number, $M_2$) can be observed and, therefore, a reduced increase in static pressure can be expected.

- The following phases give a more relevant contribution both to satisfy a more severe constraint on the reaction degree ($R > 0.5$) and to improve the impeller performance, because of the more detailed investigation within the range of geometrical parameters. It is rather evident that the third phase (and also the fourth ones in optimization #1), which operates with decreased search intervals, really allows a geometry refinement, as demonstrated by the slighter changes in the several geometric parameters. As a consequence, significant increases in the adiabatic efficiency are obtained. On the other hand, the high level of adiabatic efficiency which has been reached at the end of the first phase is nearly meaningless, because it corresponds to flow conditions very far from the final ones. Furthermore, the first example has been also developed through a fourth phase which allowed an accurate boundary layer calculation and a better localization of the peaks in blade loading. As stated above, this particular strategy is in accordance with the most modern tendency in optimized design, which often consists of the matching of rough and accurate flow calculations.

- The second optimizing process is rather different from the other ones, because the reaction degree constraint is, in practice, already satisfied at the end of the first phase. The second one does not produce significant variations in the impeller geometry, while the third one proceeds with a more accurate search.

- The tables also report the number of function evaluations within each optimization phase. It must be pointed out that the overall number is much higher than the actual function evaluations, i.e. of those which allowed a fully comprehensive fluid dynamic analysis. Many configuration are, instead, immediately discarded because of a preliminary detection of constraint violation or of unreliable situations in both impeller geometry or flow conditions. The total computational time is therefore mainly affected by the actual number of objective function estimations, whose ratio to the total number of impeller generated increases, as expected, during the latest phases. The number of function evaluation increases, of course, with the complexity of the problem. As an example, the 4th phase in optimization #1 required a greater computational time, because of the more severe limit to the reaction degree. Such a constraint results, however, violated at the end of the process, so indicating that the reaction degree value which has been reached is probably the maximum allowable one, also in relation with the other design constraints.

- Due to the progressive geometry refinement, each configuration is identified by a quite complex genetic code. The code established at the end of the first phase characterizes the impeller with respect to the reference one. The following codes must be considered as subcodes, whose genes designate the independent variable values with respect to those determined by the previous phases.

<table>
<thead>
<tr>
<th>Table 4. Results of Optimization Case #2.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>$F_{dim}$</td>
</tr>
<tr>
<td>$V_{AC}$</td>
</tr>
<tr>
<td>$V_{AD}$</td>
</tr>
<tr>
<td>$V_{AD}$</td>
</tr>
<tr>
<td>$V_{AC}$</td>
</tr>
<tr>
<td>$n$, rpm</td>
</tr>
<tr>
<td>$Z$</td>
</tr>
<tr>
<td>$Esplit$</td>
</tr>
<tr>
<td>$h_{in}$</td>
</tr>
<tr>
<td>$h_{th}$</td>
</tr>
<tr>
<td>$R_{c}$</td>
</tr>
<tr>
<td>$\beta_{lbs}$</td>
</tr>
<tr>
<td>$\beta_{lb}$</td>
</tr>
<tr>
<td>$k_{1}$</td>
</tr>
<tr>
<td>$k_{2}$</td>
</tr>
</tbody>
</table>

| Function Eval. | 463 / 6024 | 298 / 3986 | 667 / 4456 |

<table>
<thead>
<tr>
<th>Table 5. Results of Optimization Case #3.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>$F_{dim}$</td>
</tr>
<tr>
<td>$V_{AC}$</td>
</tr>
<tr>
<td>$V_{AD}$</td>
</tr>
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| Function Eval. | 306 / 3362 | 1603 / 7052 | 1722 / 5398 |

| $\beta_{1}$ | 1.984      | 1.978      | 1.988      |
| $\eta_{1}$ | 0.660      | 0.846      | 0.876      |
| $R$         | 0.171      | 0.515      | 0.549      |
| $M_{2}$     | 0.9301     | 0.6852     | 0.6609     |

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Fig. 2. Development of the Optimized Design Processes.

Fig. 4. Impeller meridional profile pattern in the optimization cases.

Fig. 3. Examples of evolution of a single optimizing phase.
The evolution of the optimizing processes is highlighted in figs 2 and 3, in terms of increasing improvement in both pressure ratio and adiabatic efficiency values. This situation is verified both by the average levels displayed in fig. 2 and by the progressive data gathering around the final optimal levels. The progressive reduction in result scattering is a significant criterion for checking the convergence of a single phase. Fig. 3 shows in further detail the behaviour of single phases of an optimizing process, so confirming that the genetic search cycles previously defined allow a quite continuous evolution of the performance parameter which have been controlled. In practice, the number of phases which are necessary for completing the impeller design depends upon the level of refinement which can be achieved in both flow investigation and constraint definition. In any case, the evolution of the design should be followed in order to detect the attainment of a stationary condition. As a matter of fact, results in figs 2 and 3 correspond to the successive modifications in the impeller geometry. As a consequence, the impeller meridional profiles undergo large variations, as shown in fig. 4. Such modifications become less evident in optimizations #1 and #2, during the following phases, and this can indicate the convergence of the process. In the case of the third optimization, significant differences still occur between the results of second and third phase and this suggests the need of further phases to complete the process.

In addition, the comparison of two impeller profiles, which result from different constraint values, may validate the effectiveness of the method. In fig. 5, the meridional profiles obtained at the end of the 4th and 4.1st phases of the first optimization mainly differ in the blade width at trailing edge. This is consistent with the need of reducing the kinetic energy level for increasing the reaction degree.

Similarly, the examination of the blade angle development in figs. 6 and 7 suggests a number of considerations. The design targets of the several optimizations lead to very different blade angle values at leading and trailing edge and to dramatic variations in its pattern along the meridional path. The blade angle often presents the typical behaviour of back-swept blades with a minimum absolute value in the middle of the blade, but the minimum blade angle location is strongly different when comparing the several optimization results and, in some cases, a continuous variation from the inlet to the discharge blade angle can be observed.

The detailed insight of the results of optimization #1 in fig. 7 shows that the design method is also able to propose solutions with positive discharge blade angles, when a low value is imposed to the reaction degree constraint (i.e., in the first phase). The progressive refinement of the impeller geometry leads to different solutions in the following phases. In particular, the final results of 4th and 4.1st phases present similar blade angle behaviours but with different values, in order to meet different values of the constraint on impeller reaction degree. This is a further proof of the consistency of the design method.

Figures 4 and 7 also show that strong differences occur among the solutions of the several design cases examined, so demonstrating that the proposed methodology is able to operate within a wide range of impeller design data. Such differences may be better appreciated by means of the 3D views in fig. 8. This picture also confirms that the optimized design parameters may be employed either within a CAD-CAM procedure or to build a computational grid for more accurate numerical simulation of the flow pattern.

In order to better emphasise the effectiveness of the proposed method, some results of flow calculations can be analyzed. Figures 9 to 13 allow, in particular, a detailed examination of the flow conditions resulting from the first optimization process. Fig. 9 shows the blade tip relative Mach number behaviour and the related trend strongly differs when proceeding from the first to the last phase of the process. The early phases suggest a typically decelerated flow through most of the flow path, while the latest ones present a more evident flow acceleration in the final part towards the impeller discharge. Different results can also be observed when comparing the 4th and 4.1st phases.

The Mach number patterns are in accordance with both the design constraint and the flow model features. In fact, the main loss source being the boundary layer thickness, the method shows its capability in adapting the flow distribution to reduce the viscous region growth. It must also be pointed out that the Mach number levels in these figures are those estimated within the inviscid region and they result from the interaction with the viscous, low-kinetic energy zone. The continuous
increase in both hub and tip static pressures (fig. 10) confirms that the design is addressed to find a reliable solution. As expected, the lowest increase results from the first optimizing phase, due to the low reaction degree, while the highest one corresponds to the 4.1st phase.

The blade-to-blade Mach number distribution is presented in fig. 12 with reference to both mid-span and tip flow surfaces. The figure highlights the strong differences which occur between the results of the second and the final phases. In particular a different control is exerted both on the flow deceleration along the blade pressure side within the first part of the flow path, and on the final acceleration which compensates the boundary layer growth along the blade surfaces. Variations also occur in terms of difference between the suction and pressure side velocity levels, which affect the local conditions of blade loading. The latter parameter is also influenced by the average velocity level which is rather lower in the flow conditions resulting from the second phase of the optimization. An increase in blade loading is obviously expected in the 4.1st phase, due to the higher increase in static pressure which has been imposed by the reaction degree constraint.

The results of blade-to-blade distribution are also summarized by the contours of the blade load coefficient, in fig. 13. The second optimizing phase leads to an almost continuous increase in this parameter along the flow path, while the fourth phase involves local peaks in blade loading in the flow region close to the hub. In particular, an excess in the load coefficient (i.e., > 2) can be observed in a limited region of the blade vane of the impeller obtained by the 4.1st phase. This confirms that the limit imposed to the reaction degree cannot be reached without producing a violation of further constraints. Additional uncertainties in the flow solution could also derive, in this case, from the presence of separated flow region.

Similar detailed flow field investigations could be carried out for the other optimization cases, in order to check the validity of the solutions found. A single example is presented in fig. 14, which presents the evolution of the blade load conditions that proceeds with a continuous improvement during the optimizing process. In particular, in the second design case (fig. 14a) the final configuration is characterized by peaks in blade loading which are satisfactorily lower than the limiting value of 2. This condition is much more favourable than those reached at the end of the previous phases and it is achieved by variations only in the blade meridional profile and blade angle development, the number of blades and the splitter blade choice being unchanged. In the third optimization case (fig. 14b) the final geometry presents a steeper rise in blade loading, compared with the intermediate situations, because of the reduction in the number of blades. Such a growth is effectively controlled by an advanced location of the splitter blade (i.e., = 0.3 and 0.15 at the end of
second and third phase, respectively, as reported in table 5). The fluid dynamic conditions throughout the impeller are, of course, also affected by the choices made for the blade angles, which, mainly in the third case, undergo strong variations when proceeding from the first to the third phase.

As a final consideration, the examples of the methodology proposed have confirmed the possibility of exploiting the well known selective capacity of the genetic algorithms, specially during the first phases of the optimization process. In the last phases, which operate with reduced search intervals, the optimizing algorithms acquires an accuracy level which is comparable with the one typical of numeric optimization methods, but the typical advantages of the genetic methods are however preserved.

CONCLUSION

Based on their recent experiences, the authors have developed a strategy for optimal design of radial flow impellers, with a genetic algorithm approach within a multi-step procedure organized into a number of succeeding phases of search of the optimum configuration.

The results have confirmed the effectiveness of this multiple-phase strategy as regards the search of the range of size and operating characteristics of the compressor in a first step and the progressive improvement in both local flow and load conditions and overall performance. Besides, the level of accuracy of the procedure is competitive with that of numerical optimization method, previously employed by the author, but with a higher selectivity.

The effectiveness of the methodology has been also demonstrated by examining the flow conditions through the optimized impellers. Flow parameter distributions are consistent with the model features and the method follows the assignment of both design targets and constraint values. The satisfactory results obtained by the final mesh refinement also suggests the possibility of adopting a fully 3-D model for flow analysis in the final optimizing phases, after a faster selection operated with a simplified model.

It is the purpose of the authors to extend, therefore, their methodology to different turbomachinery components, by also employing more accurate flow models. Really, the present paper has been mainly addressed to the definition of the optimizing procedure, but it must be reminded that the final goal is to establish an optimal design method characterized by a satisfactory computational speed, also when matched to CFD codes with more relevant CPU time requirements.

**Fig. 11. End-Wall boundary layer development at the end of each phase of the optimization #1.**

**Fig. 12. Comparison of results of blade-to-blade calculations at the end of different optimizing phases.**

**Fig. 13. Comparison of local blade loading conditions evaluated at the end of different optimizing phases.**
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