WHEN ARE NONLINEARITIES IMPORTANT AT STALL INCEPTION?

F.E. McCaughan
Department of Mechanical & Aerospace Engineering
Case Western Reserve University
Cleveland, Ohio

ABSTRACT

Recent experimental work has shown that in some compressors, the nonaxisymmetric disturbance leading to loss of stability appears as a localized phenomena, rather than a travelling sine wave which spans the entire circumference, suggesting that nonlinear effects appear very early in the evolution of the disturbance. In a regime dominated by nonlinear effects, the Fourier modes used to describe the spatial structure of non-axisymmetric disturbances, obtained from either experimental data or numerical data produced by a model, can interact very early in the rotating stall inception process. In this paper, we determine which parameters affect the rate of interaction of the various modes in a study of the Moore-Greitzer (MG) model. The relevant parameters are related back to the physics of compressors. Though the stall inception process may well be three-dimensional and involve physics not captured by the quasi two-dimensional MG model, this study is of interest to those who wish to detect and control the magnitude of non-axisymmetric disturbances, in order to decrease the stall margin in a compression system. Any control strategy which depends on eight detecting devices around the annulus of the compressor can resolve only the first three spatial Fourier modes. If disturbances leading to compression system instability develop as spikes, this approach will be completely unsuccessful at detecting the disturbances while they are still small enough to be controlled. The problem is further exacerbated by temporal nonlinearities, that is, the operating point may be linearly stable, but may lose stability to larger disturbances. It is observed in experiments and the Moore-Greitzer model that the compressor loses stability before the throttle is closed past the peak of the performance curve. Both spatial and temporal nonlinearities are discussed.

INTRODUCTION

One of the current focuses of compressor modelling is to provide a 'good enough' model for use in a control system. Ideally, at the design stage, we would like to say for a given compressor, this compressor will go into rotating stall with a large hysteresis loop, or this compressor will exhibit a large amplitude surge oscillation, should circumstances render the flow unstable. While the modelling has helped to define certain trends, such as an increased tendency to surge as the B parameter is increased, it is impossible to make exact predictions about the end result when the compressor loses stability. In fact, the very nature of modelling excludes the possibility that this may ever occur. Even if we could solve the Navier Stokes equations in such a complex domain as that found in a compressor, the arbitrariness of flow disturbances would still leave the initial conditions for the system as unknowns. Greitzer's experimental study [3] showed that whether a compression system surges or stalls depends on the initial conditions as well as on the parameters of the system. The Moore-Greitzer model has been shown to capture this behaviour [5].

We can use the information from experiments and results from various models to push the design in a particular direction, say that surge will be more likely than stall, but the bottom line for the control system engineer is that we cannot a priori predict which will happen.

As the throttle valve is closed, both experimental results and modeling results show that rotating stall starts to grow even when the compressor eventually surges. One theory suggests that rotating stall actually pushes the compressor into surge. Alternatively, it is possible that nonaxisymmetric disturbances start to develop as soon as the compressor goes unstable, whether the final result is surge or rotating stall. If rotating stall can be suppressed, the compressor performance can be improved.

Presented at the International Gas Turbine and Aeroengine Congress and Exposition
Most current control strategies, for example [7], are based on the postulate that the evolution of rotating stall is initially dominated by linear dynamics. This excludes the possibility that the stall cell may develop in a nonlinear manner before the peak of the performance curve. We use the term nonlinear here to include two different aspects of the dynamics. First we note that rotating stall may first appear as a travelling sine wave and as it grows it becomes more like a square wave. Or, the Fourier modes describing the disturbance may interact earlier, so that the disturbance that grows into rotating stall appears as a spike, or local disturbance, see Figure 1. We shall refer to this as spatial nonlinearity. On the other hand, when both rotating stall and steady axisymmetric flow are linearly stable near the peak of the characteristic, a small disturbance may be sufficient to push the operating conditions away from the design flow and into rotating stall. That is, if the steady axisymmetric point is linearly stable, it may be nonlinearly unstable. We will refer to this as temporal nonlinearity. In either case suppression of the rotating stall disturbance is outside of the range of operation of a linear control strategy. Those controls approaches which attempt to deal with the temporal nonlinearity still neglect the spatial interactions and consider only a single Fourier mode, [1]. Both spatial and temporal nonlinearities must be taken into consideration if an effective control system is to be developed.

The pressure rise \( P \) is determined by solving for the pressure rise across each control volume:

\[
P = \frac{(P_4 - P_3) + (P_3 - P_{T2}) + (P_{T2} - P_{T1}) + (P_{T1} - P_{TA})}{\rho U_w^2}
\]

A detailed discussion of the modelling of each of these components, can be obtained in [6]. Additional terms may be added to the model for the total pressure change across the inlet guide vanes in order to capture the effect of wiggling the IGV's, which is one technique suggested for controlling the amplitude of nonaxisymmetric disturbances, [7].

![FIGURE 1 Schematics of Rotating Stall Development](image)

(a)  (b)

FIGURE 1 Schematics of Rotating Stall Development: a) global development where first mode of Fourier series dominates in early stages of evolution. b) localised development, where Fourier modes interact very early to give a spike.

Md Model Development

The schematic in Figure 2 shows how the axial compression system is divided into a collection of lumped volumes. The variables of interest in the problem are the axial mass flow and the pressure rise achieved by the compressor, namely the pressure in the plenum. The total ambient pressure upstream of the compression system is used as the reference value of pressure. All pressure differences are nondimensionalised using \( \rho U_w^2 \), where \( U_w \) is the wheel speed of the compressor and \( \rho \) is the density. The subscript \( T \) denotes total pressure. Hence \( P \) is defined as follows:

\[
P = \frac{P_4 - P_{TA}}{\rho U_w^2}
\]

This modelling leads to the following nondimensional and rescaled equation:

\[
P = P_e(U) - \lambda \frac{\partial U}{\partial \theta} - (\mu + \mu_{IGV} + L_E) \frac{\partial U}{\partial t} - \frac{\partial \phi}{\partial t}(z = 0) + \frac{\partial \phi}{\partial t}(z = -L_I) + P_{TIGV}
\]

where \( P_e(U) \) is the nondimensional steady total to static pressure rise across the compressor, and \( P_{TIGV} \) is the total pressure change across the IGV's. Parameters \( \lambda \) and \( \mu \) reflect blade row inertia. Parameters \( L_E \) and \( L_I \) reflect respectively, the nondimensional length of the exit and inlet ducts of the compressor. And variable \( \phi \) is the total velocity potential associated with the flow in the inlet.

The uncontrolled system needs an additional dynamic equation to complete the set. When mass is conserved in the plenum then the rate of pressure change with time for the plenum is related to the velocity entering the plenum and the velocity leaving the plenum through the throttle. In nondimensional form this equation was originally derived by Greitzer and found to be:

\[
\frac{dP}{dt} = \frac{1}{B} (U(t) - U_T(P))
\]

where \( U_T \) is the mass flow through the throttle valve. This introduces the important \( B \) parameter. In order to complete the system we must supply the functions \( P_e(U), U_T(P) \), and \( \phi \), the compressor characteristic, the throttle characteristic and the solution for the potential flow in the inlet. The two equations, 3, 4 constitute the Moore-Greitzer model. At this stage one may proceed in either of the following ways:

1. Let the variables be continuous functions of the circumferential position \( \theta \) and time, and solve the ensuing partial differential equations. Since the annulus is naturally periodic, we can express the spatial structure of the solutions in terms...
of Fourier series. The equations then reduce to a set of ordinary differential equations where the dependent variables are the amplitudes of the Fourier modes and the pressure in the plenum. In this case the compressor characteristic, $P_c(U)$, is the pressure rise that would be obtained for a clean axisymmetric flow.

2. A simpler model neglects the details of the circumferential location and uses angle averaged variables. Since $P$ depends only on time, this simplification applies only to $U$. The angle averaged mass flow is defined as

$$U_o(t) = \frac{1}{2\pi} \int_0^{2\pi} U(\theta, t) d\theta$$

and the expression governing its evolution is obtained by integrating equation 3 from $0 - 2\pi$. The compressor characteristic, is now

$$P_c(U_o(t)) = \frac{1}{2\pi} \int_0^{2\pi} P_c(U(\theta, t)) d\theta$$

In the previous case, an axisymmetric characteristic is supplied and the model returns a rotating stall characteristic, that is the performance when the compressor drops into rotating stall. In this case we must supply a mean characteristic, $P_{c0}(U)$, which reflects the unstalled and the stalled performance.

The above equations, even in the absence of any control systems, are completely nonlinear. It is the full set of nonlinear uncontrolled equations which are numerically solved in this paper. Most strategies for control of rotating stall are based on linearised versions of these equations, with additional control law equations. In the next section we discuss the linear version of the equations and in the later sections, we show with numerical data, the shortcomings of the linear theory.

**ANALYTICAL DISCUSSION**

**Spatial Representation using Fourier Series**

The main variables of interest are the total to static pressure rise of the compressor, $P$ and the axial velocity or mass flow, $U$. The pressure rise has been nondimensionalised with $\rho U_w^2$, where $U_w$ is the wheel speed. It can be further rescaled with a characteristic pressure coefficient $H$ where $2H$ is the difference between the maximum pressure coefficient and the shutoff head. The axial velocity has been nondimensionalised with the wheel speed and it can also be further rescaled with $W$ where $2W$ is the mass flow coefficient associated with the maximum pressure coefficient. The axial velocity depends on circumferential location and time, but the pressure rise depends only on time. For convenience, the mean flow, $U_0$ which is a function of only time is considered separately. Hence

$$U = \frac{1}{W} \frac{\dot{u}}{U_w} = U_o(t) + u(\theta, t)$$

$$P = \frac{1}{H} \frac{u^2 - P_{x,s}}{\rho U_w^2}$$

where $\dot{u}$ is the dimensional velocity. The additional scaling introduces the parameter $S$ where $S = H/W$ reflects the steepness of the performance curve. For example high speeds will lead to larger values of $S$. This parameter turns out to very important in prediction of compressor performance and its use circumvents the limitations of using a specific characteristic associated with a single experiment. It allows one to extract some measure of the effects of compressor speed from the Moore-Greitzer model without introducing compressibility effects. The governing equations are now found to be:

$$P'(t) = \frac{1}{4B^2S_i} [U_o(t) - U_0(P)]$$

$$U_o'(t) = \frac{S}{1_c} \left[ -P(t) + \frac{1}{2\pi} \int_0^{2\pi} P_c(U(\theta, t)) d\theta \right]$$

$$[\phi_1(z = 0, \theta, t) - \phi_1(z = -L_1, \theta, t) + \mu u_z] + \lambda u_\theta = SP_c(U(\theta, t)) - \tilde{\mu} U_o(t) - SP_0$$

where $\tilde{\mu} = \mu + \mu_0^p + L_0$ and $l_0 = \mu + L_1$.

The velocity potential in the governing equations $\phi(x, \theta, t)$ associated with the flow in the inlet is determined from Laplace's equation:

$$\phi = U_o x + \sum_{n=1}^{N} A_n(t)e^{in\theta} + \text{c.c}$$

where $\text{c.c}$ denotes the complex conjugate. This is the analytical solution of Laplace's equation for a straight annular duct with an inner to outer radius ratio close to one, so that radial effects can be neglected. For simplicity in the equations we define

$$K_n = \frac{[e^{n\theta}]}{[ne^{n\theta} - \text{c.c}]}$$

which is the coefficient in the above solution when $z=0$, which occurs at the compressor face. The velocity potential at this location is required in the governing equations. The potential is defined as above so that the coefficient $K_n$ disappears from the solution for the axial velocity at the compressor face:

$$U(\theta, t) = \phi_{,x}(z = 0, \theta, t) = U_o(t) + \sum_{n=1}^{N} A_n(t)e^{in\theta} + \text{c.c}$$

When this solution is substituted into equation 8, the third equation becomes

$$\sum_{n=1}^{N} \left[ K_n A_n(t) + \tilde{\mu} A'_n(t) + \lambda in A_n(t) \right] e^{in\theta} + \text{c.c} =$$

$$SP_c(U) - \frac{S}{2\pi} \int_0^{2\pi} P_c(U) d\theta$$

$$\text{c.c}$$

$$\text{c.c}$$
The right hand side is a nonlinear function of the $A_n$‘s but can also be expressed in terms of a Fourier series,

$$P_e(U) = -\frac{1}{2\pi} \int_0^{2\pi} P_e(U) d\theta = \sum_{n=1}^{N} F_n e^{i\omega n} + c.c.$$  

Finally the equation for the complex amplitude of the $n$th mode in the Fourier series representation of rotating stall becomes:

$$K_n A_n(t) + \ddot{A}_n(t) + \lambda n A_n(t) = SF_n$$  

From this equation we can discern which parameters affect the linear growth of a rotating stall disturbance. The complex form can be transformed to a real amplitude using the identity

$$R_n^2 = 4((\text{Re}(A_n))^2 + (\text{Im}(A_n))^2)$$

and equation 14 becomes:

$$\frac{1}{S}(K_n + \mu) \frac{d}{dt} \left( \frac{R_n^2}{8} \right) = \text{Re}(F_n) \text{Re}(A_n) + \text{Im}(F_n) \text{Im}(A_n)$$  

This is still the full nonlinear equation as we have neither specified the form of the axisymmetric characteristic, nor have we yet linearised the nonlinear right hand side.

Substituting $P_e(U)$ into equation 16, and keeping only the linear terms we determine the equation which governs the linear rate of growth of the nth Fourier mode:

$$\frac{1}{S}(K_n + \mu) \frac{d}{dt} \left( \frac{R_n^2}{8} \right) = \frac{dP_e}{dU}(U = U_e) R_n^2$$

which can be expressed more simply as

$$\frac{d}{dt} (R_n^2) = \frac{2S}{(K_n + \mu)} \frac{dP_e}{dU}(U = U_e) R_n^2$$

To linear accuracy this equation is uncoupled from the equations for the other Fourier modes and from the changes in the mean mass flow and pressure rise. Thus in a linear analysis we can examine it separately. This assumption is fundamental to the control strategies which prevent the compressor from stalling by suppressing disturbances Fourier mode by Fourier mode. A nonlinear analysis requires the coupling of all the modes and the mass flow and pressure rise. When the compressor behaves nonlinearly, any control system based on this linear approximation will fail.

From this equation we can extract the parameters which affect the linear growth of each Fourier mode in a rotating stall disturbance. The slope function $dP_e/dU$ is the only term which can control the sign. No matter what function we use to describe the steady pressure rise, the slope will be positive for values of equilibrium mass flow smaller than the value at the peak of the compressor characteristic, and the slope will be negative, when the mass flow is greater than this critical value. As we will show later, the model reveals that a rotating stall disturbance can grow when the slope is negative if the disturbance is 'large enough', even though linear theory predicts that the disturbance will decay.

We note here that all the Fourier modes linearly lose stability at the same flow coefficient. In some compressors it has been determined that the $n=1$ Fourier mode will be the first to go unstable and a blade row time lag has been suggested in [2] to improve the model. If this were always the case, then rotating stall would always be observed to grow from global disturbances. However in some compressors, the disturbances growing into rotating stall are seen first as local spikes in the flow field. In these cases the structure of the disturbance is dominated by the higher order Fourier modes. When the compressor loses stability in a linear manner, probably at low speeds, the global disturbance theory applies and the $n=1$ Fourier mode is the first to go unstable. At high speeds, when the compressor is linearly stable, but nonlinearly unstable, the higher Fourier modes are more important than $n=1$. It is a topic of further study to investigate the effect of the additional blade row time lag model, [2], on the nonlinear results.

The growth rate, defined to be the coefficient of $R_n^2$ in equation 18, increases as the unstable side of the characteristic gets steeper. We note however that increasing $S$ or decreasing $\mu$ will have the same effect as increasing the slope of the unstable side of the characteristic. Steeper slopes lead to larger growth rates. This means faster decay of a disturbance when the mass flow is on the negatively sloped side of the characteristic. Later we will examine the possibility that parameter values which are dangerous when the operating point is on the positively sloped side of the characteristic where they promote faster growth, may be advantageous when the operating point is on the negatively sloped side, since there they will lead to faster decay of a small disturbance.

There are two points which will be discussed in the context of this analysis. First we will show that if the nonaxisymmetric disturbance grows rapidly, it will more likely be a spike than a global disturbance. We will also show that the linear analysis breaks down even for small amplitude disturbances when the operating point is on the stable side of the characteristic. If we provide a one-mode disturbance, it will decay according to the linear equation given above, but the other modes instantly start to grow because of nonlinear interactions. The energy in these modes is small, but once it has been redistributed from the first mode to the higher modes, (so that the disturbance is now a spike rather than a single mode), it is sufficiently large to push the performance away from the stable operating point into fully developed rotating stall.

**Rotating Stall and Axisymmetric Flow may Both be Stable**

Experiments have long shown that a hysteresis exists between steady operating conditions and fully developed rotating stall. We most often think of the events as occurring in the following order. As we close the throttle past the peak of the performance curve, the operating point loses stability, say at $K_{TC}$ and the compressor drops into rotating stall. As the throttle is closed further, a locus of steady rotating stall points is traced out. When the throttle is opened again the compressor does not recover until
Fourier Transform routines were developed by Paul Swarztrauber for the integration of the equations in time. The forward and reverse Fast Fourier fifth order Runge Kutta Fehlberg scheme was used for the integrations. Eight were carried out on a Sparc station, using 128 points to recover the compressor and this topic has been discussed in a previous paper on the stability of fully developed rotating stall. [4]

Spatial Nonlinearities

In the following sections, all numerical integrations of Equations 8 were carried out on a Sparc station, using 128 points to resolve the variation in the circumferential direction. An adaptive fourth-fifth order Runge Kutta Fehlberg scheme was used for the integration of the equations in time. The forward and reverse Fast Fourier Transform routines were developed by Paul Swarztrauber and are freely available from the netlib mathematical software repository.

Early in its inception, rotating stall may be observed as either a global disturbance, like a sine wave, or a local disturbance, like a spike. An example of each of these occurrences is shown in Figure 4. In this figure we show several snapshots of the axial velocity as it varies around the annulus of the compressor, from 0 to 2π. In both examples, the same initial disturbance was given, so that the different dynamical behavior is due to the system parameters. We have determined that the important parameters affecting stall inception are

\[ S, \lambda, K_T \]

and the initial amplitude of a disturbance. Each of these will be discussed in the next sections and finally the effects will be interpreted in terms of the physical parameters.

Effect of Aspect Ratio of Performance Map, S

One of the key parameters in determining the nature of the dynamics at the inception of rotating stall is the aspect ratio S. This parameter appears in the rescaling of the compressor characteristic, causing all performance curves to fall on a single characteristic curve. Parameter S is defined as \( H/W \), where H is half of the difference between the pressure rise at the peak of the characteristic and the shutoff head, and W is half of the corresponding mass flow coefficient. Larger values of S correspond to steeper performance curves, which are in turn associated with higher speeds.

The stall inception process is shown for several values of S in Figure 5. The figures on the left show the axial velocity readings from 8 probes placed at equally spaced intervals around the annulus of the compressor. The throttle is closed to the critical case where the parabolic characteristic passes through the peak of the compressor characteristic. The operating point is neutrally stable, meaning that linear theory would predict that a nonaxisymmetric disturbance would have zero growth rate. In fact when we supply such a disturbance, we see that, in all cases, the linear theory applies to the first mode for a period of time. However the second and higher modes start to grow immediately because of nonlinear interactions, that is spatial nonlinearity. When the energy in these...
modes is sufficiently large, the amplitude of the disturbance suddenly takes off. To explain the concept of the spatial nonlinearity, imagine that a disturbance of the form

\[ \epsilon \sin \theta \]

exists in the compressor. The model for the compression system is nonlinear and both quadratic and cubic terms are present in the expression for the compressor characteristic. When the quadratic nonlinearity is applied to the disturbance, then the following terms will appear:

\[ \epsilon^2 \sin^3 \theta = \frac{\epsilon^2}{2} (1 - \cos 2\theta) \]

This causes a small change in the mean mass flow and also introduces energy to the second mode, even though this was originally zero. Similarly the cubic term creates a nonzero amplitude for the third Fourier mode. If the operating point is stable, then

\[ \frac{dc}{dt} < 0 \]

so that the initial disturbance will decay and the energy in the other modes will be very small and thus negligible. If the operating point is linearly stable but nonlinearly unstable, then this result holds only when \( \epsilon \) is very small. If any nonaxisymmetric disturbances exist in the compressor that have amplitude larger than some critical value of \( \epsilon \), then they will start to grow. As \( \epsilon \) grows, the amplitude of the higher Fourier modes and the disturbance to the mean flow also grow. This is exactly the scenario that is unfolding in Figure 5.

We note several features of these plots which vary with \( S \). First, it is clear that at low speeds, small \( S \), the disturbance takes of the order of twenty revolutions to reach the critical point where the amplitude suddenly increases. When \( S \) is increased to 2, this time is reduced to around five rotor revolutions. Secondly we note that the gradient of this rapid phase is steeper at larger values of \( S \). This implies that increasing \( S \) not only decreases the length of the 'linear' phase, it also causes more rapid growth in the 'nonlinear' phase. The difference in slope of this nonlinear phase is important since this reveals whether the first mode will dominate, producing a global disturbance, or the Fourier modes mix more quickly, producing a spike disturbance.

Although the linear theory for stability of the operating point breaks down, we note that the linear prediction of more rapid growth of rotating stall at larger values of \( S \) still holds true. Larger values of \( S \) correspond to steeper performance curves, which appear at higher speeds. Thus we note that for higher speed compressors, rotating stall disturbances grow both earlier and faster. These issues require further investigation to determine how important they may be for the design of a control system for high speed compressors.

These results for \( S \) are consistent with [4], where it was shown that a fully developed stall cell is more like a square wave at large values of \( S \). Small values of \( S \) lead to stall cells which are more like travelling sine waves.

We continue to consider the operating point where the throttle characteristic goes through the peak of the compressor characteristic. As before, linear theory predicts zero growth rate for any rotating stall type disturbances. We supply a small amplitude sine wave disturbance, so that

\[ U(\theta, t = 0) = U_0(t = 0) + \epsilon \sin \theta. \]

Initially the first mode grows very slowly behaving in a manner close to the linear prediction. However as it is grows, it feeds energy to the higher spatial modes, so that at a certain critical point, the rotating stall disturbance begins to grow rapidly, Figure 6.

The trends observed previously for increasing values of \( S \) are repeated here as \( \lambda \) is decreased. When \( \lambda \) is large, the nonaxisymmetric disturbance begins rapid growth after 20 rotor revolutions. Initially this nonaxisymmetric disturbance is dominated by the first Fourier mode. For smaller values of \( \lambda \), however the nonaxisymmetric disturbance grows rapidly and even from its early stages is more spike like in appearance.
control strategy.

of these trends are highly desirable for the efficacy of a linear

performance responds more rapidly to a change in mass flow. This

is the characteristic time lag. As \( r \) increases, the compressor

holds true. We recall the definition of \( A \) as

In all of these cases, once the period of rapid growth begins, the

stall cell develops in 5 to 10 rotor revolutions. Note that we have

chosen a small value of \( S \) for these cases and if \( S \) is large and \( \lambda \) is

small, the stall cell will develop in a couple of rotor revolutions.

While the linear stability results for the operating point are

insufficient for actual operation, we note that the linear prediction

of more rapid growth of rotating stall at smaller values of \( \lambda \) still

holds true. We recall the definition of \( \lambda \) as

\[
\lambda = \frac{N(L_{\text{stator}} + L_{\text{rotor}})}{R} = \frac{N r U_w}{R}
\]

where \( R \) is the characteristic radius, \( N \) is the number of stages,

\( U_w \) is the wheel speed at the radial position of \( r \), and \( \rho \) is

the characteristic time lag. As \( r \) increases, the compressor

performance responds more rapidly to a change in mass flow. This

corresponds to an increase in the value of \( \lambda \). Further experimental

research is required to relate the time lag \( r \) to actual physical

parameters, since larger values of \( \lambda \) delay the development of

rotating stall and also limit the spatial nonlinear effects so that

the stall inception is dominated by the first Fourier mode. Both

of these trends are highly desirable for the efficacy of a linear

control strategy.

Effect of Amplitude of Initial Disturbance

Not surprisingly, we note in Figure 7 that when the initial dis-

turbance is larger, rotating stall develops earlier. The more inter-

esting result here is that when the initial disturbance, of the form

\( \epsilon \sin \theta \) is very small, not visible for the first 15 rotor revolutions,

the higher Fourier modes appear while the nonaxisymmetric dis-

turbance is still small. Therefore, we note that the precise structure

of the disturbances present in the compressor does not matter when

the disturbances are small. The Fourier modes always interact to

produce a small amplitude spike disturbance, which then grows

rapidly. Stall inception is dominated by the first Fourier mode,

leading to a global disturbance, only when the initial amplitude of

that mode in the disturbance is large. This may happen when the

compressor receives a significantly distorted flow.

Another example is shown in Figure 8, where the only change is

that \( S \) has been increased from 0.75 to 1.0. Included in this figure is

the performance map, where we see that the throttle characteristic

passes through the peak of the compressor characteristic. Also

included is the locus of points traced out as the compressor drops

into rotating stall. As the amplitude of the disturbance is

decreased, the trend is the same as the previous figure. In the last

case, we note that when the initial disturbance is extremely small,

less than one percent of the mean flow, the higher Fourier modes

are the first to be visible and the actual stall cell development is

similar to that observed in experiment. Controlling the amplitude

of the first Fourier mode would be useless in this case, as the

inception process is dominated by the higher Fourier modes.

TEMPORAL NONLINEARITIES

In the cases considered so far, the throttle characteristic was

closed to the point where it passed through the peak of the axisym-

metric characteristic. At this point the equilibrium point is only

neutrally stable so that linear theory predicts that a disturbance

will neither grow nor decay. In a nonlinear system, an equilib-

rium point that is neutrally stable is called structurally unstable,

where even a tiny nonlinear effect could be enough to render the

operating point unstable. Hence the previous results are not alto-

gether surprising. Controlling the operating point so that it will

retain its stability past the peak of the performance characteristic

would be a difficult task. In fact, it is not one that can sensibly be

tackled until we have addressed the issue of stability before the

peak, when the operating point is on the negatively sloped side

of the characteristic. It is this issue which we investigate in the

current section.

For a range of throttle settings, before the throttle is closed to

the peak of the performance curve, the operating point is linearly

stable but may be nonlinearly unstable. A typical performance

map for such a case is shown in Figure 9. The size of the initial

disturbance required to cause the compressor to go unstable even

though it is linearly stable, varies with the system parameters and

the throttle setting.

All initial conditions which are attracted to the operating point

are said to be within the basin of attraction of that point. It is

desirable to make this basin of attraction as large as possible. In
FIGURE 7 Development of Rotating stall from a small amplitude disturbance for a range of different amplitudes of initial disturbance. The initial disturbance in each case is a sine wave. The other system parameters have values: $\lambda = 2$, $S = 0.75$, $I_1 = 2.0$, $I_2 = 2.0$, $1/K_T = 1.14$, $B = 0.4$.

In each case we determine the lowest possible amplitude of initial condition which will cause the compressor to go unstable for a range of throttle settings, $K_T < K_T^c$. In our initial discussion we will assume the disturbance is of the form: $\epsilon \sin \theta$, but we will also investigate the effects of other disturbances on the stability of the operating point and on rotating stall inception.

Effect of Throttle Setting, $K_T$

Figure 10 shows the rate of growth of a specific amplitude disturbance for a range of values of throttle setting, $K_T$. When the throttle is wide open, $1/K_T = 1.7$, the first mode decays significantly over the first two rotor revolutions. During this time the energy in the other modes grow and at some critical point, all the modes begin to grow and the rotating stall disturbance appears as a spike. Note that the fully developed stall cell is narrow since the throttle is open.

As the throttle is closed, we note that the first mode decays less, until the point where it no longer obeys the linear stability criterion and begins to grow immediately. For smaller values of $1/K_T$ the fully developed stall cell is wider. When both rotating stall and steady axisymmetric flow are stable, the design flow will lose stability as a large enough disturbance is given. When $1/K_T$ is larger, say 1.7 then the size of the disturbance required to cause instability may be far beyond the noise level in the flow and more likely the effect of a severely distorted flow. We include the result here as an exaggerated example of the fact that the compressor may go unstable even though the energy in the $n=1$ Fourier mode is decaying.

In Table 1 we show the minimum amplitude of disturbance, as a percentage of the mean flow, which will render the operating point unstable for a range of parameter values. In each case we assume that the initial mass flow coefficient and pressure rise coefficient are at the operating point. The amplitude of disturbance that the operating point will tolerate and still remain stable increases significantly as the throttle is opened. Parameter $S$ has little impact on these results. Only when $S=0.5$ do we notice a small increase...
FIGURE 9 A typical case showing how the compressor may lose stability when the throttle is open before the peak and linear theory predicts that the operating point is stable. The model parameters for this case were \( \lambda = 0.5, S = 1, I_f = 2, l_E = 4, 1/K_T = 1.7, B = 0.4, P_{co} = 1.5 \).

FIGURE 10 When the throttle is closed, that is \( 1/K_T \) decreased, a disturbance of a specific amplitude will grow faster. The other model parameters had values \( \lambda = 1, S = 0.75, I_f = 2.0, l_E = 4.0, B = 0.4, P_{co} = 1.5 \).

TABLE 1 This table shows the effect of \( S \) on the maximum size of initial disturbance as a percentage of the mean, that the compressor will tolerate when the operating point is on the negatively sloped side of the characteristic.

<table>
<thead>
<tr>
<th>Throttle setting, ( K_T )</th>
<th>1.143</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S = 0.5 )</td>
<td>0.05</td>
<td>1.71</td>
<td>4.92</td>
<td>7.49</td>
</tr>
<tr>
<td>( S = 1.0 )</td>
<td>0.05</td>
<td>1.37</td>
<td>4.69</td>
<td>7.36</td>
</tr>
<tr>
<td>( S = 1.5 )</td>
<td>0.05</td>
<td>1.22</td>
<td>4.60</td>
<td>7.36</td>
</tr>
<tr>
<td>( S = 2.0 )</td>
<td>0.05</td>
<td>1.22</td>
<td>4.60</td>
<td>7.31</td>
</tr>
</tbody>
</table>

TABLE 2 This table shows the effect of \( \lambda \) on the maximum size of initial disturbance as a percentage of the mean flow, that the compressor will tolerate when the operating point is on the negatively sloped side of the characteristic.

<table>
<thead>
<tr>
<th>Throttle setting, ( K_T )</th>
<th>1.14</th>
<th>1.2</th>
<th>1.4</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda = 2.0 )</td>
<td>0.05</td>
<td>1.32</td>
<td>4.98</td>
<td>7.88</td>
</tr>
<tr>
<td>( \lambda = 1.0 )</td>
<td>0.05</td>
<td>1.22</td>
<td>4.70</td>
<td>7.32</td>
</tr>
<tr>
<td>( \lambda = 0.67 )</td>
<td>0.05</td>
<td>1.17</td>
<td>4.48</td>
<td>6.97</td>
</tr>
<tr>
<td>( \lambda = 0.5 )</td>
<td>0.05</td>
<td>1.12</td>
<td>4.29</td>
<td>6.71</td>
</tr>
</tbody>
</table>

in the permissible disturbance level. It seems that high speed machines are no more sensitive to the disturbance level than low speed machines. We recall from the linear analysis in the last section that increasing \( S \) implied faster decay of a disturbance when the operating point is on the negatively sloped side of the characteristic. We hoped that this would lead to an operating point with a larger basin of attraction. However, the trend is to decrease the size of the basin of attraction as \( S \) is increased. Thus we see that increasing the attractiveness of the operating point at the same time increases the attractiveness of the rotating stall point.

In Table 2 we show the minimum amplitude of disturbance as a percentage of the mean flow, which will render the operating point unstable for a range of values of blade row inertia. In each case we assume that the initial mass flow coefficient and pressure rise coefficient are at the operating point. We find that the permissible disturbance level is somewhat increased as \( \lambda \) increases. So as the time lag parameter for the compressor, \( r \) increases, the operating point is slightly more stable. Again we note the failure of the linear analysis in the last section to capture this behaviour. The linear analysis would have led us to believe that increasing \( \lambda \) would lead to an decrease in the size of the basin of attraction.

Other parameters, such as \( B \), have been found to have no effect on the nonlinearity stability of the operating point. However, preliminary evidence shows that increasing \( B \) causes the stall cell...
to develop slightly faster.

COUPLED SPATIAL AND TEMPORAL NONLINEARITIES

In all previous cases, we assumed that the initial condition was a small amplitude sine wave, $\sin \phi$. It was shown that initial conditions of this form often develop rapidly into a local disturbance rather than a global disturbance by funnelling the energy of the disturbance to the higher Fourier modes. However, we are also interested in the difference between a disturbance which is initially local and one which is initially global. To examine this case we have compared the evolution of two different types of disturbance. The first disturbance is a sine wave and the second disturbance is a local square wave, spanning only 5% of the annulus. For the sake of comparison, we have chosen the same amplitude for the two types of disturbances.

The nonlinear stability of the operating point appears to be relatively unaffected by the type of disturbance, that is, the operating point will lose stability once the amplitude crosses a certain threshold, regardless of the structure of the disturbance. However, it is clear from Figure 11, that a local spike initial condition, will cause rotating stall to develop much more rapidly than the global disturbance. In this figure, we have shown the stall cell profile from 0 to $2\pi$ as it is given in the initial condition and after the first few rotor revolutions. The throttle is opened so that the operating point lies on the negatively sloped side of the compressor characteristic, where it is linearly stable, but nonlinearly unstable. This figure also shows that a small amplitude disturbance can grow to an uncontrollable level in a matter in two or three rotor revolutions. The amplitude, 7.3% of the mean velocity, is the smallest disturbance for which the operating point is unstable. Only those disturbances smaller than this will obey the linear criterion and decay.

![Figure 11](image1.png)

**FIGURE 11** A spike disturbance will develop into rotating stall more rapidly than a global disturbance. The initial amplitude was 7.3% of the mean velocity. Other model parameters were $S = 1, \lambda = 2, l_1 = 2, l_2 = 2, 1/K \tau = 1.3, B = 0.4, P_{co} = 1.5$

In Figure 12, we have shown the stall cell evolution for the same parameter values as given in the last case, but here we have increased the amplitude of the initial disturbance. The sine disturbance in this figure grows faster than the one in Figure 11, but we note that after two rotor revolutions, the amplitude is still small, as opposed to the spike initial condition, which after the same period of time has reached its maximum amplitude, though only over a fraction of the annulus.

![Figure 12](image2.png)

**FIGURE 12** Comparison of evolution of spike and sine initial conditions over the first seven rotor revolutions. The initial amplitude was 8.9% of the mean velocity. Other model parameters were $S = 1, \lambda = 2, l_1 = 2, l_2 = 2, 1/K \tau = 1.3, B = 0.4, P_{co} = 1.5$

These two examples show that, when both a spike and a sine disturbance exist, the spike will grow faster and dominate the rotating stall cell development.

CONCLUSIONS

1. Both spatial and temporal nonlinearities are important at stall inception. The spatial nonlinearities lead the nonaxisymmetric disturbance to develop as a spike rather than a global phenomenon.
2. A spike disturbance cannot be resolved by three Fourier modes.
3. Increasing parameter $S$ will cause rotating stall to develop earlier and faster. This implies that a disturbance in the compressor will grow into rotating stall more rapidly in high speed machines.
4. Decreasing parameter $\lambda$ has the same trend as increasing parameter $S$. This parameter can be modified by changing the blade chord. We observe that long blade chords are more stable than shorter ones.
5. The parameter which has the strongest effect on the nonlinearity stability of the operating point is the throttle setting, $K_T$. Neither $S$ nor $B$ have a significant impact on the basin of attraction of the operating point when it is linearly stable, but nonlinearly unstable. Increasing parameter $\lambda$ does make the operating point slightly more stable.
6. Comparison of the rotating stall development from a spike initial condition and a sine wave initial condition, shows that the former grows more rapidly and will be more difficult to control than the latter.
7. When the initial disturbance is less than one percent of the mean velocity, the higher Fourier modes are the first to be visible and the inception of rotating stall is similar to that observed experimentally.
8. When the throttle is open so that the operating point lies on the linearly stable negatively sloped side of the characteristic, large enough disturbances, greater than around five percent of the mean velocity, are nonlinear and grow to an uncontrollable magnitude in a matter of one or two rotor revolutions.

The quasi two dimensional Moore Greitzer model is unable to capture the details of how rotating stall develops in the compressor. It is difficult for even a higher dimensional model to acquire the information about how disturbances grow into rotating stall and where these disturbances appear in the compression system. On the other hand, due to the limited number of sensors that can be placed in the flow, a control system will have only limited information. Current strategies use eight probes placed at equally spaced intervals around the annulus at several axial locations. If rotating stall develops from a global disturbance, then the information provided by these probes may be sufficient to permit the disturbance to be damped while it is still small, that is linear. However if the disturbance is a localised spike, it will be harder to detect by the eight annular probes. The time delay in detection allows the disturbance to grow and the larger nonlinear disturbance will be more difficult if not impossible to eliminate. Control strategies which have the central premise of delaying instability by damping the $n=1$ Fourier mode and then the $n=2$ Fourier mode will fail in these circumstances.

ACKNOWLEDGMENTS

This work was supported by NASA Lewis Research Center under Grant number NAG 1179. The author particularly appreciates the support and encouragement of Dan Buffum, Colin Drummond and Karl Owen.

BIBLIOGRAPHY