A REVIEW OF NON-STEADY FLOW MODELS FOR COMPRESSOR STABILITY

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Abstract

This paper presents a review of the different approaches to modelling the non-steady fluid dynamics associated with two-dimensional compressor flowfields. These models are used to predict the time development of flowfield disturbances and have been found useful both in the study of rotating stall and the development of active control.

The opportunity to digest the earlier investigations has now made it possible to express the modelling ideas using only a very simple mathematical treatment. Here, the emphasis is on the underlying physical processes that the models simulate and how the assumptions within the models affect predictions. The purpose of this work is to produce, in a single document, a description of compressor modelling techniques, so that prospective users can assess which model is the most suitable for their application.

Introduction

Surge and rotating stall are important considerations both in the design of compressors, where their occurrence represents a performance boundary, and in the operation of aeroengines, where they can occur prematurely due to inlet distortion. The time development of these phenomena is determined by non-steady fluid dynamic processes both within the blade passages and in the flowfield as a whole. Direct calculations of all aspects of the flow are impractical, because of the many length and time scales that are involved, and so models that represent the effective local compressor performance are used.

Calculations using these models can only ever be as good as the predicted performance, that is pressure rise as a function of mass flow, and this depends on the accuracy of the component characteristics. These characteristics are not normally known with great precision. Consequently, the greatest value of these compressor models is in identifying the important physical processes and in undertaking parametric studies using generic performance correlations. This review is concerned with the details of the compressor models that are in current use, the physical mechanisms that they simulate and the constraints on their application.

Conceptually, a compression system is represented by a series of components comprising inlet duct, compressor, exit duct, plenum and throttle, Fig. 1. Each of these components can be modelled in different ways depending on the fluid dynamic phenomena of interest. For example, "Surge-like" disturbances are primarily axisymmetric and involve variations in the mass flow through all the system components; to study these types of phenomena the plenum and throttle are necessary along with the dynamics of non-steady mass storage within the plenum. "Stall-like" disturbances, on the other hand, involve a region of the compressor annulus that has a lower through flow than the rest and only the average mass flow and average pressure rise interact with the plenum and throttle; to study these phenomena it is necessary to account for the circumferential variation of flow within the compressor.

Stall is the primary disturbance (Day 1991b) and so the modelling techniques for non-steady flow through compressors will be discussed with reference to "stall-like" flow perturbations. These are intrinsically non-axisymmetric and involve circumferential flow redistribution and static pressure fields that decay away from the compressor. A reliable model must simulate these effects and here, as in virtually all treatments to date, attention is limited to flow variations that are primarily in the axial and circumferential directions, often referred to as two-dimensional. As a result these models are useful in the study of high hub-to-tip ratio compressors for which radially uniform flow is a reasonable approximation. A schematic representation of the inlet duct, compressor and exit duct for a two-dimensional flowfield is shown in Fig. 2.

The compressor modelling techniques discussed in this paper are applicable to both large and small amplitude flowfield disturbances.

Fig. 1 Compression system components.
However, attention is limited to the latter as this is convenient for examining non-steady flow behaviour. An additional advantage of this restriction is that small amplitude non-axisymmetric disturbances do not affect the average mass flow or average pressure rise. Therefore, the compressor models and the behaviour of non-axisymmetric disturbances may be analysed without explicit inclusion of the plenum and throttle.

Although not discussed in any depth in this review, these two-dimensional compressor models are, nevertheless, applicable to non-steady axisymmetric flow and so may be used to study surge. The compression system modelling approach for surge was developed by Greitzer (1976ab), with a simple mass flow pressure rise relationship for the compressor and lumped parameter representations for the other components, the behaviour being determined by a second-order ordinary differential equation. An important result from Greitzer's modelling is that an analogy of surge behaviour is a spring-mass-damper, with surge being equivalent to a system oscillation. This type of modelling is that an analogy of surge behaviour is a spring-mass-damper, with surge being equivalent to a system oscillation. This type of compression system model, often referred to as a one-dimensional simulation, is still extensively used (see Pinsley et al 1991, Gysling et al 1991 and Simon et al 1992) and a great deal has been learnt about surge dynamics (see Moore & Greitzer 1986 and McCaughan 1988).

Review Structure. In this review the underlying constraints on the models to represent the average blade row performance over some length and time scales of interest are examined first. It is shown that for circumferential harmonics up to the fifth or sixth the spatial flow redistribution should be well modelled and that the temporal behaviour of the lowest harmonics should be quasi-steady. Here, quasi-steady is taken to mean that the instantaneous blade loss and exit angle are identical to those for corresponding steady flow.

The quasi-steady compressor model is then derived and is, in essence, the one presented by Moore (1984a). The predicted non-steady fluid dynamic behaviour of compressor flowfields is investigated with, initially, two simplifying assumptions (perfect IGVs and high solidity compressor exit blading) concerning the compressor and flowfield interaction. These simplifications facilitate the identification of the physical mechanisms that determine circumferential propagation and growth or decay of flowfield disturbances. It is then shown that in the more general case, where inlet and exit swirl variations are fully accounted for, the physical mechanisms are the same though quantitative predictions may change.

The later sections of this review consider possible consequences when the blade rows no longer respond in a quasi-steady manner. An enhanced compressor model is described that uses a simple time lag approach for non-steady blade row response. With this enhanced model the individual circumferential modes are predicted to have distinct stability boundaries and this has been experimentally demonstrated. Finally, approximate forms of this enhanced model, that are more convenient for solution, are described and their merits considered.

Spatial Flowfield Resolution. An important constraint on two-dimensional compressor models is how large the ratio of "non-uniformity wavelength" to "blade pitch" must be for the local blade performance in non-uniform flow to be similar to that which it would have been in uniform flow with the same local conditions. There is little available information on this question, though some guidance may be drawn from the studies of back pressure distortion through stationary blade rows.

For a non-uniformity with wavelength shorter than a blade pitch, flow redistribution can freely occur and appreciable decay of the pressure field is expected. For a long wavelength non-uniformity imposed on a blade row, the blades themselves would restrict the circumferential redistribution and so there would be reduced axial decay of the pressure field. This is supported by O'Brien et al (1985) who studied the interaction between a row of stators and a downstream strut. Their results showed little decay of the long lengthscale pressure field across the stators.

From this a rough constraint can be said to be that the circumferential lengthscale of the flow non-uniformity should be long enough so that a blade and its neighbours (two pitches) have similar flow conditions (within a quarter wavelength). Therefore:

\[ \frac{1}{4} \text{ non-uniformity wavelength} > 2 \text{ blade pitches} \]

and hence an estimate for the minimum wavelength-to-pitch ratio is eight. Taking 50 as a typical blade count, then circumferential harmonics up to the fifth or sixth should be well modelled.

Ham & Williams (1983) investigated this point experimentally for a primarily first circumferential harmonic back pressure distortion through eight unloaded vanes in an annulus of moderate hub-to-tip ratio. Their results, Fig. 3, indicate that although the short lengthscale pressure field is attenuated within the individual vane passages, the circumferential pressure non-uniformity transfers through the blade row. This is because each of the eight passages responds to the local pressure and therefore adequately defines the long lengthscale variation. Without the vanes the pressure non-uniformity would decay to approximately 50% along an axial duct.

Temporal Flowfield Resolution. There are two ways in which non-steady flow arises. The flow in the absolute (stator) frame can be time varying, so both rotors and stators see unsteadiness, or there can be a stationary spatial non-uniformity which appears as a time varying flow in the relative (rotor) frame. The importance of the non-steady flow effects within the blade passages is assessed by comparing the relative sizes of the blade passage convection time, ie from the leading edge to trailing edge, with the disturbance passing time, ie the passage of the blade through one wavelength. The ratio of...
these two is referred to as the reduced frequency, and the flow behaves quasi-steadily at low reduced frequencies, ie when:

\[ \text{convection time} \ll \text{disturbance passing time} \]

Taking \( \alpha \) and \( u \) as the blade axial chord and the axial velocity respectively, the inequality may be written:

\[ \alpha \frac{u}{\nu} \ll 2\pi \frac{nf}{\Omega} \]

where \( n \) is the spatial Fourier harmonic being considered and \( f \) is the fraction of rotor angular velocity (\( \Omega \)) with which the spatial pattern moves. Thus for quasi-steady blade row performance:

\[ nf \ll 2\pi \frac{\phi}{\nu} \]

where \( \phi = \frac{u}{\nu} \). Typically \( \phi \approx 0.3 \) to 0.7, \( \frac{u}{\nu} \approx 0.15 \) and non-steady phenomena of interest have \( 0 < f < 1 \). Under these conditions the right-hand side of equation (1) is approximately twelve and so for the lowest circumferential harmonics the blades are expected to behave quasi-steadily.

### Blade passage Fluid Inertia

Even when the blade flowfield responds quasi-steadily there is an associated non-steady flow effect due to the acceleration of the fluid within the blade passage. This acceleration requires a non-steady pressure difference across the blade row that may be estimated by modelling the blade passage as a parallel duct. Thus for quasi-steady blade row performance:

\[ \left( \frac{P_{in} - P_{out}}{\rho U^2} \right)_{\text{inertia}} = \frac{\rho}{c_x} \frac{\partial}{\partial \tau} \left( \frac{u_x}{\cos(\text{stagger})} \right) \]

Thus:

\[ \frac{P_{in} - P_{out}}{\rho U^2} = \Lambda \left( \frac{\partial}{\partial \tau} \frac{u_x}{\cos(\text{stagger})} \right) \]

Where the inertia parameter is:

\[ \Lambda = \frac{\rho}{c_x \cos(\text{stagger})} \]

Williams (1986) calculated that typical values of \( \Lambda \) for individual blade rows of military aeroengines are:

- \( \Lambda = 0.314 \) (±10%) for low pressure compressor
- \( \Lambda = 0.14 \) (±13%) for high pressure compressor

For civil engines the inertia parameters are usually within the range 0.1-0.2 for each row in the high pressure compressor.

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**Fig. 3** Transfer of a primarily first harmonic back-pressure distortion through eight unloaded vanes (Ham & Williams 1983).

**Fig. 4** A parallel duct at the mean stagger angle can be used to estimate the inertia driven non-steady pressure difference.

### Quasi-Steady Compressor Model

In this review the starting point for all the models will be the blade row performance. Steady-state blade row performance is usually correlated as total pressure loss coefficient and exit deviation as functions of inlet angle, Mach number and Reynolds number. An equivalent presentation, which is entire consistent for modelling purposes, is in terms of static pressure rise and exit flow angle as functions of inlet angle and flow coefficient. Reynolds number and compressibility effects are being ignored as they are not necessary to elucidate the physical mechanisms.

For some of the models it is possible to recombine the individual blade row pressure rise into a more compact form where the overall compressor pressure rise is directly evaluated in terms of, say, the local flow coefficient at inlet. Wherever possible this will be done as it simplifies the application of the model.

For quasi-steady flow through a stator passage the local instantaneous static pressure rise may be related to that in steady flow by inclusion of the inertia term, equation (2). Thus:

\[ \frac{P_{out} - P_{in}}{\rho U^2} = \frac{P_{out} - P_{in}}{\rho U^2} - \Lambda_{\text{row}} \frac{\partial \phi}{\partial t} \]

where rotor speed has been used to produce the non-dimensional time \( t = \frac{U}{\nu} \). The instantaneous exit angle, \( \alpha_{out} \), is the same as in steady flow:

\[ \alpha_{out} = \alpha_{out}^{\text{steady}} \]

In general, the steady state pressure rise and outlet flow angle for each blade row are functions of the inlet angle and flow coefficient.

For a rotor row the time rate of change for the inertia term must be evaluated in the relative frame, that is:

\[ \frac{\partial}{\partial \tau} + \frac{\partial}{\partial \theta} \]

hence for non-uniform and non-steady flow through a rotor:

\[ \frac{P_{out} - P_{in}}{\rho U^2} = \frac{P_{out} - P_{in}}{\rho U^2} - \Lambda_{\text{row}} \frac{\partial \phi}{\partial \tau} \frac{\partial \phi}{\partial \theta} \]

and:

\[ \alpha_{out}^{\text{rel}} = \alpha_{out}^{\text{rel}} \]

In order to proceed without introducing unwarranted complexity, it will be assumed that there is no flowfield redistribution within the compressor, this is the idealised case of no axial gaps between adjacent blade rows. Consequently, the constraints of mass conservation and flow at low Mach number imply that the axial...
velocity at each circumferential position is constant through the machine. The individual blade row pressure rises may, therefore, be combined to obtain:

\[ \frac{P_2 - P_1}{\rho U^2} = \sum_{\text{all rows}} \frac{P\text{out} - P\text{in}}{\rho U^2} = \lambda \frac{\partial \phi}{\partial \theta} - \mu \frac{\partial \phi}{\partial t} \quad (6) \]

where 1 and 2 refer, respectively, to the leading edge of the first blade row and the trailing edge of the last one. The overall inertia parameters are:

\[ \lambda = \frac{\text{sum of all rows}}{\text{number of rows}} \quad \mu = \frac{\text{sum of all rows}}{\text{number of rows}} \quad (7) \]

At each circumferential location, the summation of the individual blade row pressure rises may only be replaced by the overall static-to-static pressure rise, \( \psi^{(s)} \), evaluated at the local inlet conditions provided that the flow angles between all the blade rows correspond to those in steady uniform flow operation. The quasi-steady assumption implies that this is true, and so the equation (6) may be rewritten:

\[ \frac{P_2 - P_1}{\rho U^2} = \psi^{(s)}(\phi, \alpha_1) \quad \text{steady} = \lambda \frac{\partial \phi}{\partial \theta} - \mu \frac{\partial \phi}{\partial t} \quad (8a) \]

where \( \phi(\theta, t) \) and \( \alpha_1(\theta, t) \) are the local values of the flow coefficient and inlet swirl angle at the compressor. Similarly, the compressor exit flow angle, \( \alpha_2 \), may be written:

\[ \alpha_2 = \alpha_2(\phi, \alpha_1) \quad \text{steady} \quad (8b) \]

The dependence on both \( \phi \) and \( \alpha_1 \) have been explicitly included to emphasise that no assumptions have been made concerning the swirl sensitivity of the compressor.

It is convenient, as was done by Hynes & Greitzer (1987) and was implicit in Moore (1984a), to express the compressor performance in terms of the total-to-static pressure rise. This is achieved by subtracting off an inlet dynamic head from both sides of equation (8a) to give:

\[ \frac{P_2 - P_1}{\rho U^2} = \psi^{(s)}(\phi, \alpha_1) \quad \text{steady} = \lambda \frac{\partial \phi}{\partial \theta} - \mu \frac{\partial \phi}{\partial t} \quad (8c) \]

where the total-to-static pressure rise is \( \psi^{(s)} = \psi^{(s)}(\phi) \). The exit flow angle is, of course, still determined by equation (8b).

**Simplified Behaviour of Flowfield Disturbances**

The quasi-steady compressor model can be used to examine the predicted behaviour of non-axisymmetric flowfield disturbances within compressors. In this section it will be shown that as a consequence of the non-steady fluid dynamics of compressor flowfields any non-axisymmetric disturbance will be circumferentially propagated and that its amplitude may grow or decay.

The physical mechanisms that determine the circumferential propagation and growth or decay of “stall-like” phenomena will be examined and, to minimise the mathematical complexity, two-simplifying assumptions concerning compressor performance will be utilised. The first one is that the compressor pressure rise does not depend upon the inlet swirl angle, eliminating \( \alpha_1 \) from equations (8abc). This represents the idealised case of a compressor operating with perfect IGVs, ones where the flow is isentropic and the IGV exit angle is constant. The second assumption is that the compressor exit flow angle, \( \alpha_2 \), stays constant, and would be the idealised case of high-solidity exit blading.\(^6\) Equations (8bc) become:

\[ \frac{P_2 - P_{11}}{\rho U^2} = \psi^{(s)}(\phi) - \lambda \frac{\partial \phi}{\partial \theta} - \mu \frac{\partial \phi}{\partial t} \quad (9a) \]

\[ \alpha_2 = \text{constant} \quad (9b) \]

For the purposes of the present discussions it is sufficiently general to consider small amplitude flowfield disturbances and so the equation may be linearised about a uniform axisymmetric flow to obtain:

\[ \frac{\delta P_2}{\rho U^2} = \frac{\delta P_{11}}{\rho U^2} = \frac{\partial \psi^{(s)}}{\partial \phi} \delta \phi - \lambda \frac{\partial \delta \phi}{\partial \theta} - \mu \frac{\partial \delta \phi}{\partial t} \quad (10) \]

where \( \delta \phi \) denotes the perturbed quantity. Any flowfield disturbance can be expressed by a Fourier decomposition and, because of linearity, the harmonics may be considered individually. Thus the flow coefficient perturbation at the compressor may be taken as:

\[ \delta \phi = \text{REAL}(a \ e^{i(\theta + \omega t)}) \quad (11) \]

with similar expressions for other perturbed quantities. For \( n=0 \) this represents a n-lobed harmonic wave-form of the type that, depending on the associated value of \( \alpha_0 \), moves around the annulus at fraction \( \omega/\Omega_0 \) of rotor speed and therefore resembles a stall-like disturbance. (Although not of interest here, surge-like disturbances correspond to \( n=0 \) and to study these the plenum and throttle are required.)

To solve equation (10), to find the value of \( \alpha_0 \) required in equation (11), it is necessary to relate the changes in flow, \( \delta \phi \), to those in exit pressure \( \delta P_2 \) and inlet total pressure \( \delta P_{11} \). For the case of constant height annular inlet and exit ducts the linearised equations of motion can be solved analytically, see Appendix A. In the inlet duct only potential flow perturbations can be created by the compressor and these decay upstream. Solving the non-steady form of the Bernoulli equation for axial flow in the inlet duct (\( \alpha_0=0 \)) gives that at the leading edge of the first blade row (equation A-5):

\[ \frac{\delta P_{11}}{\rho U^2} = -\frac{\partial \delta \phi}{\partial \theta} \quad (12) \]

with \( \delta P_1 \) and \( \delta \phi \) each varying with time and circumferential position. In the exit duct the only allowed disturbances are a decaying potential field and vorticity associated with the variation in compressor loading around the annulus. For fixed compressor exit flow angle (\( \alpha_2 = 0 \), the high-solidity assumption), equation (A-9) gives:

\[ \frac{\delta P_2}{\rho U^2} = \frac{\partial \delta \phi}{\partial \theta} \quad (13) \]

Essentially equations (12) and (13) represent the impedances of the inlet and exit flowfields (in this case it has been assumed that the ducts are long enough for the non-axisymmetric pressure field to completely decay). Combining the above gives the differential equation that determines the time development of the small-amplitude non-axisymmetric perturbation:

\[ \left[ \left( \frac{\partial^2}{\partial \theta^2} + \mu \frac{\partial}{\partial \theta} + \lambda \frac{\partial^2}{\partial \phi^2} \right) \right] \delta \phi = \frac{\partial \psi^{(s)}}{\partial \phi} \delta \phi \quad (14) \]

**Growth or Decay.** The differential operator on the left hand side of the above equation represents the time rate of change in a frame of reference rotating around the annulus at fraction \( \lambda/(\Omega U + \mu) \) of rotor speed. In this reference frame the equation becomes:

\[ \left( \frac{\partial^2}{\partial \theta^2} + \mu \frac{\partial}{\partial \theta} + \lambda \frac{\partial}{\partial \phi} \right) \delta \phi = \frac{\partial \psi^{(s)}}{\partial \phi} \delta \phi \quad (15) \]

Solutions to the above are exponential growth or decay of a non-axisymmetric disturbance depending upon the sign of \( \partial \psi^{(s)}/\partial \phi \). The predicted stability boundary for rotating stall is therefore at the peak of the total-to-static pressure rise characteristic, a condition first presented by Dunham (1965) through considerations of distortion transfer.

The zero slope stability boundary may be explained by a static style argument based on changes in the local mass flow and pressure rise,

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\(^{6}\) Assuming constant height compressor annulus.

\(^{7}\) No assumption, other than quasi-steady and no internal flowfield redistribution, has been made concerning the blade rows within the compressor.
Local decrease in mass flow causes increased pressure rise and results in locally increased flow.

Fig. 5 Static argument showing why negative slopes are stable whilst positive ones are unstable.

Fig. 5. On the negatively sloped pressure rise mass flow characteristc a local increase in mass flow causes a decrease in pressure rise. This causes the flow to decelerate, and the disturbance decays. On a positively sloped characteristic the opposite is true, and so any circumferential disturbance is amplified. Although appealing, this argument does not consider the impact of flowfield dynamics, nor does it explain why it is the total-to-static characteristic that is important, this will be returned to later.

Experimental evidence to support the zero slope stability boundary is limited because of difficulties in estimating the slope of the characteristic. However, in experimental investigations where un-stalled operation beyond the usual stability boundary was possible (Longley & Hynes 1990, Day 1991b, Paduano et al 1991 and Haynes et al 1993) the natural stability boundary was very close to the zero slope condition.

Circumferential Propagation. The derivation of the quasi-steady compressor model has assumed only that the flow disturbance has a long circumferential lengthscale. As a consequence of the fluid dynamics any non-axisymmetric disturbance, that is of potential form in the upstream flowfield, will be circumferentially propagated at a rate that depends primarily on $\lambda$ and $\mu$, the inertia parameters, and does not involve the slope of the characteristic.

The physical processes that determine circumferential propagation may thus be examined by considering the case where the slope of the total-to-static characteristic is zero. For this condition equation (14) may be rewritten as:

$$\left(\frac{2}{\lambda} + \mu - \lambda \right) \frac{\partial \Sigma}{\partial t} \delta \phi = -\lambda \left(\frac{\partial^2 \Sigma}{\partial t^2} + \frac{\partial}{\partial \phi} \right) \delta \phi$$

The left hand side contains the inertia in the stationary components: inlet and exit flowfields ($\mu \Sigma$) and the stationary blades ($\mu - \lambda$). The right hand side contains the inertia in the rotors ($\lambda \Sigma$) and the time rate of change in their frame of reference. Thus the propagation rate is set by the constraint that the inertia induced pressure perturbations in the absolute and relative frames must balance, this was first suggested by Cumpsty & Greitzer (1982).

When viewed in the frame of reference of the flow non-uniformity, the rotors and stators pass in opposite directions, Fig. 6. As drawn, the flow in the rotor passages experience a deceleration associated with the velocity non-uniformity so there must be an inertia driven pressure increase across the blade row in addition to the pressure rise due to flow turning. In the stators (and the inlet and exit flowfields) the flow accelerates and so a decreasing axial pressure gradient is required. Thus it is possible for there to be a circumferential pressure non-uniformity within the compressor, that must accompany the velocity non-uniformity, whilst still satisfying no pressure perturbation far upstream or downstream.

Increasing the propagation rate (which is always less than rotor speed) reduces the rotor inertia effect whilst increasing the stator one, and the rotation rate of the non-uniformity is that which will balance the pressure fields. If inertia is the sole mechanism, then stall-cell propagation rates must be between zero and rotor speed since the time rate of change must be opposite for the rotors and stators.

Fig. 6 Rotors and stators pass in opposite directions through a moving flow non-uniformity. The requirement to match these pressure differences determines the propagation rate.

Evidence to support the importance$^9$ of inertia in determining circumferential propagation speeds is provided by studies of fully developed rotating stall. Cumpsty & Greitzer (1982) examined compressor performance at the trailing edge of a fully-developed rotating stall cell and, as confirmation that the inertia driven non-steady pressure fields must be matched, predicted stall cell speeds, Fig. 7. The predicted stall cell speeds are generally lower than the measured ones and this suggests that other phenomena may be important.

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$^8$ These were achieved by some form of active or passive flowfield stabilisation.

$^9$ Later it will be shown that non-steady blade row performance can also have a significant effect.
The simplified examination of the quasi-steady compressor model suggests that the propagation rate is set only by considerations of fluid inertia. This is contrary to the classical explanation for stall-cell propagation proposed by Iura & Rannie (1954) and Emmons et al (1955, Fig. 9). They argued that it was the upstream flowfield redistribution caused by increased blockage within a blade passage that was important. In this section the simplifying assumptions of perfect IGVs (isentropic, fixed exit angle) and high-solidity compressor exit blades will be relaxed so that both the effects of fluid inertia and changes in incidence can be assessed.

In the inlet flowfield the disturbances are of a potential form and so any axial velocity perturbation, \( \delta v \), has associated with it both tangential velocity (in quadrature) and angle changes. These changes of flow coefficient and inlet angle not only affect the pressure rise, \( \Delta P(\phi, \alpha) \), but also the compressor exit flow angle \( \alpha_2(\phi, \alpha_1) \). For this most general interaction between the compressor model and the inlet and exit flowfields the equation (derived in Appendix B) that determines the behaviour of small amplitude disturbances is:

\[
\frac{2}{v_i} - \mu \frac{\partial}{\partial t} + R \frac{\partial}{\partial \delta} \delta \phi = S \delta \phi \tag{17a}
\]

The rotation term is:

\[
R = \lambda - \frac{\partial \psi \cot \phi}{\partial \alpha_1} = \frac{\partial}{\partial \alpha_1} \left( \frac{1}{2} \delta^2 \sin^2 \phi \right)
\]

and the modified slope of the total-to-static pressure rise characteristic is:

\[
S = \frac{\partial \psi \cot \phi}{\partial \alpha_1} - \frac{\partial \psi \cot \phi}{\partial \alpha_1} \cot \phi = \frac{\partial \psi \cot \phi}{\partial \alpha_1} \cot \phi
\]

Equation (17a) is of the same form as that derived earlier, (equation 14), and so the qualitative behaviour of flowfield disturbances is unchanged though propagation speed and growth rate will be altered.

Whether growth or decay of a disturbance occur is determined by \( S(\psi(\alpha_i + \mu)) \), where \( S \) may be thought of as the effective slope of the total-to-static characteristic. The stability boundary is no longer at zero slope on the total-to-static pressure rise characteristic, it is however still the same for all circumferential harmonics. The rate of propagation is now determined by \( R(\psi(\alpha_i + \mu)) \) and depends on both inertia and swirl sensitivity. The propagation mechanism is, however, still one of balancing pressure perturbations that now have two causes. The inertia term generates a pressure perturbation corresponding to the velocity change, whilst swirl sensitivity has a similar effect as it creates a pressure perturbation associated with the incidence change.

Example of Swirl Sensitivity. As modelled above the blade passage fluid inertia is an easily defined quantity whilst those of swirl sensitivity and the slope of the pressure rise characteristic depend strongly on the compressor design. For instance, compressors with high-solidity IGVs are usually regarded as insensitive to small

\[10\] The blade are still assumed to respond quasi-steadily and there in no internal flowfield redistribution.
The pressure rise characteristic depends on the values of the blade row deviation and total pressure losses (increasing deviation and loss make the slope of the characteristic becomes more positive) and these are influenced by blade design. In this section, a hypothetical compressor design will be used to demonstrate the probable sizes of these effects.

For the present purposes only a representative compressor is required, and so it is sufficient to use a simple empirical correlation (Howell 1965) to determine the variations in blade loss coefficient and exit deviation. The compressor stage design parameters are listed in Table 1 and the inertia parameter was set at \( A = 0.2 \) per row, in accordance with the earlier discussion. Both a single-stage and four-stage configuration will be examined, and their predicted pressure rise characteristics are shown in Fig. 10, note the difference in slopes between the total-to-static and static-to-static curves.

The stability boundary and the circumferential propagation rate is calculated for four configurations of each compressor. The reference case is with perfect IGVs and high-solidity exit blading (the propagation rate is therefore given by equation 14) and the other cases correspond to relaxing either or both of these constraints. When the perfect IGVs are removed (so there is no upstream blade row) the bulk inlet swirl of the flow is changed to the value that was produced at the exit of the IGVs. The rotor blades, therefore, operate with the same mean incidence and the comparison is an assessment of the effects of flowfield redistribution rather than changes due to modified pre-swirl. Without the high-solidity exit constraint the compressor exit angle is taken to be the air outlet angle of the last blade row.

For the single-stage compressor the calculated propagation rate and the slope of the total-to-static characteristic at the stability boundary are shown in Fig. 11. Removing the perfect IGVs increases the predicted propagation rate. For this hypothetical compressor, as with many others, increasing the inlet swirl angle decreases the flow turning (the propagation rate) and also a marginally decreased flow angle perturbation causes an increase in the propagation rate (positive term in equation 17b) and also a marginally decreased stability of the flowfield (positive term in equation 17c). This is confirmed by the negative slope of the total-to-static characteristic at instability and corresponds to approximately a 1% increase in flow coefficient for the characteristics of Fig. 10.

Relaxing the high solidity exit blading constraint similarly increases the propagation rate (through the \( \partial \phi_{VT}/\partial \alpha_i \) term) but has very little effect on the stability boundary because, for this compressor, \( \partial \phi_{VT}/\partial \alpha_i \approx 0.06 \) which is small compared to the other terms.

To investigate whether these effects are important in multi-stage compressors, a similar study for a four-stage configuration is shown in Fig. 12. The results are comparable, the first mode propagation rate is approximately doubled and the stability boundary is at an increased flow coefficient of \( 1/2 \% \).

### Table 1 Parameters at design for the hypothetical single-stage and four-stage compressors used in the illustrative examples.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow Coefficient</td>
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<tr>
<td>Stage Loading (( \Delta \beta/\beta ))</td>
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<tr>
<td>Reaction</td>
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<tr>
<td>Inlet Flow Angle (( \alpha_i ))</td>
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<tr>
<td>Rotor Inlet Angle (( \alpha_{ir} ))</td>
<td>-53.13</td>
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<tr>
<td>Rotor Exit Angle (( \alpha_{or} ))</td>
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<tr>
<td>Stator Inlet Angle (( \alpha_{s} ))</td>
<td>45.00</td>
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<tr>
<td>Stator exit Air Angle (( \alpha_{es} ))</td>
<td>18.43</td>
</tr>
<tr>
<td>Inertia Parameter (( A ) per row)</td>
<td>0.2</td>
</tr>
</tbody>
</table>

11 For large amplitude flow disturbances, such as inlet distortion and fully developed rotating stall, a circumferential portion of the IGVs can have separated flow and result in high loss and exit angle deviation.

12 The small change in exit angle has only a minor effect on the pressure rise characteristic because the flow is normally near axial.

13 The stability boundary is independent of the mode number.

### Enhanced Compressor Model

The quasi-steady compressor model has been examined both with simplifying assumptions concerning the flowfield interaction (perfect IGVs and high-solidity exit blading) and also in its most general form. For the first four circumferential modes the predicted propagation rates varied substantially between the different configurations, there was no preferential circumferential lengthscale and the stability boundary was always near the peak of the total-to-static characteristic.
However, the higher the circumferential harmonic, the more likely it is that the blade pressure losses and exit deviation will respond non-steady. An enhanced compressor model will now be examined that incorporates some account of these non-steady flow effects and accounts for axial gaps between adjacent blade rows.

Non-Steady Loss and Deviation. The steady state pressure rise across a blade row can be written as:

$$P_{\text{out}} - P_{\text{in}} = \frac{1}{2} \phi^2 (\tan^2 \alpha_{\text{in}} - \tan^2 \alpha_{\text{out steady}}) - \text{LOSS}_{\text{steady}} \quad (18a)$$

where

$$\text{LOSS}_{\text{steady}} = \frac{P_{\text{in}} - P_{\text{out}}}{\rho U^2} \quad (18b)$$

and all quantities are in the frame of reference of the blade. An approach for modelling the non-steady response of a blade row is to apply a time lag to the loss and deviation. (This lag law was originally used by Emmons et al 1955 to predict the time rate of change of flow.) Thus the static pressure rise across a blade row in non-uniform non-steady flow may be modelled by:

$$P_{\text{out}} - P_{\text{in}} = \frac{1}{2} \phi^2 (\tan^2 \alpha_{\text{in}} - \tan^2 \alpha_{\text{out}}) - \text{LOSS} \quad (19a)$$

with:

$$\tau_{\text{loss}} \frac{\partial}{\partial t} \text{LOSS} = \text{LOSS}_{\text{steady}} - \text{LOSS} \quad (19b)$$

and:

$$\tau_{\text{dev}} \frac{\partial}{\partial t} \alpha_{\text{out}} = \alpha_{\text{out steady}} - \alpha_{\text{out}} \quad (19c)$$

Some investigations into these lag laws for non-steady blade row performance have been undertaken for disturbances of large amplitude. Nagano et al (1971) experimentally studied the variation of blade loss and deviation for a compressor operating in rotating stall. They concluded that whilst the pressure loss time lag ($\tau_{\text{loss}}$) was approximately the convection time, the deviation time lag ($\tau_{\text{dev}}$) varied across the range of zero to 3 times convection time.

Mazzawy (1977) developed the multiple segment parallel compressor model and included a pressure loss time lag. The justification for doing so was experimental measurements of rotor loss as a function of incidence both in steady and non-steady (non-uniform) flow, these are shown in Fig. 13 along with the variation predicted using the lag law (equation 19). The validity of this time lag model for small amplitude disturbances is inferred by the comparison of Haynes et al (1993) shown later.

Example of Time Lags. The effects of pressure loss and blade deviation time lags are compared using the hypothetical four-stage compressor in the configuration with perfect IGVs and high-solidity exit blading as the reference case. The time lag was set approximately equal to the convection time, so $\tau_{\text{loss}} = \tau_{\text{dev}} = 0.3$ (assuming $c_{\tau}/c_{\tau_{\text{c}}} = 0.15$ and $\phi = 0.5$). For the cases with and without time lags, the propagation rate and slope of the characteristic at the stability boundary are shown in Fig. 14. Both pressure loss and deviation time lags increase the propagation rate and stabilise the higher modes, with their combined effect being approximately the sum of the individual ones. The higher the mode the greater the stabilising effect of non-steady blade row response, as indicated by a more positive slope of the total-to-static characteristic. This ordering is consistent with the higher modes having higher reduced frequency. This separation of the individual mode stability boundaries was first suggested by Stenning & Kriebel (1958) and experimentally verified by Haynes et al (1993).

The increased propagation rate may be explained by interpreting the reduced amplitude of response due to time lag as, in some sense (see later section), a larger blade passage fluid inertia and thereby increasing the speed. The reduction in the amplitude of the total pressure loss perturbation has the effect of making the time-average characteristic more negatively sloped, Fig. 15, and hence more stable (deviation time lag has a similar effect).
Fig. 14 Effects of pressure loss and deviation time lags on the hypothetical four-stage compressor (with perfect IGVs and high-solidity exit blading).

Example of Blade Row Gaps. In all of the models described above it has been assumed that there is no internal flowfield redistribution, i.e. $\delta_1 = \delta_2$. This can only ever be an approximation as there must be some clearance between adjacent blade rows. The larger the gaps the less the coupling (non-axisymmetric and non-steady flowfield interaction) between the stages, see Turner (1959), Ham & Williams (1983) and Longley & Hynes (1990).

The presence of gaps has several effects, firstly internal circumferential flow redistribution changes both the amplitude of the perturbation through the compressor and the incidences on the blade rows within the compressor (Longley 1990). Secondly, the interaction pressure fields between adjacent components can decay resulting in an effective "decoupling" of the compressor blade rows. This can affect the stability of the system in either a beneficial or detrimental way (see Longley & Hynes).

Conceptually, including the effects of gaps into a compressor model is straightforward. Each gap between the blade rows is considered as an annular volume and the equations of motion linearised to obtain solutions, similar to what was done in the inlet and exit ducts. Within each gap there are three unknowns corresponding to the vortical perturbation and the two potential perturbations (upstream and downstream decaying pressure fields). There are three boundary conditions specified at the upstream end of the gap, namely matching the flow angle, mass flow and pressure perturbations, and so the unknowns may be found. The implementation of the model however becomes more complicated than the preceding compact models, e.g. equation (9), as each blade row and gap must be explicitly analysed.

For the four-stage compressor, with perfect IGVs and high-solidity exit blading, the effect of gaps of length $x/r=0.05$ $(c_{v}/c=0.15)$ are shown in Fig. 16. The propagation rate is slightly increased and the individual modes have different stability boundaries. The gaps appear longer for the higher modes and therefore the compressor behaves more like four individual compressor stages. An individual stage has its total-to-static pressure rise peak at a lower flow than the overall compressor (assuming identical stages) and therefore the decoupling stabilises the higher modes.

Fig. 15 Lag law reduces the effect of the pressure losses and thereby making the effective slope of the characteristic more negative (stable).
Simplified Forms of Compressor Model

The inclusion of time lags and inter-blade-row gaps increases the complexity of the model and it becomes both computationally expensive and requires extensive information about the individual compressor blade rows. In many instances this information is either not available or the application does not warrant such effort. Therefore, approximate methods have been developed that include some of the advantages of the enhanced model.

Lagged Losses and Quasi-Steady Deviation. When both pressures and flow angles differ from their steady state values an explicit row by row analysis is required. If, however, only the total pressure losses are lagged, ie \( \tau_{\text{dev}}=0 \), then all flow angles are quasi-steady and the individual blade rows can be recombined (see Chue et al 1989 and Hendrix & Gysling 1992) to give:

\[
\frac{P_2 - P_1}{\rho U^2} = \psi'_{\text{ideal}} - L_{\text{rotor}} - L_{\text{stator}} - \frac{\lambda_{\theta}}{\partial \theta} - \mu_{\theta} \frac{\partial \Phi}{\partial t} \tag{21a}
\]

\[
\alpha_2 = \alpha_2(\Phi, \alpha_1) \bigg|_{\text{steady}} \tag{21b}
\]

with

\[
\tau_{\text{loss}} \frac{\partial}{\partial t} L_{\text{stator}} = \frac{L_{\text{stator}}}{L_{\text{stator}}} - L_{\text{stator}} \tag{21c}
\]

and

\[
\tau_{\text{loss}} \left( \frac{\partial}{\partial t} + \frac{\partial}{\partial \theta} \right) \frac{L_{\text{rotor}}}{L_{\text{rotor}}} = \frac{L_{\text{rotor}}}{L_{\text{rotor}}} - L_{\text{rotor}} \tag{21d}
\]

In the above \( \psi'_{\text{ideal}} \) represents the ideal (isentropic) pressure rise due to flow turning (first part on right hand side of equation 18) and \( L_{\text{stator}}, L_{\text{rotor}} \) are the overall total pressure losses for the rotors and stators respectively (\( \psi'_{\text{ideal}} = \psi'_{\text{ideal}} - \psi'_{\text{ideal}} - \psi'_{\text{ideal}} - \psi'_{\text{ideal}} \)).

A comparison between the calculated and measured propagation and growth rates for small amplitude flowfield disturbances was undertaken by Haynes et al (1993) for a three-stage compressor that appeared to have little non-steady deviation effect, Fig. 17. The calculations were based upon decomposing the measured pressure rise performance, \( \psi'_{\text{ideal}} \), into the ideal pressure rise and loss terms and then solving the set of equations (21). The experimental measurements were obtained by actively controlling the compressor and extracting the flowfield dynamics from the controlled system. The comparisons indicate that with an appropriate choice of the time constant (1.5 x convection time) the growth rates and propagation speeds for the first three modes were well predicted across a range of flow coefficients.

Effective Inertia Parameters. As mentioned above one interpretation of the effects of time lags is to increase the blade passage fluid inertia. In this section approximate relationships are derived that determine the modified, or effective, inertia parameters.

For the case of quasi-steady exit angles but time lagged pressure losses equation (21c) can be rearranged as:

\[
L_{\text{stator}} = \frac{L_{\text{stator}}}{1 + \tau_{\text{loss}} \frac{\partial}{\partial t}} \tag{21c}
\]

For a single temporal frequency and small values of \( \tau_{\text{loss}} \) this can be approximated for the stator by:

\[
L_{\text{stator}} = L_{\text{stator}} - \tau_{\text{loss}} \frac{\partial}{\partial \theta} \frac{\partial \Phi}{\partial t} \tag{21c}
\]

and for the rotor by:

\[
L_{\text{rotor}} = L_{\text{rotor}} - \tau_{\text{loss}} \frac{\partial}{\partial \theta} \frac{\partial \Phi}{\partial t} \tag{21c}
\]

Substituting these into equation (21a) and rearranging gives:

\[
\frac{P_2 - P_1}{\rho U^2} = \psi'_{\text{ideal}} - L_{\text{rotor}} - L_{\text{stator}} - \frac{\lambda_{\theta}}{\partial \theta} - \mu_{\theta} \frac{\partial \Phi}{\partial t} \tag{22a}
\]

where:

\[
\lambda_{\text{eff}} = \lambda - \tau_{\text{dev}} \frac{\partial}{\partial \theta} \frac{\partial \Phi}{\partial t} \tag{22b}
\]

and:

\[
\mu_{\text{eff}} = \mu - \tau_{\text{loss}} \frac{\partial}{\partial \theta} \frac{L_{\text{stator}} + L_{\text{rotor}}}{L_{\text{stator}} + L_{\text{rotor}}} \tag{22c}
\]

These effective inertia parameters were first presented by Chue et al (1989).

Following a similar approach, Strang (1991) suggested that the effects of time lags on blade deviation could be approximated by:

\[
\lambda_{\text{eff}} = \lambda - \tau_{\text{dev}} \sum_{\text{rows}} \frac{\sin \alpha_{\text{out}}}{\cos \alpha_{\text{out}}} \frac{\partial \alpha_{\text{out}}}{\partial \theta} \tag{23a}
\]

\[
\mu_{\text{eff}} = \mu - \tau_{\text{dev}} \sum_{\text{all rows}} \frac{\sin \alpha_{\text{out}}}{\cos \alpha_{\text{out}}} \frac{\partial \alpha_{\text{out}}}{\partial \theta} \tag{23b}
\]

The assumption made in deriving the above is that the change in flow angle due to time lag affects only the pressure rise in that blade row and not any of the downstream ones.

In both of these approximations modifying the values of the inertia parameters to \( \lambda_{\text{eff}} \) and \( \mu_{\text{eff}} \) yields a model of identical structure to the quasi-steady one. Therefore the only differences will be modified propagation and amplitude of the growth rates but no preferential circumferential length scale (all the modes will share the same stability boundary near the peak of the total-to-static characteristic).
Discussion

Compressors are predicted to behave like circumferential wave propagators for the time development of non-axisymmetric (stall-like) disturbances. This oscillatory nature associated with axisymmetric surge-like phenomena within compression systems. The circumferential propagation rate is determined by the requirement to balance the non-steady pressure perturbations across the moving and stationary blade rows with those in the inlet and exit flowfields.

The mechanisms that drive these pressure perturbations are the inertia of the fluid within the flowfield, the effects of incidence on the blade row pressure rise and non-steady viscous effects. The inertia effect will always be present whilst the incidence and viscous effects will, to some extent, depend on the blade design. In the examples shown the propagation rate can be approximately doubled depending upon how the blade rows are modelled.

The growth or decay of circumferentially propagating flowfield disturbances is related to the slope of the total-to-static pressure characteristic, the swirl sensitivity of the compressor and the non-steady blade row response. For the first circumferential mode the predicted stability boundary is close to the peak of the total-to-static characteristic and has little dependence on non-steady flow effects. In the examples included, the non-steady blade response has a stabilising influence on the higher modes and therefore only the low order circumferential modes are expected to be detected prior to instability.

Velocity and Pressure Perturbations. In this review much has been made of the relationships between velocity and pressure perturbations, and the different types will now be summarised. Whenever the pressure perturbation is determined by the time derivative of the absolute axial velocity perturbation, eg equation (13), the quantity is inertial and affects the amount of the compressor flowfield that the other forces (propagation and growth or decay) must drive. If the relationship is a simple proportionality, eg \( \delta P = c \delta \phi \), then it affects the stability of the flowfield, with a destabilising effect if they are in-phase (\( c > 0 \)) and stabilising effect when they are in anti-phase (\( c < 0 \)). In effect used for positive slopes, to ensure that the amount of the compressor flowfield that the other forces (propagation and growth or decay) must drive is always present whilst negative one are stable. Whenever pressure and velocity are in quadrature (\( \delta P = \partial \delta \phi / \partial \delta \phi \)), the effect is to circumferentially propagate the disturbance. An example of this latter effect is found in a rotor blade passage when it passes through a spatial flow non-uniformity.

These ideas concerning velocity and pressure relationships will now be used to demonstrate that some physical mechanisms, in this case inlet swirl sensitivity, may affect both propagation and stability depending on the compressor design.

In the inlet flowfield the disturbance is of potential form and so the axial and tangential velocity perturbations are in quadrature. For small values of mean inlet swirl angle, the inlet angle perturbation (\( \delta \omega \)) is determined by the tangential velocity change and is therefore in quadrature with the axial velocity. For this case, the pressure rise perturbation associated with \( \delta \omega \) will affect the propagation rate. For compressors with large values of inlet swirl, the angle perturbation will be primarily associated with the axial velocity change. Therefore the swirl sensitivity pressure perturbation will be in-phase or anti-phase with the velocity change and consequently will affect the stability.

The dual effect of swirl sensitivity is confirmed by both equation (17b), propagation rate, and (17c), growth or decay, having a \( \delta P / \delta \omega \delta \phi \) term with a coefficient depending on the size of \( \omega \).

Stability Argument. Gysling (1992) examined compressor stability in terms of the net mechanical energy input into the flowfield. A compressor puts energy into a non-axisymmetric flowfield disturbance whenever the total-to-total characteristic has a positive slope (\( \partial^2 \phi / \partial \phi^2 > 0 \)). Mechanical energy can escape since some of it is convected away by the non-steady vortical disturbance in the exit flowfield, and instability will only therefore occur when the compressor puts in more energy than can be convected away (or dissipated in losses). In rough terms, the vortical field carries energy at a rate \( \phi \), the convection speed, and so instability occurs approximately when:

\[
\frac{d \phi}{d \phi} = 0
\]

or equivalently, since \( \phi = \phi^2 + 1/2 \phi^2 \), when:

\[
\frac{d \phi^2}{d \phi} = 0
\]

This is the zero slope of the total-to-static pressure rise characteristic condition that was deduced from equation (14) and observed in the later studies of low order modes in the hypothetical compressor. Relaxing the high-solidity exit blading constraint can change (depending on the size of \( \delta \phi / \delta \phi \)) the balance between the decaying potential field (which carries no net energy) and the shed vorticity, thus affecting the stability boundary. It is not yet known whether or not this could be a feasible approach to active control, which as implemented by Paduan (1991) and Haynes & Greitzer (1993) has only used \( \delta \phi / \delta \phi \) in equation (17c).

Other Flow Perturbations. The stability of the flowfield has been considered by analysing two-dimensional (stall-like) disturbances that have a long circumferential lengthscale. However flowfield instability requires that all types of flow disturbances are stable. It can be shown (for instance see Haynes & Greitzer 1987) that one-dimensional (surge-like) disturbances go unstable at points on the positively sloped part of the total-to-static characteristic. This would suggest that stall-like disturbances are expected to grow prior to surge and this has been observed (see Day 1991b).

Other types of flowfield disturbances that have not yet received extensive modelling are short lengthscale (similar to blade pitch) two-dimensional and three-dimensional (with radial variation of flow properties) ones. If any of these become unstable before long lengthscale two-dimensional ones then the compressor will have a different stability boundary.

Day (1991a) exhibited results that show rotating stall developing from a "spike" or "pulp" (three-dimensional and short circumferential lengthscale) on some cases whilst from a "modal-wave" (two-dimensional) in others. These short lengthscale disturbances are observed to propagate at approximately 75% of rotor speed (much faster than long lengthscale two-dimensional ones) and it is not yet known what mechanisms determine their behaviour.

Other Effects. Emmons et al (1955) observed that the propagation rate for linearised disturbances have a weak dependence on the spatial mode, see equation (14). They argued that the propagation rate for fully developed rotating stall must be set by non-linear effects as much as one spatial harmonic is involved. Although examined in terms of small amplitude disturbances, the models described above have been successfully used to study nonlinear phenomena, in ones with large amplitude flow disturbances (see Moore 1984bc, Emmons & Greitzer 1987). Longley (1990) experimentally verified that the non-linear flowfield redistribution associated with inlet distortion could be adequately simulated using the quasi-steady model. Longley also identified the circumferentially propagating, growing and decaying waves that had been predicted by Haynes & Greitzer.

The effects of compressibility have not been investigated in this review. However, the fluid dynamic phenomena (inertia and non-steady response) that these models simulate are not restricted to low Mach number flows, though their implementation will differ. Consequently compressible versions of these models have been developed (LINEARB 1980, Kodama 1986, Bonnaurc 1991) though verification of their predictions is harder. Compressibility only introduces additional physical phenomena at high Mach numbers (shocks and choking), and these have been discussed by Mazzawy (1980) and modelled by Cargill & Freeman (1990).
Conclusions

Important contributions towards understanding the behaviour of flowfield disturbances can be made by examining the models for non-steady flow within compressors. These models have also been successfully used to calculate the time development of flowfield disturbances though, as shown by the included examples, the propagation rate depends strongly on the assumptions made within the model. The calculated stability boundary for the first mode is, however, always near to the peak of the total-to-static characteristic whilst for higher circumferential modes the non-steady blade row response may have a substantial stabilising effect.

The final choice of compressor model depends upon the amount of information that is available, though it appears that non-steady blade row response must be modelled for quantitative comparisons. Experimentally it has been observed that rotating stall can result from a short length scale three-dimensional disturbance and these types of phenomena are not addressed by the models described above. Areas where further research could be beneficial are the modelling of non-steady blade row response to moderate reduced frequency disturbances and the modelling of three-dimensional flow phenomena.

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In this appendix the relationships between the flow coefficient perturbation, \( \delta \phi (\xi, \theta, r) \), and the pressure and flow angle perturbations in the inlet and exit flowfields are derived. The assumption made is that the equations of motion may be linearised, and a single Fourier disturbance of the form \( e^{i \omega t - k r} \) will be considered.

**Inlet Flowfield.** The compressor can only create localised flow disturbances and these must be of the potential form that decays upstream. The non-dimensional axial (SO) and tangential (Sy) velocity perturbations in the inlet flowfield may thus be expressed:

\[
SO = \frac{\partial \phi}{\partial r} \quad \text{and} \quad Sy = \frac{\partial \phi}{\partial \theta}
\]

where the velocity potential is \( \phi (\xi, \theta, r) = A(\xi) e^{i \omega t - k r} \).

Since \( \nabla^2 \phi = 0 \), then for upstream decaying disturbances \( A(\xi) = ae^{i \alpha \xi} \), and so \( \delta \phi = \ln \delta \phi \).

The tangential perturbation is:

\[
\delta \phi = \frac{1}{i \alpha} \delta \phi_1
\]

To evaluate the corresponding perturbation in angle, \( \delta \alpha \), the definition \( \nu = \tan \alpha \) is linearised:

\[
\delta \nu = \delta \tan \nu + \nu \sec^2 \nu \delta \nu
\]

and so:

\[
\delta \alpha = \cos^2 \alpha \left( \frac{1}{i \alpha} \delta \nu_1 \right) - \delta \tan \alpha
\]

The non-steady form of the Bernoulli equation (for disturbance of assumed form) is:

\[
\frac{\partial \phi}{\partial t} + \frac{\nabla \phi}{c_p U^2} = \text{constant}
\]

and therefore:

\[
\frac{\delta p}{\rho U^2} = -\frac{1}{i \alpha} \delta \phi_1
\]

The above equations (A-1 to A-5) are valid everywhere in the upstream flowfield, and so to specify the relationships at the leading edge of the first blade row it is only necessary to apply a subscript "1" to the appropriate quantities.

**Exit Flowfield.** In the exit duct there is a convected vortical disturbance along with a downstream decaying potential field. Two of the equations of motion are:

Axial Momentum:

\[
\frac{\partial \phi}{\partial r} + \frac{\phi}{\partial \theta} + \nu \frac{\partial \phi}{\partial \theta} = -\frac{\partial}{\partial \theta} \frac{\delta p}{\rho U^2}
\]

Continuity:

\[
\frac{\partial}{\partial r} \delta \phi + \frac{\partial}{\partial \theta} \delta \nu = 0
\]

Eliminating the axial derivative and using \( \nu = \tan \alpha \) gives:

\[
\frac{\delta p}{\rho U^2} = \frac{\delta \phi}{\delta \alpha}
\]

The pressure field decays downstream and therefore is of the form:

\[
\frac{\delta p}{\rho U^2} = b e^{-i \alpha \xi} + \nu + i \omega t
\]

Substituting equation (A-2) and rearranging gives:

\[
\frac{\delta p}{\rho U^2} = -\frac{1}{i \alpha} \delta \phi_1
\]

This equation is valid everywhere in the downstream flowfield, but will only be applied at the trailing edge of the last blade row of the compressor, denoted by subscript "2". (It is worth noting that this equation is also valid in the relative frame provided that all the quantities are relative and that the time rate of change is interpreted in the relative frame.)

**Appendix B: Compressor and Flowfield Interaction**

Derived in this appendix is the most general form of interaction between the inlet and exit flowfields and the quasi-steady compressor model. The total-to-static pressure rise of the compressor is now \( \psi(\phi, \alpha) \) and so the linearised form of equation (9a) becomes:

\[
\frac{\delta p_2}{\rho U^2} = \frac{\delta p_1}{\rho U^2} + \phi \frac{\delta \nu_1}{\theta} - \lambda \frac{\partial}{\partial \nu} (\delta \phi_2) - \mu \frac{\partial}{\partial \nu} (\delta \phi_1)
\]

An expression for \( \delta \nu_1 \) in terms of \( \delta \phi \) is given by equation (A-3) and the inlet flowfield impedance \( \delta p_1/\rho U^2 \) in terms of \( \delta \phi \) is still given by equation (A-9). In the exit flowfield, however, \( \alpha_2(\phi, \alpha_1) \) and so when linearised becomes:

\[
\delta \alpha_2 = \frac{\partial \alpha_2}{\partial \theta} \delta \phi_1 + \frac{\partial \alpha_2}{\partial \alpha_1} \delta \alpha_1
\]

Therefore, the exit impedance becomes (from equation A-9):

\[
\frac{\delta p_2}{\rho U^2} = \left( \frac{1}{i \alpha} \delta \phi_1 - \frac{\phi^2}{i \alpha} \sec^2 \alpha_2 \left( \frac{\partial \alpha_2}{\partial \theta} \delta \phi_1 + \frac{\partial \alpha_2}{\partial \alpha_1} \delta \alpha_1 \right) \right)
\]

---

14 Non-dimensional axial distance is \( \xi = (axial \ distance)/radius \).
Combining equations (B-1), (B-2) and (B-3) with (A-3) and (A-5) gives, after rearrangement:

\[
\left(\frac{1}{\alpha_1} + \mu \right) \frac{\partial}{\partial t} + \mathbf{R} \frac{\partial}{\partial \theta} \delta \phi = S \delta \phi \quad \text{(B-4a)}
\]

where:

\[
\mathbf{R} = \lambda - \frac{\partial \psi^a}{\partial \alpha_1} \phi \ln \frac{\psi}{\psi^a} + \phi \sec^2 \alpha_2 \left( \sin \alpha_1 \cos \alpha_1 \frac{\partial \alpha_2}{\partial \alpha_1} - \phi \frac{\partial \alpha_2}{\partial \phi} \right) \quad \text{(B-4b)}
\]

and:

\[
S = \frac{\partial \psi^a}{\partial \phi} - \frac{\partial \psi^a \sin \alpha_1 \cos \alpha_1}{\partial \alpha_1} - \phi \frac{\partial \alpha_2 \cos^2 \alpha_2}{\partial \alpha_1 \cos^2 \alpha_2} \quad \text{(B-4c)}
\]