ABSTRACT

Many models and formulae have been put forward, over the years, for the determination of the toughness necessary for the arrest of propagating ductile fracture in gas pipelines. One of the first, and most prominent, was that developed by Battelle Columbus Laboratories for the Pipeline Research Committee of the American Gas Association. As originally embodied, the model involved the comparison of curves expressing the variation of fracture velocity and of decompression wave velocity with pressure (the "two-curve model" - TCM). To aid in analysis, at a time long before a computer was available on every desk, a "short formula" (SF) was developed that provided a good fit to the results of the TCM for a substantial matrix of conditions. This SF has subsequently been adopted by several standards bodies and used widely in the analysis of the results of full-scale burst tests. Since the original description of the derivation of the SF is to be found only in a report to the PRC dating back to the Seventies, many in the pipeline industry today are left without a full appreciation of its range of validity. The present paper briefly discusses the original intent of the SF as a substitute for the TCM, and presents the results of extensive calculations comparing the results of the two. It can be concluded that the SF provides an excellent estimate of the results of the TCM over a very wide range of design and operating parameters, within the limitations inherent in the method.

INTRODUCTION

Over the years, many models and formulae have been developed with the purpose of determining some measure of the toughness that is necessary to arrest a propagating ductile fracture in a gas pipeline (see, for example, Ref. [1]). Some of these approaches are analytical, and explicitly consider both the driving force for fracture propagation, provided by the pressure of the expanding gas within the bursting pipeline, and the resistance to fracture offered by the pipe and by any backfill. Others are merely statistical fits to full-scale test results. In recent years, much attention has been paid to ways of dealing with the difficulties of measuring fracture propagation resistance in modern steels with high levels of strength and apparent toughness. The effect of rich gases, high pressures and low temperatures on gas decompression behaviour have also received renewed attention, as projects involving new, long-distance, frontier pipelines are once again under active consideration (see, for example Ref. [2]). Nevertheless, a need remains for a relatively simple method to predict ductile fracture arrest requirements for conventional pipelines, both to serve the requirements of pipeline standards and to allow for engineering analysis. The Battelle "short formula" (SF), which is based on the results of the Battelle two-curve model (TCM), has been widely used for this purpose, but there is not a wide appreciation of its background, accuracy and range of validity. In the following sections, the background to the TCM will be briefly reviewed, the development of the SF will be discussed, and a fairly comprehensive comparison between the results of the two methods will be presented. Some commentary will be provided on the limitations of each method, and on the limitations of methods based on the application of ideal gas equations in general.

THE BATTELLE TWO-CURVE MODEL

The basis of the Battelle TCM was first presented publicly by Maxey, some 25 years ago [3], and it has recently been reviewed in detail in Ref. [4]. In order to appreciate fully the
relationship between the TCM and the SF, however, it is necessary at least to summarize the method and the equations on which the former is based.

The TCM considers the driving force for ductile fracture propagation to be uniquely related to the gas pressure in the plane of the crack tip. When a full-bore opening occurs in a pipe, the pressure clearly cannot immediately fall to zero, since the gas outflow velocity cannot exceed the acoustic velocity. In fact, an expansion wave begins to travel down the pipe, away from the rupture site, whose leading edge moves at the acoustic velocity under the original operating conditions. Each successively lower pressure in the wave travels at a lower velocity, reflecting the algebraic sum of the local outflow velocity and the local acoustic velocity (as influenced by temperature and pressure). The wave propagation will continue uninterrupted until some obstruction (e.g. a closed block valve) is reached. In the steady state, the crack tip pressure will be that pressure which corresponds to the same propagation velocity as the fracture itself. The TCM thus functions by comparing curves expressing the variation of fracture velocity and decompression velocity.

The decompression problem is simplified if the expansion can be considered adiabatic and isentropic (reasonable assumptions within tens of metres of the origin, for typical gas transmission diameters) and if the gas approximates ideal behaviour, under pipeline conditions. Pure methane and lean mixtures at pressures and temperatures typical of pipelines in the Seventies could reasonably be modelled as ideal. For these conditions, there is a closed-form solution for the relationship between decompression velocity within the expansion wave and pressure, given by

\[ P_d = P_i \left[ 2 + \left( \frac{\gamma - 1}{\gamma + 1} \right) V_o \right]^{2y} \] (1)

where

\( P_d \) is the decompressed pressure within the wave;

\( P_i \) is the initial pressure;

\( V \) is the velocity of propagation of pressure \( P_d \);

\( V_o \) is the original acoustic velocity; and

\( \gamma \) is the original specific heat ratio of the gas.

This relationship can easily be rearranged to be explicit in \( V \), and then produces curves like that labelled “gas decompression” in Figure 1.

The determination of the fracture velocity curves is a two-step process. First, it is necessary to determine the “arrest pressure”, \( P_a \), that is, the pressure below which no propagating fracture can be sustained, the intercept on the pressure axis in Figure 1. This was achieved by modification of a fracture mechanics equation for initiation that had already been developed, to give

\[ \sigma_a = \frac{2\sigma}{3.33\pi} \cos^3 \exp \left[ -\left( \frac{\pi R E}{24\sigma^2 V_d^2} \right)^{1/2} \right] \] (2)

where

\( \sigma_a \) is the arrest stress, corresponding to \( P_a D/2t \);

\( E \) is Young's modulus;

\( \sigma \) is the material flow stress;

\( D \) and \( t \) are the pipe diameter and wall thickness; and

\( R \) is the fracture propagation resistance.

In the original formulation of the TCM and for the purposes of this paper, \( R \) can be equated to the Charpy shelf energy per unit area.

The second step in the determination of the fracture curve is the calculation of fracture velocity, for values of pressure higher than \( P_a \). The equation developed for this purpose is

\[ V_f = \frac{C\sigma}{\sqrt{R}} \left( \frac{P}{P_a} - 1 \right)^x \] (3)

where

\( V_f \) is the fracture velocity corresponding to pressure \( P \);

\( C \) is a constant depending on the presence and type of backfill; and

\( x \) is an exponent, originally determined to be 1/6 when \( R \) is equated to Charpy shelf energy per unit area.

This equation allows a series of curves showing the variation of fracture velocity with pressure, for specific levels of toughness, to be developed (see Figure 1).

The relative position of the fracture and decompression velocity curves determines whether ductile fracture will arrest or continue to propagate. For all rupture events, the initial...
condition corresponds to the operating pressure, and for all practical designs, the initial decompression velocity will exceed the fracture velocity. The pressure in the plane of the crack tip thus declines, and the fracture velocity decreases along the appropriate curve (this discussion ignores transient effects close to the origin, where the opening in the pipe is not fully developed). If, however, an intersection exists between the fracture and decompression velocity curves (e.g. Curve 1 in Figure 1), the fracture and the pressure level that is driving it run together at the same velocity, and no further decompression is possible. The fracture will thus run (at least) until the next discontinuity in toughness or geometry is encountered. On the other hand, if the toughness is increased such that no intersection exists (e.g. Curve 3 in Figure 1), decompression velocity exceeds fracture velocity for all pressure levels. The pressure in the plane of the crack tip thus declines until it falls below the arrest pressure, and the fracture rapidly arrests. Tangency between the two curves (Curve 2 in Figure 1) thus represents the boundary between arrest and propagation, and the corresponding toughness level is referred to as the arrest toughness.

THE BATTELLE SHORT FORMULA

In the mid-Seventies, when the TCM was developed, the determination of arrest toughness in the way described above was a relatively laborious undertaking; for design studies, fracture velocity curves for several different Charpy energies, and sometimes for different combinations of strength and wall thickness, needed to be calculated. In addition, pipeline standards bodies were considering the framing of requirements intended to prevent long ductile fractures. The format of the TCM does not lend itself to such an application, although presumably tabulated values of arrest toughness for a matrix of design conditions could have been adopted. However, to aid in the application of the Battelle approach, a “short formula” was quickly developed that provided arrest toughness on the basis of a statistical fit to a matrix of results from TCM calculations. The initially published equation, which is still by far the most widely known, was that given by Maxey in 1974 [3]. In its most familiar form (in US customary units) it gives the arrest toughness for a 2/3-size Charpy specimen, as

\[ CV_{ar} = 0.0072 \sigma_H^2 (D t/2)^{1/3} \]  

where

\( CV_{ar} \) is the arrest toughness in ft lb; and
\( \sigma_H \) is the hoop stress corresponding to the operating pressure, in ksi.

In SI units, again for the 2/3-size specimen, this becomes

\[ CV_{ar} = 2.382 \times 10^4 \sigma_H^2 (D t/2)^{1/3} \]  

where

\( CV_{ar} \) is in J and \( \sigma_H \) is in MPa.

This equation was originally presented [3] as having been developed for an initial acoustic velocity of 396 m/s, yield strength from 414 to 552 MPa, hoop stress from 60 to 80% of SMYS and a \((D t/2)\) product from 1290 to 12900 mm². The specific heat ratio, \( \gamma \), of course, is not included in the SF, but a standard value of 1.4 was adopted for the TCM calculations. The calculations also applied only to backfilled pipe (i.e. the leading constant for soil backfill was used in Equation (3)) and, of course, to all-gas decompression.

Later, Maxey et al. provided an equation based on a somewhat broader range of variables, including acoustic velocity (but not specific heat ratio) as a variable in the SF [5]. In SI units, this version, again for the 2/3-size specimen, is

\[ CV_{ar} = \left[ 4.747 \times 10^5 - (5.967 \times 10^8 V_s) \sigma_H^2 (D t/2)^{1/3} \right] \]  

As can be quickly verified, if \( V_s \) is placed equal to 396.34 m/s, equation (5) is reproduced.

With the computing power that is readily available today, it is easy to conduct a comprehensive validation of equation (6) against the predictions of the TCM. In fact, at least for carrying out engineering analysis, it could be said that the SF no longer serves a useful purpose, since it is now so simple to use the TCM (particularly assuming ideal gas behaviour). However, the SF is still used and quoted in a number of standards, world-wide [6-8], so its accuracy remains of considerable interest. Note that the range of validity of the TCM itself is a separate issue that is continuing to receive widespread attention.

Figure 2 shows the results of a very wide-ranging validation exercise. It covers:

- OD from 12 to 48 in (305 – 1219 mm) in steps of 4 in;
- Pressure from 4.1 to 15.2 MPa in steps of 2.6 MPa;
SMYS from 345 to 552 MPa in steps of 69 MPa; 
\( \sigma_0 \) from 0.64 to 0.80 SMYS in steps of 0.08; 
\( V_a \) from 366 to 427 m/s in steps of 30.5 m/s. 
\( \gamma \) constant at 1.4.

It can be seen that some SMYS and OD values have been adjusted from standard specification values to allow simpler looping procedures, but the overall correlation covers virtually the entire range of practical designs.

The high accuracy of the fit between the results of the SF and of the TCM is readily apparent, in particular from the slope and square of the correlation coefficient which are shown in the figure. Note that the fit has been forced through the origin, but this makes a negligible difference to the quality of the correlation. In reality, the fit is even better than is apparent from a superficial view of the data points, since a large number of the points are superimposed on each other, centred about the 1:1 line. In fact, of the 1749 results incorporated in Figure 2:
785 are within 1 J (i.e. the results of the TCM and SF calculations are within 1 J of each other);
1284 are within 2 J;
815 are within 2%;
there are only 162 outside 5%;
there are only 29 outside 10%.

Of the latter 29, all relate to the smallest diameters and to thicknesses less than 4.3 mm. It should be noted that all combinations resulting in thicknesses less than 2.5 mm have been eliminated from Figure 2. The related designs, for the most part, were impractical; in this thickness range, there was a systematic tendency for the SF to under-predict the TCM by some 10-20%.

Perhaps a clearer appreciation of the relationship between the predictions of the SF and the TCM can be gained from the consideration of a realistic series of design cases. Figure 3 compares the results of the two methods for a matrix of cases based on a D/t ratio of 80, with:
OD 508, 762 and 914 mm;
SMYS 359, 448 and 552 MPa; and
\( \sigma_0 \) 0.60, 0.72 and 0.80 SMYS.
This parameter range leads to pressures in the range 5.38 to 11.03 MPa.

The relationship shown in this figure is based on a constant acoustic velocity of 396 m/s and specific heat ratio of 1.4. The slope is slightly different from that of the overall relationship in Figure 2, but, for practical purposes, is indistinguishable. The quality of the correlation is also equivalent. Also shown in Figure 3 is the relationship between the arrest toughness from the short formula and that (labelled “GASFRAC”) calculated using a gas decompression model that iteratively calculates the thermodynamic parameters of the gas at each step of the decompression process, based on the BWRS equation of state. This model was developed to allow rich gases, in which the phase boundary could be intersected during the decompression process, to be addressed by the TCM. It can be applied to single-phase behaviour, however, and is appropriate when initial conditions (in terms of gas composition, pressure and temperature) are such that the assumption of ideal gas behaviour is inapplicable. The gas temperature assumed in the present case was 20°C. It can be seen that the difference between the two predictions is only about 5%, and that the SF is slightly conservative, relative to the GASFRAC results. However, if the predictions of the SF are corrected for acoustic velocity (only) using equation (6), they become some 6% non-conservative, relative to GASFRAC. This largely results from the fact that the factor for \( V_a \) in equation (6) somewhat overestimates the effect of acoustic velocity, for the higher values (the predicted acoustic velocity for the 27 cases considered ranged from 433 to 446 m/s). In fact, as can be seen from Figure 4, the results of the SF, using the predicted acoustic velocities for the pressures and temperatures considered, are some 4% lower than those of the TCM, using the same acoustic velocities. In addition, the results of the TCM itself, using a constant specific heat ratio of 1.4 and the predicted acoustic velocities, were found to be lower than those of GASFRAC by about 2.3%. It should be noted that including the actual specific heat ratio as well as the predicted acoustic velocity in the ideal gas TCM calculations results in a significant further reduction in calculated arrest toughness (since the calculated specific heat ratios are substantially higher than 1.4 for all the cases considered). The relationship between the TCM and SF results becomes non-linear, with the deviation between the two reaching almost 30% for the highest levels of arrest toughness.
conservatism inherent in the choice of "standard" thermodynamic parameters is effective in offsetting the effects of such factors as the presence of higher hydrocarbons and inert gas, and perhaps, to some extent, an increasing over-estimation of fracture propagation resistance from Charpy shelf energy with increasing arrest toughness (even below 90 J).

CONCLUSIONS
The basis of the TCM has been reviewed, and the way in which the SF was derived has been discussed. On the basis of a comparison of the results of the two calculation methods covering a very wide range of inputs, it can be concluded that the SF gives an exceptionally good fit to the results of the TCM, and that errors greater than a few percent will rarely be encountered. A comparison of the predicted arrest toughness from the SF and from the TCM incorporating an iterative gas decompression model instead of ideal gas behaviour showed that the SF is very slightly conservative, provided the "standard" acoustic velocity is used. It can become non-conservative if a correction for higher acoustic velocity is incorporated. It is suggested that caution should be exercised in the use of the SF incorporating acoustic velocity as a variable (Equation (6)), since some of the inherent conservatism in the original choice of thermodynamic parameters appears to have been necessary in order to achieve good separation between propagation and arrest points for the original test matrix, incorporating real pipeline gas.

It should also be reiterated that the question posed, and answered, in the current paper was simply "Does the SF fulfil the purpose for which it was developed?" In other words, does it accurately reproduce the results of the TCM, assuming ideal gas behaviour and, in the case of the TCM, a constant specific heat ratio of 1.4? Issues regarding the deviation of gas behaviour from ideal, the accuracy of various decompression models in both single- and two-phase regions, and the appropriate means of measuring ductile fracture propagation resistance, for modern, high-strength, high-toughness steels, are still topics of extensive discussion that are completely unaffected by the present conclusions. However, it can be confirmed that the continued use of the SF in such applications as standards requirements and general guidelines for conventional pipelines, with the necessary limitations on material type and gas behaviour, is justified.

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