PREDICTIONS OF 3-D STEADY AND UNSTEADY INVISCID TRANSONIC STATOR/ROTOR INTERACTION WITH INLET RADIAL TEMPERATURE NON-UNIFORMITY *

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ABSTRACT
Numerical predictions of 3-D inviscid, transonic steady and periodic unsteady flow within an axial turbine stage are analyzed in this paper. As a first case, the unsteady effects of the stator trailing edge shock wave impinging on the downstream rotor are presented. Local static pressure fluctuations up to 60% of the inlet stagnation pressure are observed on the rotor suction side. The second case is an analysis of the rotor-relative radial secondary flow produced by a spanwise parabolic non-uniform temperature profile at the stator inlet. The generation of local hot spots is observed on both sides of the rotor blade behind the passing shock waves. The magnitude of the unsteady stagnation temperature fluctuations is larger than the time-averaged rotor inlet disturbance. In both cases, steady, unsteady and time-averaged solutions are presented and compared. From these studies, it is concluded that the steady-state solution in static pressure matches well with the time-averaged periodic unsteady flow field. However, for the stagnation temperature distribution only the trend of the time-averaged solution is modeled in the steady-state solution.

NOMENCLATURE
- $c$: speed of sound
- $h_t$: stagnation enthalpy
- $p$: static pressure
- $p_t$: stagnation pressure
- $p_t^*$: rotary stagnation pressure, Eq. (7)
- $R$: radius ($R = \sqrt{y^2+z^2}$)
- $s$: entropy
- $t$: time (non-dimensionalized with period)
- $T_t$: stagnation temperature
- $u_x, u_y, u_z$: Cartesian velocity components in $(x, y, z)$
- $u_r, u_{θ}, u_R$: Cylindrical velocity components in $(x, θ, R)$
- $V$: absolute velocity (stator)
- $W$: relative velocity (rotor)
- $α_r, α_θ$: tangential and radial flow angles
- $γ$: ratio of specific heats
- $ρ$: static density
- $φ$: characteristic variable
- $Ω$: angular speed
- $ω$: vorticity
- $(i)^n$: time index
- $(i)^F$: flux-averaged quantity
- $(i)^{inl}$: inlet
- $(i)^{out}$: outlet
- $(i)^{rot}$: rotor, stator
- $(i)^{rel}$: relative frame (rotor)
- $(x,n)$: streamwise, normal and binormal components
- $(\cdot)$: circumferential arithmetic average

1 INTRODUCTION
The demand for an increase in the cycle performance of today's gas turbines creates severe heat loads in the first turbine stage since higher operating temperatures are required. The mean flow temperature is usually well above the limit supported by the surrounding material. Cooling of both the endwalls and the blades of the first stage is thus usually necessary. Consequently, midspan streaks of hot, less dense gas, pass through the first stator row and become hot jets of fluid.

This and the inherent unsteadiness of a turbomachine flow field created by the relative motion between the stationary blades (stator) and the rotating rotor blades, requires the designer to account for 3-D as well as unsteady effects. For example, the thermal analysis of a turbine airfoil requires the knowledge of the local heat loads, which means that the knowledge of the average driving temperature in the blade passage is not sufficient to optimally design the cooling system. Hence, time-accurate values are required as well as deviations from the average.

In this context, this paper presents a numerical methodology for analyzing the 3-D inviscid, transonic, steady and periodic unsteady flows within an axial turbine stage. In particular, the computations of steady and unsteady flow fields in a complete industrial first tur-
bine stage under different, though realistic inlet conditions, will serve to evaluate the extent of the changes that may occur with respect to design techniques (throughflow procedures AGARD, 1981, Turner, 1991) based on uniform inlet conditions and steady flow field.

Another motivation for analyzing and comparing the steady solution with the time-averaged periodic unsteady flow solution stems from the emergence of numerical methods that incorporate 'corrections' to the baseline steady flow in order to account for deterministic periodic unsteadiness, see for instance the work of Adamczyk (1985) and Giles (1992). Adamczyk derived a set of average-passage flow equations for a multi-stage turbomachinery by sequentially applying an ensemble-averaging, a time-averaging and a passage-to-passage averaging operator to the governing equations. Giles proposed an asymptotic approach for multi-stage unsteady flow computations in which the effect of periodic unsteadiness on the steady flow is included through quadratic terms. Compared to the full non-linear unsteady flow methods, these techniques offer potentially great savings in computer time, though still retaining the global effects of unsteadiness. However, the use of these improved through-flow solution procedures needs to be justified by evaluating the extent of the changes resulting from unsteadiness. These are examined here for two flow cases occurring in a highly loaded transonic first turbine stage.

Finally, from a CFD point of view an underlying motivation for comparing the steady-state and the time-averaged solutions is to highlight the importance of the formulation of the numerical boundary conditions at the inlet, at the exit as well as at the stator/rotor interface.

This paper is structured as follows. After a brief description of the numerical procedure given in Section 2, the first case is presented in Section 3. It focuses on the effects resulting from the impact of the stator trailing edge shock wave off the adjacent rotor, for which a 2-D flow computation was previously performed by Giles (1990a).

Here, the 3-D effects and the extent of the unsteadiness are addressed, and a comparison between steady-state and time-averaged solutions is performed. It has been experimentally observed that the periodic unsteady interaction between a shock wave and a turbine airfoil can cause considerable effects in terms of blade loading and heat transfer, see for instance Doorly and Oldfield (1985), Johnson et al. (1988), and Collie et al. (1992). Then, in Section 4, the unsteady rotor-relative secondary flow produced by a vane inlet spanwise temperature gradient is presented. The comparison between the steady-state and the time-averaged solutions allows to extract the extent of the unsteady temperature migration. The problem of temperature redistribution in an axial flow turbine stage has been analyzed by several authors (see for instance Butler et al., 1986, Krouthén and Giles, 1988, Ni and Sharma, 1990, Harasgama, 1990, and Dorney et al. 1990). In particular, the numerical studies of Ni and Sharma, and Dorney et al. tend to reproduce the migration of one midspan, circular hot streak of fluid, experimentally investigated by Butler et al. Notice that in contrast to the present analysis, all of the above mentioned studies report on a turbine operating under subsonic flow conditions, where the level of unsteadiness is very much lower than for the transonic cases. Finally, the essential conclusions are given in Section 5.

2 NUMERICAL PROCEDURE

The governing relations considered here are the time-dependent 3-D Euler equations solved in conservation form. The numerical methodology uses a node-based, explicit Ni-Lax-Wendroff discretization scheme (Ni, 1981) implemented on an unstructured grid composed of hexahedral cells (Saxer, 1992). Relative flow variables attached to each individual blade row are used. The mesh itself is first generated in a structured fashion by iteratively solving a 3-D Poisson system, in which the source-terms are automatically evaluated in order to provide a control of the cell size and of the skewness at the blade boundary. Then, the structured mesh is transformed into an unstructured grid. The Euler algorithm presented here requires the addition of a numerical smoothing, whose purpose is to capture shocks and to prevent unwanted high frequency oscillations in the solution. It has the form of a combined fourth- and second-difference operator acting on the state vector. The fourth-difference smoothing exploits the advantages of a pseudo-Laplacian to ensure second-order accuracy in shock-free regions even in the presence of grid irregularities. This is an important property when comparing different solutions and when studying the effects of prescribed inlet distortions, which must not be smeared out numerically in the computational domain. It is an extension to 3-D of the method introduced by Holmes and Connell (1989). The shocks are captured using a non-linear second-difference operator which includes an artificial bulk viscosity parameter tailored by the local flow divergence and the Mach number to avoid large shock overshoots, and not to alter the global accuracy of the scheme in smooth flow regions.

2.1 Boundary Conditions

For steady-state flow computations, the quasi-3-D non-reflecting boundary conditions formulation developed by the authors (Saxer and Giles, 1991) is used at the inlet and at the outlet as well as at the stator/rotor interface, and is designed to avoid numerical reflections. In this technique, the solution at the boundary is circumferentially decomposed into Fourier modes, the 0-th mode corresponding to the average solution. This part is treated according to the standard 1-D characteristics theory, which allows the user to specify certain physical quantities at the boundaries by setting average changes in the incoming characteristics. Using a Lax-Wendroff type algorithm to time-march the solution to the steady-state, the changes in the boundary values from time level n to time level n + 1 are required. Thus, the characteristic variables are defined in terms of perturbations to the average inflow or outflow at the time level n. For example, at the inflow, the average characteristic changes are calculated from the requirement that the average entropy, radial and tangential flow angles, and stagnation enthalpy have certain values.

\[
\begin{align*}
(\bar{s})^{+H} &= \bar{s}_{inl}, \\
(\bar{\alpha}_p)^{+H} &= \alpha p_{inl}, \\
(\bar{\alpha}_R)^{+H} &= \alpha R_{inl},
\end{align*}
\]
\[ (\tilde{h})^{\text{out}} = \tilde{h}_{\text{ini}}. \]  

\[ \tilde{s} = \log(\eta p) - \gamma \log \rho, \]

\( \tilde{h} \) is an entropy-related function defined by

\[ \tilde{s} = \frac{\log(\eta p) - \gamma \log \rho}{(\gamma - 1) R}, \]

and \( \tilde{h} \) is the mean total enthalpy. \( \alpha_{\text{ini}} \) and \( \alpha_{\text{ini}} \) together with \( \tilde{s}_{\text{ini}} \) and \( \tilde{h}_{\text{ini}} \) are user-specified average inflow angles, entropy and total enthalpy, respectively, which are usually a function of the radius. For an axially subsonic outflow, the first four characteristics are outgoing, so only the fifth characteristic variable needs to be set. The average change in the characteristic is determined to achieve the user-specified average exit pressure \( \tilde{p}_{\text{out}} \) at a certain radius together with the requirement that the outflow is in radial equilibrium. The latter condition is expressed by

\[ \frac{\partial \tilde{p}(R)_{\text{out}}}{\partial R} = \tilde{u}^2 \]

together with the specification of \( \tilde{p}_{\text{out}} \) at some particular radius. At the inflow and outflow, the changes in the outgoing characteristics are obtained from the changes distributed by the Lax-Wendroff algorithm. The remaining part of the solution, represented by the sum of the harmonics, is treated according to the 2-D non-reflecting boundary conditions theory which prevents spurious reflections at the boundaries (Giles, 1990b).

In a steady-state calculation of a stator/rotor interaction, a circumferential stream-thrust flux-averaging technique is used in order to conserve mass, momentum and energy across the mixing plane between a stator and a rotor row. Hence, radial variations are automatically accounted for and the stator and the rotor flow fields are matched at the interface. In this technique, the average characteristic changes at the stator outflow and the rotor inflow are set to eliminate the following characteristic jumps, taking note of the direction of propagation of each characteristic.

\[ \begin{pmatrix} \Delta \tilde{\phi}_1 \\ \Delta \tilde{\phi}_2 \\ \Delta \tilde{\phi}_3 \\ \Delta \tilde{\phi}_4 \end{pmatrix} = \begin{pmatrix} -\varepsilon^2 & 0 & 0 & 0 \\ 0 & \tilde{\rho} \tilde{c} & 0 & 0 \\ 0 & 0 & \tilde{\rho} \tilde{c} & 0 \\ 0 & \tilde{\rho} \tilde{c} & 0 & \tilde{\rho} \tilde{c} \end{pmatrix} \begin{pmatrix} \rho_F - \rho_F \gamma \\ u_F - u_F \gamma \\ u_F - u_F \gamma - \Omega R \\ \rho_F - \rho_F \gamma \end{pmatrix} \]

where \( \rho_F, u_F, u_F \) and \( \rho_F \) represent the stream-thrust flux-averaged values of density, axial, circumferential and radial velocity components and pressure, respectively. Note that because of the use of relative flow variables, the rotor wheel speed \( \Omega R \) has to be introduced into the condition of matching circumferential velocities. Once this is done for both sides of the interface, the remainder of the boundary condition treatment (i.e. harmonics) is exactly the same as for a standard inflow and outflow boundary, (Saxer, 1992).

For time-accurate calculations of stator/rotor flow fields, Eq. (4) is used on a 1-D local basis to calculate the changes in the incoming characteristics on both sides of the interface. The outgoing characteristics changes are calculated by the Lax-Wendroff algorithm. The combined five characteristic changes on both side of the interface are then converted back to primitive and finally conservation variables before the flow field is updated. The importance of formulating non-reflecting boundary conditions will be highlighted later in Section 4.

### 3 Shock Interaction in an Axial Turbine Stage

In this section, the numerical procedure is applied to investigate both the steady and the unsteady flow fields occurring in a generic (i.e. the stator-to-rotor pitch ratio is 1) highly loaded transonic first turbine stage, see Fig. 1. For supersonic vane exit conditions, a system of oblique shock waves is generated at the trailing edge of the stator. For small axial gaps, the suction side trailing edge shock extends to the downstream rotor and impinges on the suction side. This produces reflected waves on both the adjacent rotor and on the upstream stator causing unsteady blade loading. For these closely spaced blades, the formulation of the boundary conditions at the stator/rotor interface becomes a key point in the numerical simulation.

#### 3.1 Unsteady Shock Motion

Figure 2 shows the instantaneous static pressure contours at a constant radius \( R = R_{\text{mid}} \) at eight intervals during one blade-passing period, i.e. from \( t = 0 \) at the beginning of the period to \( t = 1 \) at the end of the period, which also corresponds to \( t = 0 \) by periodicity. Note that the unsteady pressure contours match well across the stator/rotor boundary at any time during the period.

At \( t = 0 \), the stator trailing edge oblique shock has hit the crown of the rotor suction surface. As the rotor blade moves upward, the location of impingement moves forward towards the leading edge. A reflection is clearly visible at \( t = 0.375 \). In addition to this primary reflection, the portion of the reflected wave which moves towards the pressure surface of the adjacent rotor is reflecting a second time and is moving back towards the original rotor. At \( t = 0.5 \), the primary shock wave reflection has reached the rotor leading edge and the secondary reflected wave has crossed back to the original rotor. It has sufficiently intensified to be now visible on the pressure contours.

At \( t = 0.625 \), the stator oblique shock no longer impacts on the rotor, and from \( t = 0.625 \) to \( t = 0.875 \) the length of its straight portion is increased (due to a velocity component approximately \( \sqrt{\varepsilon^2 - \varepsilon^2} \) along the shock front) until it refracts with the secondary reflected shock. Also at \( t = 0.625 \), the primary reflected wave has left the rotor and is propagating upstream towards the stator suction surface. At \( t = 0.75 \), the primary reflected shock has just struck the suction side of the upstream stator in a region close to the trailing edge. The secondary reflected shock wave is still moving upstream towards the crown of the rotor.

At \( t = 0.875 \), the primary oblique shock has almost regained its maximum strength. The primary reflected shock is reflecting from the upstream stator suction sur-
face and moves back to the adjacent rotor. This reflection is also visible at \( t = 0 \), but then it disappears as it strikes the rotor leading edge at \( t = 0.125 \). However, this effect is recorded in the history of the rotor leading edge pressure in Fig. 3. In addition, this figure clearly illustrates the impact of the stator oblique shock on the rotor leading edge at \( t = 0.5 \).

Figure 4 shows the instantaneous tufts distribution on the rotor suction surface for one blade-passing period. A secondary flow is visible in the aft part of the blade that drives fluid from hub towards midspan. Reverse flow occurs behind the secondary reflected wave as it moves forward towards the leading edge and strengthens. However, recirculation occurs only partially during the cycle.

For the case examined here, the unsteady stator/rotor shock interaction is essentially a two-dimensional process driven by the oblique shock leaving the stator trailing edge and impinging on the moving rotor blade. This unsteady shock interaction is similar in nature to the result presented by Giles (1990a) in a 2-D time-accurate numerical simulation which included quasi-3-D source terms. However, some differences appear in the timing of the events because Giles’ computation was performed on the actual configuration with a stator-to-rotor pitch ratio of 1.69.

3.2 Steady and Time-Averaged results

The flow conditions for the steady-state and for the time-averaged periodic unsteady computation are listed in Table 1. The Mach and angle values represent flux-averaged quantities. Starting from the steady-state solution, the unsteady computation took 12 periods to converge to a periodic solution, using 400 iterations/period. A CPU time of \( 385 \times 10^{-6} \) seconds/iteration/grid point was required on a Stardent GS-2000 in vector mode.

The stator static pressure distributions around the blade root and at midspan are given in Fig. 5. Also shown are the maximum and the minimum unsteady pressures on both the suction and the pressure sides. Because the flow is choked at the throat, and since the stator inlet conditions are steady, no unsteadiness is present upstream of that area and so the mass flow is the same in both cases. Although the unsteady pressure envelope can locally account for 50% of the inlet stagnation pressure, the time-averaged solution almost matches the steady-state result over the entire span. It should be pointed out that the time-averaged solutions presented in this paper were computed during the blade-passing period from all the time-steps, i.e. they represent the following equation.

\[
\tilde{\phi}(x, y, z) = \frac{1}{T} \int_0^T \phi(x, y, z, t) \, dt,
\]

with \( \tilde{\phi} \) representing a quantity that has been time-averaged over a period \( T \). On the other hand, the maxima and minima values were extracted from eight unsteady snapshots during the final blade-passing period. This is why a certain amount of discreteness in those quantities is observed in Figs. 5 and 6. The rotor blade pressure distributions at three spanwise locations are shown in Fig. 6. Here, the unsteadiness is much more intense, especially on the suction side of the blade due to the unsteady shock motion and shock reflection described in the previous section. At the hub, the steady-state shock located at the front part of the suction side is intensified during a portion of the period. On the suction side near the leading edge, the peak-to-peak pressure variation accounts for up to 60% of the (stator) inlet reference stagnation pressure. It is surprising to notice that although the variations in pressure (and Mach number) are large, the time-averaged solution is similar to the steady-state result over all but the root section of the blade. In particular, the maximum variation in the blade static pressure between the steady-state and the time-averaged unsteady solution is less than 4% of the stator inlet stagnation pressure. In general, the level of unsteadiness is larger on the suction side than on the pressure side, and also at the hub and at the tip compared to midspan.

In the unsteady solution, a shock-induced secondary flow creates a slight flow blockage at the lower radii (hub), see Fig. 7. This is compensated by an increase in mass flow through the outer rotor radii streamtubes. Tufts on the rotor suction side are plotted in Fig. 8 for the steady and the time-averaged solutions. The cross-flow seen in Fig. 8(b) is a direct consequence of the entropy production caused by the unsteady shock interaction and can be qualitatively understood by using secondary flow theory applied on the time-averaged solution. By assuming locally incompressible flow, it can be shown that the generation of a streamwise component of vorticity can be written as a function of gradients of rotary stagnation pressure in the binormal and axial directions, respectively (Hawthorne, 1974, Johnson, 1978, Horlock and Lakshminarayana, 1973).

\[
\frac{\partial}{\partial \xi} \left( \frac{\omega_v}{\rho W} \right) = \frac{2}{\rho W^2} \left( \frac{1}{\sigma} \nabla p^* \cdot \nabla \left( \frac{1}{W} \nabla \cdot \rho \phi^* \right) \right)
\]

Equation (6), is written in intrinsic coordinates where \( \hat{s}, \hat{n}, \hat{b} \) represent the unit vectors in the direction of relative streamline, inward of principal normal and in the binormal direction, respectively. \( \sigma \) is the principal radius of curvature of the relative streamline and \( p^* \) is the rotary stagnation pressure defined by

\[
p^* = p + \frac{1}{2} \rho \left( (u^2+v^2+w^2)_{rel} - \Omega^2 R^2 \right).
\]

In the steady-state calculation \( p^* \) is almost constant over the entire blade except behind the hub shock, see Fig. 9(a). In the time-averaged solution however, gradients of rotary stagnation pressure show up not only near the location of the steady-state shock but over the entire near-hub suction surface, see Fig. 9(b). The comparison clearly shows that a strong axial \( p^*_s \) gradient exists in the time-averaged solution which, when dot-produced with the rotation vector, produces a positive component of streamwise vorticity. This tends to develop a radially outward component of velocity.

On the stator there is a 6% unsteady peak-to-peak variation in torque, despite the fact that the pressure field is steady on all but a small portion of the suction side near the trailing edge. This variation is essentially caused by
the primary rotor-reflected shock wave moving upstream and striking the stator suction side. Notice that in the 2-D simulation (Giles, 1990a) the unsteady stator lift has a peak-to-peak variation of 6%, which compares well to the 3-D case. However, as opposed to a rotor unsteady peak-to-peak torque variation of 66%, 'only' 40% variation in lift is experienced by the rotor in the 2-D case. In general the agreement between the time-averaged and the steady-state axial torque is excellent with differences less than 0.5% of the time-averaged value.

4 RADIAL TEMPERATURE MIGRATION

The objective of this section is to analyze the migration of a spanwise non-uniform inlet temperature distribution in an axial flow turbine stage (similar to the one discussed in the previous section). The design of this stage was performed by Rolls-Royce and is representative of a high pressure (4:1 stage pressure ratio), cooled aircraft turbine operating in the transonic regime. The flow simulation uses a 3-to-5 stator-to-rotor blade count ratio, which corresponds closely to the actual configuration of (36 stators)/(61 rotors), see Figs. 10 and 11. 56 x 36 x 21 nodes are used in each of the three stator channels, and 56 x 22 x 21 in each of the five rotor passages.

The stator inlet conditions are set by two flow angles and by assuming a uniform stagnation pressure. A radially parabolic stagnation temperature profile is set in which the temperature is 21% higher at midspan than at the hub and the tip endwalls. Notice that these conditions produce a radially sheared velocity field, hence vorticity. However, under the invariance of the inlet stagnation pressure and by using the Munk and Prim substitution principle (Munk and Prim, 1947), it can be shown that no 3-D secondary flow associated with the temperature gradient occurs in the vane (Saxer, 1992). This is because the Munk and Prim principle states that if a steady isentropic flow field without body forces is determined for a specific geometry and a total pressure distribution, then the streamlines pattern, the Mach number and the static pressure fields remain unchanged for any stagnation temperature distribution. Another way of checking this statement is to use again secondary flow theory (Hawthorne, 1974, Johnson, 1978, Horlock and Lakshminarayana, 1973). Rewriting Equation (6) in the absolute frame of reference would show that the growth of the absolute streamwise vorticity is dependent upon the gradient of stagnation pressure, which is set to zero at the inlet, i.e. the vortex lines introduced at the inlet have to remain perpendicular to the flow in the vane. In fact, the secondary flow developed by the turning of the inlet normal vorticity is exactly balanced by the secondary flow introduced by the temperature gradient.

These arguments are not valid in the rotor frame of reference where a radial secondary flow occurs. It can be qualitatively explained by using vorticity and velocity triangles arguments and simple dynamics (Butler et al., 1986, Saxer, 1992). Figure 12 is a schematic of the vorticity and the velocity triangles at the stator/rotor interface. In the relative frame, the stator vorticity (normal to the flow) can be decomposed into a normal and a streamwise component, which will strengthen as the normal component turns into the rotor passage producing a 3-D secondary flow. From the velocity triangle it can be inferred that in the rotor frame the hot midspan fluid is streaming with a relative higher velocity than the cold endwall fluid, and with an incidence angle oriented more towards the pressure side of the rotor. This implies that relative to a uniform inlet conditions solution, the temperature distortion calculation exhibits an excess of rotary stagnation pressure (or rotor-relative inlet stagnation pressure) at midspan and a deficit at the hub and the tip of about 12% of the rotor-relative inlet dynamic head, which contributes to the generation of secondary flow. The overall effect is the collection of hotter gas on the rotor pressure side than on the suction side. On the pressure side, the hot fluid is spreading from midspan towards the hub and the tip endwalls resulting in the heating of the upper and lower walls. On the suction side, cool endwall gas flows towards midspan. Also a comparison between steady-state flow solutions without and with inlet radial temperature distortion shows that the rotor-relative secondary flow affects the vane flow field by reducing the stator/rotor interface pressure (about 17% of the rotor inlet dynamic head for the case examined), see Saxer and Giles (1990). In turn this produces an increase in both the stator exit and the rotor inlet Mach numbers and also the rotor inlet angle. This is sufficient to trigger a shock at the rotor root, see Fig. 10a).

4.1 Unsteady Rotor-Relative Temperature

Instantaneous rotor-relative stagnation temperature contours and static pressure contours on the suction side (of the lower rotor blade) are plotted in Fig. 13 for four snapshots during one period. Notice that for the case examined here, the period, starting at $t = 0$ in the configuration shown in Fig. 10, is defined as the time for one rotor passage to rotate by an amount of five rotor pitches to $t = 1$, which also corresponds to $t = 0$. The suction side radial migration of the cooler gas at the hub and the tip towards midspan is small at any time during the period. However, under the shock interaction, the local instantaneous surface stagnation temperature can exceed the time-averaged midspan rotor inlet value by about 10%, which corresponds to half of the stator inlet distortion. This shows up as local hot spots of elliptical shape created behind the passing shock waves. Notice that in the rotating frame of reference the time-averaged parabolic inlet distortion is reduced from 21% to 17%. The local envelope of these temperature fluctuations has a magnitude of the order of the absolute distortion, hence larger than the rotor-relative inlet variation. The rotor-relative pressure surface stagnation temperature distribution together with the static pressure contours during one period are illustrated in Fig. 14. As explained in Section 3, the rotor pressure side is less affected by the shock interaction, thus the local unsteady temperature fluctuations are smaller than on the suction side. However, under the influence of the radial secondary flow, which is stronger on the pressure side than on the suction side, the hot spots cover a broader portion of the blade.

4.2 Steady and Time-Averaged Results

Steady-state and time-averaged periodic unsteady static pressure contours at the hub are represented in Fig.
waves of both the stator and the rotor are reflected back face and at the rotor exit where a uniform pressure has been set. In particular, the trailing edge oblique shock waves of both the stator and the rotor are reflected back into the computational domain as expansion waves. The use of the improved (non-reflecting) boundary conditions in the calculation of the steady-state stator/rotor interaction avoids these numerical reflections, and thus allows for a fair comparison between steady-state and time-averaged solutions.

Steady-state and time-averaged periodic unsteady rotor-relative stagnation temperature contours are compared in Fig. 15 for the suction side. Clearly the pattern is the same, with a slightly stronger secondary flow in the time-averaged solution. The local difference of one contour between the two solutions corresponds approximately to 6% of the (stator) inlet temperature variation (i.e. radial distortion). At midspan, the time-averaged surface temperature downstream of the leading edge on the suction side is higher than the time-averaged inlet temperature. This is not really predicted by the steady-state code. A similar conclusion was found by Takahashi and Ni (1990) for the migration of a hot spot in a subsonic turbine stage. Also shown on Fig. 15 are the corresponding static pressure distributions. Except for some minor differences at the root near the weak shock, the agreement is excellent. On the rotor pressure side, see Fig. 16, the differences are larger with steady-state temperature contours showing a stronger radial flow in the tip region. For instance, at the tip near the trailing edge, the steady-state heating reaches 9% of the inlet stagnation temperature reference taken at the hub, which corresponds to 42% of the inlet distortion. On the other hand, the time-averaged tip heating corresponds only to 2.5% of \( T_p \), i.e. 12% of the inlet radial temperature variation. This means that compared to the time-averaged solution, the steady-state result locally overpredicts the temperature migration by about one third of the inlet distortion. However, the static pressure contours plots of Fig. 16 show a nearly perfect agreement for this quantity on the pressure side. Hence the steady-state code is adequate for blade loading information, a conclusion similar to the one found by Takahashi and Ni. The close agreement between the steady-state and the time-averaged flow solutions is a consequence of the similar distributions of rotor-relative inlet stagnation enthalpy (a local difference less than 0.5% of the stator inlet distortion) and flow angles (less than 1.5°). Moreover, the difference in rotary stagnation pressure between the steady-state solution and the time-averaged result, which could explain a different secondary flow behaviour, is less than 2% of the rotor inlet dynamic head.

Finally, the steady-state rotor-relative stagnation temperature contours on the pressure side are compared in Fig. 17 with Pappas's result (Pappas, 1990) using Denton's inviscid solver. Although some differences exist in the definition of the blading (discussed by Saxer, 1992), the agreement is good.

5 SUMMARY AND CONCLUSIONS

This study has demonstrated the importance of 3-D inviscid periodic unsteady flow in a generic transonic axial turbine. A periodic stator/rotor shock wave interaction has been described for uniform vane inlet conditions. The unsteady static pressure envelope can exceed the level of the time-averaged local value. This leads to 3-D shock-induced unsteady secondary flow, not present in the steady-state solution. Although the computed level of unsteadiness is higher at the hub and at the tip compared to midspan, the behavior in time of the stator/rotor shock wave interaction is similar in nature across the span. The integrated blade surface pressure coefficient fluctuates as much as 2/3 of the time-averaged value. This is 60% larger than in the 2-D case.

The steady-state solution agrees extremely well with the time-averaged result (especially for the static pressure distribution), which is a surprising result considering the fact that for the cases examined here the unsteady fluctuations can be one order of magnitude larger than the ones occurring in standard designed subsonic axial flow turbines. It is believed that such good agreement between the two solutions is greatly due to the use of the non-reflecting boundary conditions in the steady-state calculation of the stator/rotor interaction. By avoiding spurious reflections and by providing a relieving effect for the flow to account for the presence of the leading and the trailing edge of neighboring blades, these steady-state conditions represent, to a good approximation, the time-averaged physical phenomena. However, in terms of the blade stagnation temperature distribution, while the steady-state solution retains the same trend as the time-averaged flow field, differences appear in the local values. This has been examined in the second flow case by looking at the rotor-relative secondary flow caused by the introduction of a spanwise temperature distortion at the stator inlet (21% over-temperature at midspan compared to the endwalls, with a parabolic variation). The time-averaged rotor surface temperature radial migration on the pressure side of the blade near the tip is reduced compared to the steady-state solution. This may be due to the essentially 2-D pattern of the shock interaction mentioned in the first case. The unsteady temperature field is very different from the steady-state solution. Unsteady hot spots of elliptical shape, whose strength correspond to half of the inlet temperature distortion, are periodically created behind the passing shock waves on both the blade suction and pressure sides. The magnitude of the local unsteady rotor-relative stagnation temperature envelope reaches the level of the stator inlet distortion. This is larger than the time-averaged rotor inlet distortion.

ACKNOWLEDGMENTS

This work has been partially funded through a research grant from Rolls-Royce PLC, Dr. P. Stow technical supervisor. Additional funding provided by the Air
REFERENCES


Figure 1: Side view of the scaled transonic first (top) turbine stage with stator pressure and rotor suction sides and mean height blade-to-blade mesh (bottom) (stator: 80 x 30 x 30, rotor: 80 x 30 x 30 nodes).
Figure 2: Unsteady static pressure contours at $R = R_{mid}$. 
Figure 3: Pressure history for the last two periods.

![Pressure history graph](image)

Figure 4: Computed tufts on rotor suction side at different times.

![Tufts image](image)

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<th>parameter</th>
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<th>time-averaged</th>
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Table 1: Steady and time-averaged flow parameters for scaled transonic turbine stage.
Figure 5: Steady and unsteady stator blade static pressure.

Figure 6: Steady and unsteady rotor blade static pressure.

Figure 7: Spanwise distribution of mass flow at rotor inlet.
Figure 8: Computed tufts on rotor suction side: a) steady-state, b) time-averaged.

Figure 9: Rotary stagnation pressure ($p_r/p_{t, in}$) on rotor suction side: a) steady-state, b) time-averaged (inc. = 0.01).

Figure 10: Static pressure field at the hub: a) steady-state, b) time-averaged (increment $\Delta p/p_{t, in} = 0.025$).

Figure 11: Steady-state static pressure at the hub using reflecting boundary conditions.

Figure 12: Velocity and vorticity triangles at stator/rotor interface.
Figure 13: Rotor-relative stagnation temperature contours and static pressure contours on the suction side at different times. $\Delta T_i / T_{int} = 0.01$, $\Delta p / p_{int} = 0.025$.

Figure 14: Same as Fig. 13 but for rotor pressure side.
Figure 15: Rotor-relative stagnation temperature contours \( (T_i/\gamma T_{i \text{ ini}}) \) and static pressure contours \( (p/p_{i \text{ ini}}) \) on the suction side: a) steady-state, b) time-averaged.

Figure 16: Rotor-relative stagnation temperature contours \( (T_i/\gamma T_{i \text{ ini}}) \) and static pressure contours \( (p/p_{i \text{ ini}}) \) on the pressure side: a) steady-state, b) time-averaged.

Figure 17: Rotor-relative stagnation temperature contours on the pressure side with \( (T_{i \text{ mid}}/T_{i \text{ hub}})_{\text{ini}} = 1.30 \).