INTRODUCTION

The oscillating cascade problems have received significant attention in recent years. A number of Euler solutions of the oscillating transonic cascade flows have been presented. These solutions, in a Cartesian coordinate system, are used to study the unsteady Euler equations. To validate the accuracy of the present approach, transonic flows around single NACA 0012 airfoil pitching harmonically about the quarter chord are computed first. The calculated instantaneous pressure coefficient distributions during a cycle of motion compare well with the related numerical and experimental data. To further evaluate the present approach, calculations of transonic flow around oscillating cascade of two unstaggered NACA 0006 blades with interblade phase angle equal to 180 deg are performed. From the instantaneous pressure coefficient distributions and time history of lift coefficient, the present approach, where a simple spatial treatment is utilized on the periodic boundaries, gives satisfactory results. By using the above solution procedure, transonic flows around oscillating cascade of four biconvex blades with different oscillation amplitudes, reduced frequencies, and interblade phase angles are investigated. From the distributions of magnitude and phase angle of the dynamic pressure difference coefficient, the present numerical results show better agreement with the experimental data than those from the linearized theory in most of the cases. For every quarter of one cycle, the pressure contours repeat and proceed one pitch distance in the upward or downward direction for interblade phase angle equal to -90 deg or 90 deg, respectively. The unsteady pressure wave and shock behaviors are observed. From the lift coefficient distributions, it is further confirmed that the oscillation amplitude, interblade phase angle, and reduced frequency all have significant effects on the transonic oscillating cascade flows.

ABSTRACT

The modified total-variation-diminishing scheme and an improved dynamic triangular mesh algorithm are presented to investigate the transonic oscillating cascade flows. In a Cartesian coordinate system, the unsteady Euler equations are solved. To validate the accuracy of the present approach, transonic flow around single NACA 0012 airfoil pitching harmonically about the quarter chord is computed first. The calculated instantaneous pressure coefficient distributions during a cycle of motion compare well with the related numerical and experimental data. To further evaluate the present approach involving nonzero interblade phase angle, the calculations of transonic flow around oscillating cascade of two unstaggered NACA 0006 blades with interblade phase angle equal to 180 deg are performed. From the instantaneous pressure coefficient distributions and time history of lift coefficient, the present approach, where a simple spatial treatment is utilized on the periodic boundaries, gives satisfactory results. By using the above solution procedure, transonic flows around oscillating cascade of four biconvex blades with different oscillation amplitudes, reduced frequencies, and interblade phase angles are investigated. From the distributions of magnitude and phase angle of the dynamic pressure difference coefficient, the present numerical results show better agreement with the experimental data than those from the linearized theory in most of the cases. For every quarter of one cycle, the pressure contours repeat and proceed one pitch distance in the upward or downward direction for interblade phase angle equal to -90 deg or 90 deg, respectively. The unsteady pressure wave and shock behaviors are observed. From the lift coefficient distributions, it is further confirmed that the oscillation amplitude, interblade phase angle, and reduced frequency all have significant effects on the transonic oscillating cascade flows.

INTRODUCTION

Recently, significant attention has been given to the vibration problems. A number of Euler solutions of the oscillating cascade flows were presented (Gerolymos, 1988; He, 1990; Huff et al., 1991; Bendiksen, 1991; Huff, 1992; Hsiao and Bendiksen, 1992). By utilizing the explicit MacCormack scheme, Gerolymos (1988) investigated both started and unstarted supersonic flows in vibrating cascade of fan blades. He (1990) developed sinusoidal shape and high-order shape corrections for phase-shifted periodic boundary condition, so that the large computer storage required by the conventional direct parameter storage approach can be reduced. Huff et al. (1991) introduced a high resolution wave-split scheme to predict the unsteady aerodynamics associated with transonic flows over oscillating cascades. Implementing five-stage Runge-Kutta finite volume scheme on a deformable cascade mesh, Bendiksen (1991) calculated the unsteady transonic cascade flows. Based on Beam-Warming, block ADI algorithm, Huff (1992) studied the flows around oscillating flat-plate cascade and biconvex airfoil cascades. Hsiao and Bendiksen (1992) introduced a Runge-Kutta Galerkin finite element method to solve the unsteady flow in cascade. Even though several works have been done, further research on the Euler solution procedure for solving the oscillating cascade flows with nonlinear effects is still necessary and worthwhile. To accurately resolve the moving shocks and compression waves, a modified locally implicit TVD scheme on the dynamic triangular mesh is presented in this work. Instead of using periodic boundary conditions from block to block for traditional structured grid systems, only single block is used, and two different approaches to treat the periodic boundary conditions are evaluated. It is found that the simple spatial approach achieves more numerical stability than direct parameter storage method.

In the oscillating cascade flow calculations, considerable efforts have been expended to develop the dynamic mesh algorithms on the structured (Gerolymos, 1988; He, 1990; Bendiksen, 1991; Huff, 1992) and unstructured grid systems (Hsiao and Bendiksen, 1992). Gerolymos (1998) presented a grid displacement procedure to conform with the position of vibrating blades. To save the computing effort, He (1990) created a sonal moving grid technique, where only the grids in local regions around
blades are moved. In the calculations of oscillating cascade flows, Bendiksen (1991) indicated that the H-mesh solution produced numerical oscillation in the leading edge. Huff (1992) presented a deformig grid technique to generate C-grid to fit with the moving blade. Even though the aforementioned structured grids can be applied to the oscillating cascade flow problems, the use of unstructured triangular meshes is more suitable for domain with complicated boundaries. By using the advancing front technique and conformal mapping procedure, Hsiao and Bendiksen (1992) created a globally unstructured but locally structured blade-fitted deformable mesh. In the present paper, a dynamic mesh algorithm, which was originally developed for solving oscillating airfoil flow (Hwang and Yang, 1992), is improved, so that the dynamic mesh can be in time to respond to the oscillating cascade of blades.

The objectives of this work are: (1) to develop a numerical solution procedure for studying the transonic oscillating cascade flows and (2) to investigate the effects of oscillation amplitude, interblade phase angle and reduced frequency on unsteady flow phenomena. In the present numerical solution procedure, modification of a locally implicit TVD scheme on dynamic triangular mesh, improvement of a dynamic triangular mesh algorithm, triangular mesh generation, spatially periodic boundary treatment and nonreflecting inlet/outlet boundary conditions are included.

For the transonic flows around single oscillating airfoil and oscillating cascade of two blades with 180 deg interblade phase angle, satisfactory results are obtained. In the calculations of transonic flows around oscillating cascade of four biconvex blades, different values of oscillation amplitude, reduced frequency and interblade phase angle are employed. The distributions of magnitude and phase angle of the first harmonic dynamic pressure difference coefficient, instantaneous pressure contours and lift coefficient distributions are presented. In the comparison of magnitudes and phase angles with the related data of experiment and linearized theory, the present Euler solutions are acceptable. For every quarter time period, the instantaneous pressure contours repeat and move one pitch distance in the upward or downward direction for interblade phase angle equal to -90 deg or 90 deg, respectively. The unsteady shock and pressure wave behaviors are observed. From the present numerical results, it is concluded that the oscillation amplitude, interblade phase angle and reduced frequency all significantly affect the physical phenomena of transonic oscillating cascade flows.

NUMERICAL APPROACH

For unsteady inviscid flows in a moving domain, the two-dimensional Euler equations including moving cell effects (Hwang and Yang, 1992) are solved in the $X - Y$ Cartesian coordinate system. On the dynamic triangular mesh, a locally implicit TVD dynamic mesh (Hwang and Yang, 1992) are solved in the one-dimensional Euler equations including moving cell effects (Hwang and Yang, 1992), is modified by using an improved limiter function (Hwang and Liu, 1992). Instead of calculating $K_1$ explicitly (Hwang and Yang, 1992), the spring stiffness $(K_1)$ is updated implicitly during each symmetric iteration cycle in the present work, so that the dynamic mesh can be improved to respond to the moving boundary immediately. In addition to the mass, momentum and energy conservation laws which govern the physics of the flow, the geometric conservation (Marcel Vinokur, 1989) is satisfied numerically in the present paper.

RESULTS AND DISCUSSION

To validate the accuracy of present approach, transonic flows around single oscillating NACA 0012 airfoil and oscillating cascade of NACA 0006 blades with $\sigma$ equal to 180 deg are studied first. In the calculations of NACA 0006 cascade flow, the direct parameter storage method and the present spatial treatment are implemented on the periodic boundaries to compare the numerical characteristics. For transonic flows around oscillating cascade of four biconvex blades, the computations with different oscillation amplitudes, reduced frequencies and interblade phase angles are performed. The magnitude and phase angle of the first harmonic dynamic surface pressure difference coefficient $(C_{p1})$ are obtained by using the Fourier transform for the last cycle of oscillation. The dynamic surface pressure difference
The coefficient is normalized by the oscillating amplitude and the phase angle is referenced to the blade motion. In this work, the calculated magnitudes and phase angles are compared to those from experiment and linearized theory. Also, the lift coefficient distributions and unsteady behaviors related to moving shocks and pressure waves are presented.

**Code Validation**

**Transonic Flow Around Oscillating NACA 0012 Airfoil.** To evaluate the present solution algorithm, the transonic flow, which is around a NACA 0012 airfoil pitching harmonically about the quarter chord with an oscillation amplitude of \( \alpha_1 = 2.51 \) deg and a reduced frequency of \( k = 0.0814 \) based on semichord, is investigated. The motion of the oscillating airfoil is governed by the relation

\[
a = a_0 + \alpha_1 \sin(2\pi f t)
\]

where \( a \) is the instantaneous angle of attack. The frequency \( \omega \) is related to the reduced frequency by the relation \( k = \omega c / 2V_{\infty} \), and \( c \) is the airfoil chord length. The computational domain is taken to be \( 21c \times 20c \), and the mesh (Fig. 1) contains 4888 elements and 2516 nodes. In this case, the solution for the airfoil motion is obtained with direct parameter storage method for phase-shifted periodic boundary conditions. The flow is around a NACA 0012 airfoil pitching harmonically about the quarter chord with an oscillation amplitude of \( \alpha_0 = 0.016 \) deg as the initial condition. By choosing CFL as 30, three cycles of airfoil motion are processed to obtain a periodic solution. Comparing with the related numerical results (Kandil and Chuang, 1989; Batina, 1991) and experimental data (Landon, 1982), the calculated instantaneous pressure coefficient distributions during the third cycle of motion are shown in Fig. 2. Except the pressure distributions for \( a = 1.09 \) deg, the results during the first part (a is positive) of the cycle demonstrate a shock wave on the upper surface of the airfoil, and the flow over the lower surface is predominantly subcritical. During the latter part (a is negative) of the cycle (except \( a \) equal to \(-1.25 \) deg), the flow over the upper surface is subcritical, and a shock forms on the lower surface. The flow phenomena for \( a \) equal to \( 1.09 \) deg and \(-1.25 \) deg are due to the fact that the shock can’t move with the airfoil motion in the same speed. In general, the trend of present results agrees well with the experimental data (Landon, 1982), and the profiles of pressure distribution are closer to the solutions of Batina (1991) than those of Kandil and Chuang (1989). Also, the presently predicted shock position \( a = -2.0 \) deg matches with the adaptive solution (Rausch et al., 1992) than that of the nonadaptive result given by Batina (1991). From the above discussion, it is concluded that the present solution algorithm is accurate for studying the unsteady flow problems with moving boundaries.

**Transonic Flow Around Oscillating Cascade of Four Biconvex Blades.**

For the transonic flows around oscillating cascade of biconvex blades, the inlet Mach number and exit pressure ratio (static exit pressure divided by total pressure) are equal to 0.8 and 0.7322, respectively. The computational domain comprises four uncambered biconvex blades, in which the values of thickness-to-chord ratio, solidity (chord length divided by blade pitch) and stagger angle are 0.076, 1.3 and 53 deg, respectively. The motion of these four blades, which is executing torsional mode oscillations about midchord, is governed by Eq. (2), and \( m = 1, 2 \) and 3 represent each blade from the lowest to the highest one, respectively. By setting \( a_0 \) to be 7 deg, different values of oscillation amplitude \( \alpha_1 \), reduced frequency \( k \) and interblade phase angle \( \sigma \) are employed. The mesh (Fig. 6a) contains 12462 elements and 6598 nodes, and there are 134 points that lie on each blade surface. From the pressure coefficient distribution of the initial solution (Fig. 6b), the weak leading edge shock appears on the upper surface of each blade. The present numerical results compare well with the experimental data given by Buffum and Fleeter (1990). In the computation of transonic flow with \( \sigma \), \( k \) and \( \alpha_1 \) equal to \(-90 \) deg, 0.462 and 1.2 deg, respectively, six cycles are processed to obtain a periodic solution. By choosing a constant marching time step of \( \Delta r = 0.0236 \) (CFL is about 20), it only takes 360 time steps to complete one cycle of motion. During the first cycle of blade motion, only the lowest blade \( m = 0 \) is set to motion at the beginning, whereas the second \( m = 1 \), the third \( m = 2 \) and the highest \( m = 3 \) blades are set to motion when \( 2Mr \) reaches \( \pi / 3 \), and \( 2\pi / 3 \), respectively. From the time history of lift coefficient in Fig. 7, the periodic solution is quickly achieved. The efficiency of the present numerical approach is confirmed.
When one value of $\alpha_t$ (1.2 deg) and two values of $\sigma$ (90 deg and -90 deg) are chosen, the transonic oscillating cascade flows with $k$ equal to 0.185 and 0.462 are studied. Magnitudes and phase angles of the first harmonic dynamic surface pressure difference coefficient ($\Delta C_p$) are calculated and plotted in Figs. 8 and 9. To evaluate the present numerical solutions, the experimental data and results obtained by linearized theory (Buffum and Fleeter, 1990) are introduced. By choosing the experimental data as the reference values, the distributions of magnitude (Fig. 8a) indicate that the present Euler solver provides the better results than does the linearized theory. For the phase angle distributions with $\sigma$ equal to -90 deg (Fig. 8b), the same conclusion is drawn. If the value of $\sigma$ is replaced by 90 deg, the difference between experimental data and present solution is significant (Fig. 8b). The similar phenomenon was observed by Huff (1992), and he mentioned that it was difficult to access how much of the quantitative differences were due to the numerical error and how much was due to experimental error. For the magnitude distributions with $k=0.462$ (Fig. 9a), the agreement with experimental data is worse than that in Fig. 8a for both interblade phase angles. In addition, the values of magnitude of present calculation are closer to those from linearized theory rather than those from experiment when $\sigma$ is equal to 90 deg (Fig. 9b). Comparing the results of phase angle, the present calculation ($\sigma=-90$ deg, Fig. 9b) indicates better agreement with experimental data than those of linearized theory. Considering the case with $\sigma$ replaced by 90 deg, the values of phase angle from both linearized theory and present calculation deviate a distance from those from experiment. As determined from the formula provided by Buffum and Fleeter (1991), the calculation ($k=0.462, \sigma=90$ deg and $\alpha_t=1.2$ deg) lies within the super-resonant region. In super-resonant flow, the pressure waves will propagate upstream and downstream to infinity without decay, which may account for some of the discrepancies. Furthermore, in this super-resonant case, the comparison between present calculation and linearized theory is satisfactory. Except the super-resonant case ($\sigma=90$ deg and $k=0.462$) and one subresonant phase angle distribution ($\sigma=90$ deg and $k=0.185$), the present Euler solutions show better agreement with the experimental data than those from linearized theory. In the present calculations, no viscous effects are included. It is well known that the viscous effects may cause flow separation and change the unsteady behavior, and some of the above discrepancies between present calculations and experimental data may be attributed to viscous effects. Even Though tremendous computational efforts will be involved, further research on Navier-Stokes solver is inevitable in the future to accurately resolve the unsteady shock/boundary layer interaction and flow separation.

To further understand the effects of oscillation amplitude and interblade phase angle on unsteady flow phenomena, instantaneous pressure contours for the transonic oscillating cascade flows with $k=0.462$ are plotted in Figs. 10-13. It is observed that the contours repeat and proceed one pitch distance in the upward or downward direction for $\sigma$ equal to -90 deg (Figs. 10 and 11) and 90 deg (Fig. 13), respectively. From the contours given in Fig. 10a and Fig. 11a, the compression wave on the upper surface of the lowest blade in Fig. 10a becomes a shock which is located at the midchord of the upper surface of the lowest blade in Fig. 11a. Also, the leading edge weak shock still appears on the upper surface of lowest blade. There is an additional shock on the lower surface of the second blade in Fig. 11a. The weak shock on the upper surface of the third blade in Fig. 10a becomes stronger, and it shifts slightly in the downstream direction. On the upper surface of the highest blade in Fig. 10a, the wave compresses more closely and the original shock is getting stronger and moves slightly in the downstream direction. Since different unsteady behaviors are observed in both figures, the nonlinearity of the oscillation amplitude with respect to the unsteady flow phenomena is confirmed. To further understand the unsteady behaviors during each quarter time period, the instantaneous pressure contours at four instant times ($2\pikr/10\pi$) - $\frac{1}{10}$, $\frac{3}{10}$, $\frac{7}{10}$, and $\frac{9}{10}$ are plotted in Fig. 12. Due to the periodic characteristics, the pressure contour at Fig. 11d is the same as that at $2\pikr$ equal to $10\pi$. From the sequence of contours given in Fig. 11d, Figs. 12a - d and Fig. 11a, the shock on the lower surface of the lowest blade in Fig. 11d keeps moving close to the leading edge and eventually passes through the leading edge to form a weak shock on the upper surface of the lowest blade in Fig. 11a. Initially, there's no shock between the lowest and the second blades in Fig. 11d. A shock on the upper surface of the second blade in Fig. 11d moves close to the leading edge, and it finally goes around leading edge to interact with the pressure wave to form shocks on the lower surface of the second blade and on the midchord of the upper surface of the lowest blade (Fig. 11a). As shown in Fig. 11d, there is one shock and one compression wave on the upper surface of the third blade. The latter compression wave moves close to the former shock, and eventually combines together to form a stronger shock on the upper surface of the third blade in Fig. 12c. Then, this strong shock keeps moving upstream to be close to the leading edge (Fig. 11a). On the upper surface of the highest blade in Fig. 11d, there exist a weak leading edge shock, a shock on the midchord and the compression wave. During time evolution, this weak leading edge shock goes downstream and combines with the midchord shock which is moving upstream. Then a strong shock is formed on the upper surface of the highest blade in Fig. 12d. In the meantime, the rear compression wave merges to be stronger (Fig. 12b). Considering the case where $\sigma$ and $\alpha_t$ are equal to 90 deg and 4.8 deg, respectively, the instantaneous pressure contours are shown in Fig. 13. The instantaneous locations and motion for the lowest blade are the same as those in the above case with $\alpha_t$ and $\sigma$ equal to 4.8 deg and -90 deg, respectively. In Fig. 13a, there exists a shock on the front portion of upper surface of the lowest blade. Also, a compression wave stands between the front part of lower surface of the highest blade and the rear part of upper surface of the third blade. On the lower surface of the third blade and on the upper surface of the highest blade, two compression waves are observed, respectively. Furthermore, a strong shock locates on the front part of the upper surface of the second blade. Comparing the results shown in Figs. 11a and 13a, it is obvious that the unsteady flow behaviors are strongly related to the interblade phase angle.

To investigate the unsteady aerodynamic characteristics, the lift coefficients on the lowest blade during the sixth cycle are calculated. For the present oscillating cascade of four biconvex blades, the results given in Fig. 14 demonstrate the effects of $k$, $\sigma$ and $\alpha_t$. At any instant positions (except the highest and lowest ones) of the lowest blade, the value of $C_l$ for blade moving upward is smaller than that for blade moving downward. Also, the smallest $C_l$ occurs a little late after blade passes through
it's lowest position for all cases in Fig. 14, and the largest $C_l$ happens a little late after blade passes its highest position (Fig. 14b, c). From the aforementioned magnitudes and phase angles of dynamic pressure difference coefficient, instantaneous pressure contours and lift coefficient distributions, it is found that oscillation amplitude, interblade phase angle and reduced frequency all have significant effects on the oscillating cascade flows. The nonlinear phenomena, such as the shock formation, migration, strengthening, attenuation and interaction with pressure wave, are observed in the present calculations. No matter how complicated the flowfields are, the periodic physical phenomena still preserve. For every quarter time period, the pressure contours repeat and proceed one pitch distance in the upward or downward direction for $\alpha$ equal to -90 deg or 90 deg, respectively.

CONCLUSIONS

A numerical solution procedure, which includes modification of a locally implicit total-variation-diminishing scheme on dynamic triangular mesh, improvement of a dynamic triangular mesh algorithm, triangular mesh generation, spatially periodic boundary treatment and non-reflective inlet/outlet boundary conditions, is introduced to study the transonic oscillating cascade flows. In a Cartesian coordinate system, the unsteady Euler equations are solved. For the transonic flow around isolated NACA 0012 airfoil pitching harmonically about the quarter chord, the instantaneous pressure coefficient distributions during a cycle of motion are presented. By comparing with the related numerical and experimental results, the accuracy of the present approach is validated. In the computations of transonic flow around oscillating cascade of NACA 0006 blades with interblade phase angle equal to 180 deg, two kinds of computational domains, which include one and two blades, are used. To understand the numerical characteristics of the present spatial periodic boundary treatment, the direct parameter storage method for phase-shifted periodic boundary conditions is employed. From the time history of lift coefficient, the spatial periodic boundary treatment is more numerically stable than direct parameter storage method. In the comparison of instantaneous pressure coefficient distributions with those of two other numerical methods, the present solution procedure is satisfactory. For the transonic flows around oscillating cascade of four bi-convex blades, different values of oscillation amplitude, reduced frequency and interblade phase angle are chosen to study the unsteady phenomena. Comparing the distributions of magnitude and phase angle of the dynamic pressure difference coefficient, the present Euler solutions show better agreement with experimental data than those from linearized theory in most of the cases. The nonlinear phenomena including shock formation, migration, strengthening, attenuation and interaction with pressure wave are observed in the present calculations. For every quarter time period, the pressure contours repeat and proceed one pitch distance in the upward or downward direction for interblade phase angle equal to -90 deg or 90 deg, respectively. From the lift coefficient distributions and the above discussion, it is obvious that the oscillation amplitude, interblade phase angle and reduced frequency all significantly affect the transonic oscillating cascade flows.

REFERENCES


Fig. 1 Partial view of mesh (4888 elements and 2516 nodes) for the oscillating NACA 0012 airfoil.
Fig. 4 Time history of lift coefficient for oscillating cascade of NACA 0006 blades ($M = 0.77, k = 0.2, \alpha_0 = 0$ deg, $\alpha_1 = 2.0$ deg and $\sigma = 180$ deg).

Fig. 5 Initial and instantaneous pressure coefficient distributions for oscillating cascade of NACA 0006 blades ($M = 0.77, k = 0.2, \alpha_0 = 0$ deg, $\alpha_1 = 2.0$ deg and $\sigma = 180$ deg).
Fig. 6 (a) Partial view of mesh (12462 elements and 6598 nodes) and (b) pressure coefficient distribution of the initial solution for oscillating cascade of four biconvex blades.

Fig. 7 Time history of lift coefficient for oscillating cascade of four biconvex blades ($M=0.8$, $k=0.462$, $\alpha_0 =7$ deg, $\alpha_f =1.2$ deg and $\sigma = -90$ deg).

Fig. 8 (a) Magnitude and (b) phase angle of dynamic pressure difference coefficient ($\Delta C_p$) for oscillating cascade of four biconvex blades ($M=0.8$, $k=0.462$, $\alpha_0 =7$ deg and $\alpha_f =1.2$ deg).

Fig. 9 (a) Magnitude and (b) phase angle of dynamic pressure difference coefficient ($\Delta C_p$) for oscillating cascade of four biconvex blades ($M=0.8$, $k=0.462$, $\alpha_0 =7$ deg and $\alpha_f =1.2$ deg).

Fig. 10 Instantaneous pressure contours for oscillating cascade of four biconvex blades ($M=0.8$, $k=0.462$, $\alpha_0 =7$ deg, $\alpha_f =1.2$ deg and $\sigma = -90$ deg): $(2Mkr - 10\pi)$ equal to (a) $\frac{\pi}{2}$, (b) $\pi$, (c) $\frac{3\pi}{2}$ and (d) $2\pi$. 

Fig. 11 Instantaneous pressure contours for oscillating cascade of four biconvex blades ($M = 0.8, k = 0.462, \alpha_0 = 7$ deg, \(\alpha_I = 4.8\) deg and \(\sigma = -90\) deg): \(2(Mkr - 10\pi)\) equal to (a) \(\frac{\pi}{2}\), (b) \(\pi\), (c) \(\frac{3\pi}{2}\) and (d) \(2\pi\).

Fig. 12 Instantaneous pressure contours for oscillating cascade of four biconvex blades ($M = 0.8, k = 0.462, \alpha_0 = 7$ deg, \(\alpha_I = 4.8\) deg and \(\sigma = -90\) deg): \(2(Mkr - 10\pi)\) equal to (a) \(\frac{\pi}{2}\), (b) \(\pi\), (c) \(\frac{3\pi}{2}\) and (d) \(2\pi\).

Fig. 13 Instantaneous pressure contours for oscillating cascade of four biconvex blades ($M = 0.8, k = 0.462, \alpha_0 = 7$ deg, \(\alpha_I = 4.8\) deg and \(\sigma = 90\) deg): \(2(Mkr - 10\pi)\) equal to (a) \(\frac{\pi}{2}\), (b) \(\pi\), (c) \(\frac{3\pi}{2}\) and (d) \(2\pi\).

Fig. 14 Lift coefficient vs instantaneous angle of attack for the lowest blade of oscillating cascade of four biconvex blades: (a) \(k = 0.462, \sigma = -90\) deg, (b) \(k = 0.462, \sigma = 90\) deg, (c) \(k = 0.185, \sigma = -90\) deg and (d) \(k = 0.185, \sigma = 90\) deg.