Abstract.

When averaging the Navier-Stokes or Euler equations in the circumferential direction of a turbomachine, some so-called "spatial fluctuation terms" appear in the averaged conservation equations. We give the transport equations for these kinematic spatial fluctuation correlations. We evaluate the various terms in these equations, using the results of a three-dimensional Navier-Stokes computation. The test case is a highly loaded subsonic turbine stator.

We discuss the magnitudes of the various terms. It is shown that blade force terms and diffusion terms have dominant effects, as respect to the production by the averaged field and the shear stress terms. The dissipation of the fluctuation kinetic energy occurs mainly in the passage vortex area, and in the blade wake.

Nomenclature.

- \( b \) flow area between two consecutive blades in the \( z \) direction
- \( c_v \) heat capacity at constant volume
- \( g \) blade pitch
- \( K \) kinetic energy of the spatial fluctuation in the \( z \) direction
- \( P \) static pressure
- \( Re \) Reynolds number
- \( T \) Temperature
- \( u \) velocity
- \( x, y, z \) axial, spanwise, and blade-to-blade directions
- \( \rho \) static density
- \( \tau \) shear stress
- \( n \) unit vector normal to the blade surface

Superscripts

- \( A \) Density weighted average in the \( z \) direction (see equation 1)
- \( A' \) Fluctuation in the \( z \) direction around the density weighted average (see equation 1)
- \( A^* \) Area average in the \( z \) direction
- \( A^{**} \) Fluctuation in the \( z \) direction around the area average

Subscripts

- \( ps \) blade pressure side
- \( ss \) blade suction side

Introduction.

The computational process of the flow in multistage turbomachines includes at some step the use of through-flow methods. This is a necessary procedure to take account of interactions of blade rows, with minimum costs. Consistent through-flow models with three dimensional computations may be developed when the flow equations are averaged in the circumferential directions of the cylindrical frame of reference (Hirsch, Dring, 1987). If we compare the resulting averaged equations with the two-dimensional ones, some supplementary terms appear. These are the so-called "blade" terms, which introduce the influences of blade pressure and shear forces as well as the heat fluxes on the blades. As a consequence of the non-linearity of the convective terms in the conservation equations, some new unknowns appear from the point of view of the averaged quantities. These are the "spatial fluctuation terms," which account for the non-uniformity in the blade-to-blade direction. These terms introduce a direct influence of the three-dimensional nature of the flow.

Perrin and Leboeuf (1993) estimated the relative importance of the various terms that occur after applying a blade-to-blade average on the Navier-Stokes equations. They computed the flow in an axial flow turbine. The basic solution was three-dimensional, steady and turbulent. It included three-dimensional Reynolds stresses, but no time fluctuations as they considered an isolated rotor cascade geometry. They averaged the results of the Navier-Stokes computation in the blade-to-blade direction and computed the various terms of the through-flow momentum equations. They showed how the three-dimensional effects disturb the averaged blade-to-blade quantities. Their main conclusions were: (a) The radial fluctuation kinetic energy \( K_y \) is the most important component of the three-dimensional convection effect; it balances the radial static pressure gradient. In the axial and blade-to-blade component of the momentum equation, fluctuation terms account for 10 to 20% of the balance of averaged momentum. (b) The averaged static pressure has a distribution role among the three components of momentum equation. It allows the fluctuation terms to strongly influence the blade pressure force. (c) The blade shear stress terms and the averaged shear stress terms are negligible almost everywhere except very near the endwalls. Consequently, no source of dissipation appears in the average momentum equations.
The objective of this paper is to show the underlying mechanisms that drive the evolutions of the fluctuation correlations. Shear terms, compared to the pressure, seem to play a minor role in the average momentum equations. It is then of particular interest to understand (a) where the dissipation finally acts, and (b) how the static pressure influences the evolutions of the fluctuation correlations. We perform this study in the frame of through-flow computations, and we give first the transport equations of the kinetic spatial fluctuation correlations. Second, we estimate the relative importance of the various terms in these equations. For that purpose, we achieve a numerical experiment with a three-dimensional Navier-Stokes code. We use the results of a computation, performed with a three-dimensional Navier-Stokes computation at ONERA. For the test case, we use a linear cascade, figure 1. The experimental conditions are: overall turning angle of the flow is 92.4°; inlet Mach number is 0.5; isentropic exit Mach number is 0.71; inlet Reynolds number is 2.3 $10^5$. The test case and the Navier-Stokes computation.

We use the test turbine proposed by Denton, Hodson and Dominy(1990). This is a turbine root section from a low-pressure rotor blade, tested in a linear cascade, figure 1. The experimental conditions are: overall turning angle of the flow is 92.4°; inlet Mach number is 0.5; isentropic exit Mach number is 0.71; inlet Reynolds number is 2.3 $10^5$. We use the same test case and Navier-Stokes computation as in Perrin and Leboeuf (1993).

![Blade to blade view of the turbine.](image)

Figure 1: Blade to blade view of the turbine.

The computation was performed at ONERA with the CANARI code (Cambier, Couaillier and Veuillot, 1988). We use the code here with a fine mesh. The computational domain is split in an O-type mesh around the blade and two H-type sub-domains upstream and downstream to allow an accurate description of the rounded leading and trailing edges. The sub-domains have respectively 28875, 262605 and 42875 nodes, from upstream to downstream, on the half blade span. The location of the mesh points in the laminar sub-layer enables the capture of very small vortex structures. The turbulence model is a mixing length model (Michel, Quemard and Durant,1969). All the flow quantities, and the mesh terms in the equations presented in this paper, are non-dimensionalised by reference quantities: temperature $T_{ref} = 293$K, density $\rho_{ref} = 1.293$ kg/s, velocity $u_{ref} = 341.1$ m/s, length $L_{ref} = 0.05253$ m. The fluid is air.

This code was already validated with various fundamental and turbomachine test cases, including turbines with similar meshes (Cambier and Escande, 1989, Escande and Cambier, 1991, Billonnet, G., Couaillier, V., Veuillot A.M., Heider R., 1992). It produces good predictions of the flow, although the losses may be slightly over predicted. It is not our purpose to check the accuracy of the predictions against experimental results in this paper. We shall then assume the validity of the numerical simulation.

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The equations for the fluctuation correlations.

We use a Cartesian frame of reference where x is the axial direction and y is the direction parallel to the blade span. The spatial averaging process is applied along the blade-to-blade direction z, which stands for the circumferential direction $\theta$ in a cylindrical frame of reference.

If $A(x,y,z)$ is a flow variable, $\overline{A}$ is the related density weighted spatial average and $A'$ is the spatial fluctuation of $A$ in the z direction

\[
\overline{A}(x,y,z) = \overline{A}(x,y) + A'(x,y,z)
\]

\[
\overline{pA} = \overline{pA'} = \frac{1}{b} \int_{0}^{b} pA \, dz
\]

$g$ is the blade pitch and $b$ is the flow area between two consecutive blades, in the z direction. The variable $A$ stands for any flow variables, except for the density $p$, the pressure $P$ and the shear stress $\tau$ for which equation [2] is replaced by a simple area average. In that case, the following notations are used:

\[
\overline{p}(x,y,z) = \overline{p}(x,y) + p'(x,y,z)
\]

\[
\overline{P}(x,y,z) = \overline{P}(x,y) + P'(x,y,z)
\]

Using this averaging process on the gradient operator, we get by Leibnitz rule:

\[
\overline{\nabla A'} = \frac{1}{b} \overline{\nabla (bA')} + \frac{1}{b} \Delta [A \overline{\nabla}]
\]

where $\overline{V}$ is the gradient operator, $\Delta$ is the difference operator. The last term in equation [4] stands for a blade effect; it is the origin of the blade pressure and shear forces in the momentum equations.

The average of the non-linear terms in the transport equation will introduce some new unknowns, from the point of view of the averaged quantities:

\[
\overline{pA} = \overline{b^2 A'} + \overline{pA'}' = \overline{b^2 A'} + \overline{pA'}'
\]

The last term in equation [5] is derived according to equation [2]; it accounts for the non-uniformity of the flow in the blade-to-blade direction. Note that according to [1] and [2]:

\[
\overline{pA'} = \overline{pA'} = -\overline{pA'}
\]

To get the equations for the fluctuation correlation, we use the following steps. We multiply the $pV_j$ equation by $v'_k$, and the $pV_k$ equation by $v'_j$, we add and area-average along z. We obtain then the transport equation for the spatial fluctuation correlations.

This process is very similar to the development of the Favre formulation of the turbulent correlations.

We give below the resulting transport equation for the $b^2 v'_j v'_k$ transport equation.

\[
\frac{\partial}{\partial x_1} \left( b \overline{b^2 A'} v'_j v'_k \right)
\]

\[
+ b \overline{b^2 A'} \frac{\partial v'_j}{\partial x_1} + \overline{b^2 A'} \frac{\partial v'_k}{\partial x_1}
\]
The terms (a) to (j) in equation [7] have the following meanings:

(a) Mean velocity transport.
(b) Mean velocity production.
(c) Fluctuating velocity transport.
(d) Compressibility effects.
(e) Pressure-deformation term.
(f) Pressure diffusion term.
(g) Blade pressure force influence.
(h) Diffusion by the shear stress tensor \( \tau \) including turbulent effects.
(i) Dissipation term including turbulent effects.
(j) Blade shear force influence.

Note in equation [7] that the subscripts stand for \( x \) and \( y \) when they appear in a derivative outside an average quantity, as in \( \partial \hat{p}^s / \partial x_k \) and for \( x, y \) and \( z \) elsewhere. \( \Delta \) stands for a difference between blade suction (ss) and pressure (ps) sides. Compared to the transport equations of the classical turbulent stresses, the equation [7] has a similar structure, except for blade force terms (g) and (j).

We shall use from then on the transport equation for the kinetic energy of the spatial fluctuation \( \hat{K} \):

\[
\hat{K} = \hat{K}_x + \hat{K}_y + \hat{K}_z = \frac{1}{2}(\hat{\nu}_x^2 + \hat{\nu}_y^2 + \hat{\nu}_z^2)
\]

(8)

We obtain this equation by putting \( j=k \) in equation [7], and then summing over \( x, y \) and \( z \).

The kinetic energy \( \hat{K} \) of the spatial fluctuation

Perrin and Leboeuf (1993) give a detailed description of the relationships between \( \hat{K} \) and the non-uniformities in the blade-to-blade direction. We give here the most important conclusions of this work and reproduce two of the figures from this paper to clarify the present work.

The axial evolution of \( \hat{K} \) is given at mid-span in figure 2, and figure 3 displays its evolution in the \((x, y)\) plane. The non-uniformity of \( \hat{K} \) has two origins. The blade curvature and the boundary layers on the blade walls induce a two-dimensional effect. Three dimensional flow structures exist near the end-wall. Their effects on \( \hat{K} \) near the end-wall are described shortly here in relation with figure 3. Just after the leading edge \( (x/c < 0.3)\), a leading edge vortex occurs very near to the wall. Then \( \hat{K} \) increases in the initial part of the passage vortex grow \( (0.3 < x/c < 0.6)\). An area of high value of \( \hat{K} \) may be observed near the endwall after mid-chord. \( \hat{K} \) decreases in the last third of the blade passage. This is a result of the interaction among the passage vortex, the suction side wall, and the corner vortex. \( \hat{K} \) is finally eliminated in the blade wake. However, the passage vortex is still detected at 50\% of the chord downstream. One of the objectives of this paper is to understand the phenomena that drive the evolution of \( \hat{K} \).

![Figure 2: Averaged kinetic energy \( \hat{K} \) of the spatial fluctuation in the \((x, y)\) surface.](image)

![Figure 3: Axial evolution of the averaged kinetic energy \( \hat{K} \) at mid-span.](image)

The transport of the fluctuation correlation of kinetic energy \( \hat{K} \)

The kinetic energy of the fluctuation \( \hat{K} \) is of particular interest in computing the average static temperature \( T \) from a three-dimensional set of conservative variables \((p, pV, pE)\), where \( E \) is the total energy.

\[
\hat{T} = \frac{1}{C_v} \left( \frac{E}{2} - \hat{\nu}^2 \right)
\]

(9)
For this blade row, \( \bar{K} \) reaches a maximum value of 1.6% of the dynamic head, which introduces a correction of 1.2 degree on \( \bar{T} \).

We analyse from then on the evolutions of the various terms of the equation [7] for the transport of the fluctuation correlation of kinetic energy \( \bar{K} \). The various terms are presented as follow:

\[
\text{Transport by convection (a) + Production (b) } + \text{Diffusion by triple correlations (c) + Pressure terms (d+e+f+g) } + \text{Shear stress terms (h+i+j)} = 0.
\]

A positive value of the convection term (a) means an increase of \( \bar{K} \), and then of the blade-to-blade variations, in the direction of the mean flow. For the other terms, a negative value leads to an increase of \( \bar{K} \).

We consider the distributions of these terms along the axial direction \( x \), at mid-span in figures 4 to 6. Figures 7 to 15 give their distributions in the meridian plane (x, y).

**Convection by the mean velocity field**

The axial evolution of the averaged kinetic energy \( \bar{K} \) is given at mid-span in figure 2. In the same section, the plot of the convection term by the mean velocity field is given in figure 4. Owing to the symmetry of the blade cascade at mid-span, the flow has a two-dimensional behaviour. The non-uniformity of the flow in the blade-to-blade direction is there a consequence of the blade curvature and of the boundary layers on the walls. At mid-span, \( \bar{K} \) increases up to \( x/c = 0.6 \), and decreases afterwards, particularly after the trailing-edge. This behaviour agrees with the evolution of the mean velocity transport term (a), which is positive up to mid-chord. We mentioned already the high value of \( \bar{K} \) observed near the end-wall, under the influence of the passage vortex (figure 3). The contours of the convection term (a) show variations of signs in this passage vortex area (figure 7). They are illustrations of strong gradients of \( \bar{K} \).

**Mean velocity production**

The mean velocity production term (b) results from a superposition of mean velocity gradients and spatial velocity fluctuation along z. At mid-span, its evolution is mainly a result from a potential effect, induced by the blade blockage and loading. Near the end-wall, its value results from the deviation of the wall boundary layer, between the pressure and the suction sides. These different origins induce different behaviours. The sign of term (b) changes along the y direction, approaching the end-wall (figure 8). It reduces the convection of \( \bar{K} \) in the range 40% < \( x/c < 80% \) at mid-span, whereas the reverse is true near the end-wall.
In figure 4, the mean velocity transport (a) has a very similar evolution at mid-span as the global pressure term (d+e+f+g). The development of $K$ is strongly driven by the pressure effects at mid-span, particularly after 60% of chord. This is a major difference with the turbulent stress transport equations, where the mean velocity production term (b) plays a dominant role. There are two major physical phenomena that generate the non-uniformity in the blade-to-blade direction: the blade curvature, and the endwall boundary layer deviation. In terms of the averaged quantities, these mechanisms are very much influenced by the pressure and particularly the blade pressure force.

We present at mid-span in figure 5 the various parts of the overall pressure term: the pressure diffusion term (f), the blade pressure force influence (g), and the compressibility effects (d+e). The contours of these terms in the meridian plane are given in figures 9 to 12.

In the blade passage, the terms (f) and (g) have a same order of magnitude, but they are of opposite signs. Note that they have a common origin. After averaging the three-dimensional equations, the work introduced by the pressure force that writes $\text{div}(V \mathbf{f})$ is split in the two terms (f) and (g). The so-called blade pressure force term (g) is a work exchange between the averaged flow and the fluctuation field by the blade pressure force. This is better seen by considering the expression of this pressure term in the $\mathbf{K}$ equation:

$$
(g) = \nabla_x \Delta \left[ p^e \frac{\partial^2}{\partial x^2} \right] + \nabla_x \Delta \left[ p^e \frac{\partial^2}{\partial y^2} \right] - \nabla_x \Delta \left[ p^e \frac{\partial^2}{\partial z^2} \right] - \nabla_x \Delta (p_x^e)_{\text{avg}}. 
$$

For classical through-flow computations, this term is ignored, since the blade pressure force is assumed normal to the mean velocity vector $V$. This hypothesis is wrong even at mid-span in this cascade (figure 4), as the number of blades is finite and the mean flow is not perfectly guided by the blades.

The pressure diffusion term (f) is a work associated with the fluctuating fields. It redistributes the energy among the streamlines in the meridian plane, as the fluctuating velocity is zero on the end-walls. Upstream of the leading edge, the flow is strictly potential, and the global pressure term is restricted to the diffusion induced by the pressure (f). In the first half of the blade passage, the pressure force term (g) is positive that means a reduction of $K$. However, $K$ still increases under the pressure diffusion influence (f). The reverse behaviour is seen in the second half of the blade passage, where the kinetic energy of the spatial fluctuations is partly transferred to the averaged field.

The sign of the blade pressure force term (g) at mid-span is probably not typical in this cascade of all the turbine blades particularly in the first half of the blade passage. The scalar product of the mean velocity and the blade pressure force depends on the flow incidence, and may be either positive or negative. However near the end-wall, the blade pressure force pushes the viscous flow towards the blade suction.
side, creating a flow over-deflexion. This gives a positive sign to this scalar product, for \( \vec{n} \) pointing from the pressure to the suction sides of the flow passage. According to the negative sign of the pressure difference \( (P^*_s - P^*_u) \) between the two blade sides, the term \( (g) \) must then be positive as observed near the end-wall in figure 12. A negative sign is also observed for \( (g) \), in the second half of the blade passage, away from the end-wall. It is induced by the passage vortex, which tends to introduce an under-deflexion in the flow, in the outer part of the viscous wall layer. At mid-span, the negative value for \( (g) \) is also introduced by a non-perfect flow guidance.

There is also an approximated balance between the two terms \( (f) \) and \( (g) \) at mid-span, as well as near the end-wall. Their sum is then five times smaller than each term (figures 9, 11 and 12). This can introduce strong difficulty in the modelling of the global pressure term. The extreme values for the global pressure term \((d+e+f+g)\) appear in the passage vortex area. This agrees with the previous discussion. Note also that the compressibility terms \((d+e)\) are negligible in this cascade at mid-span (figure 5), and near the end-wall (figure 10). They introduce a slight contribution at mid-chord in the growing part of the passage vortex.

**Fluctuating velocity transport.**

This term can be interpreted as the influence of the fluctuating field on itself. Although it is small at mid-span (figure 4), it has the same order of magnitude as the global pressure term near the end-wall (figure 13), and in the blade passage. It has however a reverse sign. It starts growing at 30% of chord and seems to be very much connected with the extend of the passage vortex development (figure 3).

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in the blade wake. Perrin and Leboeuf (1993) have shown for this test case, that the global shear stress terms in the averaged momentum equation are negligible almost everywhere, except in the endwall region. In this paper, we have shown that the shear stresses have also an indirect effect on the averaged quantities through the dissipation of the spatial fluctuations.

**About through-flow model**

At this stage, it is useful to comment about the relative success of radial mixing models in through-flow computation. The purpose of these models is to take into account of some of the spatial fluctuation terms that occur from the averaging process applied on the conservation equations. The radial mixing models describe these fluctuation terms as diffusion of the averaged quantities, directly into the averaged through-flow equations. The diffusion coefficients correspond to scales that are representative of the secondary flows. Perrin and Leboeuf (1993) show that a main spatial fluctuation effect exists in the y component of momentum equation (the 'radial equilibrium equation'), in the term \( \frac{\partial (\nu^2)}{\partial y} \). This is a gradient of a part of the fluctuation kinetic energy \( \tilde{T} \). We show in this paper that pressure terms \((\nu+g)\) and the triple velocity correlations \(c\) are the main sources of the evolution of \( \frac{\partial (\nu^2)}{\partial y} \). Note also that in a turbulent approach, the term similar as the fluctuation term \(c\) is modelled as a diffusion of \(\tilde{K}\) multiplied by a typical scale of large structure. This model is similar to the radial mixing model that is employed in through-flow models. However, the exact transport of fluctuation quantities is more difficult to model, as shown in this paper.

**Conclusions**

The through-flow model is still an important tool for analyzing the flow in multiscale turbomachines. However it requires some closure relationships for spatial correlation terms that appear after an averaging process is applied to the transport equations. We derived the transport equations for these spatial kinematic fluctuations. We noticed a great structural similarity with the turbulent stresses transport equations, except for blade force terms.

We presented an analysis of the respective importance of the various terms that appear in these transport equations. We choose a subsonic turbine stator as a test case for our analysis. We computed the basic informations from a three-dimensional numerical simulation.

We showed that triple velocity correlations and the pressure terms have major influences on the convective transport of the fluctuation kinematic energy \( \tilde{K} \). The mean velocity term \(b\) is not the main production of spatial fluctuations. The blade force term \(g\) is also the main source of energy exchange with the average field. It results from the non-orthogonality of the blade pressure force and of the average velocity field. It appears very important each time the blades do not properly guide the flow.

The shear stress terms have weak influences in the global energy transport as compared to the other terms. However they can contribute up to 20% of the energy balance in the blade wake. The direct action of the dissipation is underlined in the fluctuation kinetic energy equation. This is important as the average momentum equations do not show a significant influence of the shear stresses, except very near the end-wall. The main areas of dissipation are the passage vortex, and the blade wake. However, we showed also that most of the exchanges are reversible between the mean velocity and the spatial fluctuations.

A correct through-flow model requires that the blade pressure force is not normal to the average velocity vector. The work of this pressure force is precisely the single source of direct energy exchange between the average and the fluctuation velocity fields. The simple diffusive structure of the radial mixing models is very far from the complicated mechanisms underlined in the analysis of the transport of the fluctuation kinetic energy \(\tilde{K}\). The pressure terms play dominant roles in the redistribution of the energy among the various spatial fluctuation correlations. As this is a non local phenomenon, it will be a main source of modelling difficulty.

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