COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS FOR UNSTEADY TRANSONIC CASCADE FLOW AT DESIGN AND OFF-DESIGN CONDITIONS

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ABSTRACT

This paper presents experimental and theoretical results for turbine cascades performing harmonic oscillations in transonic flow at design and off-design conditions. The experimental investigations were performed in an annular test facility where unsteady blade pressures were measured in two different test cascades, one operating at the nominal inlet flow angle, the other at an incidence angle exceeding the normal value by more than 20 degrees. The corresponding theoretical results were computed with a 2D Euler code which makes use of flux vector splitting in combination with a time-dependent grid generation.

The present data were all obtained for tuned bending modes where the blades performed heaving oscillations with the same frequency and amplitude, but with a constant interblade phase angle. For the cascade operating at design conditions, the steady flow was purely subsonic. The other test cascade was run in transonic flow, and a normal shock appeared on the rear part of the blade's suction surface.

It was found that measured unsteady pressure and damping coefficients are well reproduced by the computed results for the first test cascade. In the case of steady off-design flow (the second test cascade), significant differences between experimental and theoretical results are observed.

INTRODUCTION

For aeroelastic investigations on highly loaded oscillating transonic turbomachine cascades, the first essential is exact knowledge of the unsteady airloads. For this reason, much theoretical and experimental work has been done in the past to compute and measure unsteady pressure, lift and moment coefficients of turbomachinery bladings vibrating in transonic flow.

A literature survey shows that theoretical research has progressed steadily over the past three decades. First of all, incompressible flow solvers based on conformal mapping or integral equations were used to study the unsteady flow through a cascade of vibrating blades. Examples of this method are the publications by Whitehead [1], Atassi [2] and Carstens [3]. In order to take into account compressibility effects, the next step was the formulation of the unsteady flow problem as an acoustic radiation problem. Typical representations of this manner of treating the problem are the papers of Smith [4], who analyzed flat plate cascades with zero steady loading, and Verdon and Caspar [5], who developed an analysis for vibrating cascades with mean flow deflection.

In the last decade, increasing efforts were made to extend the flow calculations to the transonic regime. Besides further development of linearized methods which make use of special simplified forms of either the Full Potential equation (Whitehead and Grant [6], Verdon and Caspar [7]) or the Euler equations (Hall and Crawley [8]), time marching methods for solving the full nonlinear equations have also been developed. Kau [9] investigated the unsteady flow through single vibrating blade rows of turbines and compressors, while Gerolymos [10] calculated the unsteady airloads in oscillating compressor cascades for a variety of blade vibration modes. Both Kau and Gerolymos computed the unsteady flow by a rather time-consuming explicit Euler solver. Recently, Carstens contributed a publication [11] based on an implicit Euler code, where arbitrary blade vibrations are admitted.

In the meantime, the state of the art for computing unsteady transonic cascade flow has reached the point where prediction models for viscous flow come into use. An example to mention is the work of Rai [12], who calculated the transonic flow through a complete rotor-stator configuration with a Navier-Stokes code.

The progress in theoretical work has been accompanied by corresponding development of measuring techniques for unsteady cascade flow. For detailed information, see the publications of Bölcs and Fransson ([13], [14]), where a set of experimental standard test cases is presented in connection with the "Workshop on Aeroelasticity in Turbomachines".

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The objective of the present paper is to give an insight into the validity and reliability of computed results in comparison with measured ones for oscillating turbine cascades operating at different incidence flow angles. The experimental investigations were performed in the annular test facility at EPF-Lausanne, where unsteady blade pressures were measured in oscillating turbine cascades at design and off-design conditions. The corresponding theoretical results were computed with the 2D Euler code mentioned above, which was developed by Carstens [11]. Experimental and computational results of unsteady pressure and aerodynamic damping coefficients are presented for two different test cascades operating at moderate (design case) and high (off-design case) steady-state incidence flow angles. These results are investigated with respect to the aerodynamic stability of the cascades.

EULER ALGORITHM

Flux Vector Splitting in Cartesian Coordinates

The present Euler code, written in conservative form, makes use of flux vector splitting. This technique, in which the flux vectors are decomposed into positive and negative contributions such that the corresponding Jacobian matrices have either positive or negative eigenvalues, more or less simulates the method of characteristics. Performing the spatial differences of the split flux vectors with backward and forward difference operators, respectively, the arising algorithm belongs to the class of upwind methods and has the advantage of being naturally dissipative. The special type of decomposition used in the present algorithm is van Leer splitting [15]. The excellent shock capturing properties of van Leer splitting have already been elaborated in the paper of Anderson, Thomas and van Leer [16].

The two-dimensional Euler equations for an ideal gas expressed in Cartesian coordinates and conservative form are

\[
\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0 ,
\]

where

\[
Q = \rho \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho e \end{bmatrix}, \quad F = \rho \begin{bmatrix} u \\ u^2 + \frac{a^2}{\kappa} \\ v u \\ u H \end{bmatrix}, \quad G = \rho \begin{bmatrix} v \\ v^2 + \frac{a^2}{\kappa} \\ u v \\ v H \end{bmatrix}
\]

with the specific total energy

\[
E = \frac{a^2}{\kappa (\kappa - 1)} + \frac{1}{2} (u^2 + v^2) .
\]

where

\[
\kappa = \frac{c_p}{c_v}, \quad \text{ratio of specific heats}
\]

and the specific total enthalpy

\[
H = \frac{a^2}{\kappa - 1} + \frac{1}{2} (u^2 + v^2) .
\]

The dependent primitive variables \( \rho, u, v \) and \( a \) are the density, the Cartesian velocities, and the local speed of sound, respectively.

In a flux vector splitting algorithm, the flux vectors are decomposed into positive and negative contributions \( F = F^+ + F^- \) and \( G = G^+ + G^- \) such that the Jacobian matrices \( \partial F^+/\partial Q, \partial G^+/\partial Q \) have only positive and \( \partial F^-/\partial Q, \partial G^-/\partial Q \) have only negative eigenvalues. According to the artificial characteristic directions of signal transport, introduced by the split flux vectors, the calculation of the spatial derivatives of \( F^+, G^+ \) and \( F^-, G^- \) has to be performed with backward and forward difference operators, respectively. This upwind method yields a robust algorithm with excellent shock capturing properties.

In the splitting suggested by van Leer [15], \( F^\pm \) and \( G^\pm \) are given in terms of the local one-dimensional Mach numbers \( M_x = u/a \) and \( M_y = v/a \), respectively. The decomposition of \( F \) runs as follows:

\[
F^\pm = \begin{bmatrix}
\pm \frac{\rho a}{4} (1 \pm M_x)^2 \\
\frac{a}{\kappa} [(\kappa - 1)M_x \pm 2] f_1^\pm \\
v f_1^\pm \\
2 (\kappa^2 - 1) f_1^\pm + \frac{a^2}{2} f_1^\pm
\end{bmatrix}, \quad (5)
\]

Supersonic flow \( |M_x| \geq 1 \)

\[
F^+ = F , \quad F^- = 0 \quad \text{for} \quad M_x \geq + 1 \\
F^+ = 0 , \quad F^- = F \quad \text{for} \quad M_x \leq - 1 
\]

and the decomposition of \( G \) is obtained by:

Subsonic flow \( |M_y| \leq 1 \)

\[
G^\pm = \begin{bmatrix}
\pm \frac{\rho a}{4} (1 \pm M_y)^2 \\
\frac{a}{\kappa} [(\kappa - 1)M_y \pm 2] g_1^\pm \\
u g_1^\pm \\
\frac{a^2}{2} g_1^\pm + \frac{a^2}{2} g_1^\pm
\end{bmatrix}, \quad (6)
\]

Supersonic flow \( |M_y| \geq 1 \)

\[
G^+ = G , \quad G^- = 0 \quad \text{for} \quad M_y \geq + 1 \\
G^+ = 0 , \quad G^- = G \quad \text{for} \quad M_y \leq - 1 .
\]

Flux Vector Splitting in Boundary-Fitted Coordinates

Introducing a boundary-fitted moving coordinate system

\[
x = x(\xi, \eta, \tau) \quad \xi = \xi(x, y, t) \\
y = y(\xi, \eta, \tau) \leftrightarrow \eta = \eta(x, y, t) \\
\tau = t
\]

The present Euler code, written in conservative form, makes use of flux vector splitting. This technique, in which the flux vectors are decomposed into positive and negative contributions \( F = F^+ + F^- \) and \( G = G^+ + G^- \) such that the Jacobian matrices \( \partial F^+/\partial Q, \partial G^+/\partial Q \) have only positive and \( \partial F^-/\partial Q, \partial G^-/\partial Q \) have only negative eigenvalues. According to the artificial characteristic directions of signal transport, introduced by the split flux vectors, the calculation of the spatial derivatives of \( F^+, G^+ \) and \( F^-, G^- \) has to be performed with backward and forward difference operators, respectively. This upwind method yields a robust algorithm with excellent shock capturing properties.

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v f_1^\pm \\
2 (\kappa^2 - 1) f_1^\pm + \frac{a^2}{2} f_1^\pm
\end{bmatrix}, \quad (5)
\]

Supersonic flow \( |M_x| \geq 1 \)

\[
F^+ = F , \quad F^- = 0 \quad \text{for} \quad M_x \geq + 1 \\
F^+ = 0 , \quad F^- = F \quad \text{for} \quad M_x \leq - 1 .
\]

and the decomposition of \( G \) is obtained by:

Subsonic flow \( |M_y| \leq 1 \)

\[
G^\pm = \begin{bmatrix}
\pm \frac{\rho a}{4} (1 \pm M_y)^2 \\
\frac{a}{\kappa} [(\kappa - 1)M_y \pm 2] g_1^\pm \\
u g_1^\pm \\
\frac{a^2}{2} g_1^\pm + \frac{a^2}{2} g_1^\pm
\end{bmatrix}, \quad (6)
\]

Supersonic flow \( |M_y| \geq 1 \)

\[
G^+ = G , \quad G^- = 0 \quad \text{for} \quad M_y \geq + 1 \\
G^+ = 0 , \quad G^- = G \quad \text{for} \quad M_y \leq - 1 .
\]
the Euler equations are given in the new coordinate system in conservative form by

\[ \frac{\partial Q}{\partial \tau} + \frac{\partial F}{\partial \xi} + \frac{\partial G}{\partial \eta} = 0 \]  

(8)

with the transformed vectors

\[ \hat{\mathbf{F}}(\hat{Q}) = \sqrt{x_\xi^2 + y_\eta^2} \mathbf{T}_F \mathbf{F}(\mathbf{Q}) \]

\[ \hat{\mathbf{G}}(\hat{Q}) = \sqrt{x_\xi^2 + y_\eta^2} \mathbf{T}_G \mathbf{G}(\mathbf{Q}) \]  

(10)

from which the transformed split flux vectors follow as

\[ \hat{\mathbf{F}}^\pm = \sqrt{x_\eta^2 + y_\eta^2} \mathbf{T}_F \mathbf{F}^\pm \]

\[ \hat{\mathbf{G}}^\pm = \sqrt{x_\eta^2 + y_\eta^2} \mathbf{T}_G \mathbf{G}^\pm \]  

(11)

For the details of vectors \( \mathbf{Q}, \mathbf{F}, \mathbf{G} \) and matrices \( \mathbf{T}_F \) and \( \mathbf{T}_G \), see [11].

**Solution Algorithm**

After flux splitting, the Euler equations in boundary-fitted coordinates are given by

\[ \frac{\partial \hat{Q}}{\partial \tau} + \frac{\partial \hat{F}}{\partial \xi} + \frac{\partial \hat{G}}{\partial \eta} = 0 \]  

(12)

The method of solution is the approximately factored Beam-Warming implicit algorithm [17] given in delta form with first-order time accuracy by

\[ \left[ 1 + \Delta \tau \left( \delta_\xi \frac{\partial \hat{F}}{\partial \hat{Q}} + \delta_\xi^+ \frac{\partial \hat{F}}{\partial \hat{Q}} \right) \right] \Delta \hat{Q} = \]

\[ = -\Delta \tau \left[ \delta_\xi \hat{F}^+ + \delta_\xi^+ \hat{F}^- + \delta_\eta \hat{G}^+ + \delta_\eta^+ \hat{G}^- \right] \]  

(13)

The numerical scheme is written in a cell-centered finite volume formulation, where the spatial derivatives are approximated by so-called MUSCL-type differencing (MUSCL = Monotic Upstream centered Scheme for Conservation Laws) [18], i.e. they are generated indirectly by extrapolating the Cartesian solution vector with backward or forward formulas. The split-flux differencing in the \( \xi \)-direction, for example, is written as

\[ \delta_\xi^+ \hat{F}^+ = \frac{1}{\Delta \xi} \left[ \hat{F}^+ \left( Q_{j+\frac{1}{2},k} - M_{\pm\frac{1}{2},k} \right) \right] \]  

(14)

In (14) the terms \( M \) represent the metric coefficients \( x_\xi \) and \( y_\eta \) which have to be calculated at the cell interfaces. A corresponding differencing is used in the \( \eta \)-direction. The extrapolated values of the solution vector \( \mathbf{Q} \) are determined by the formulas

\[ Q_{j+\frac{1}{2},k} = Q_{j,k} + f_{jk} \left( Q_{j,k} - Q_{j-1,k} \right) \]

\[ Q_{j+1,k} = Q_{j+1,k} + f_{jk} \left( Q_{j+1,k} - Q_{j,k} \right) \]

(15)

with the control factors

\[ f_{jk} = 0 \quad \text{for first order accuracy} \]

\[ f_{jk} = \frac{1}{2} \quad \text{for second order accuracy} \]  

(16)

Further investigation of the flux splitting technique described above shows that differencing (14) together with (15) is the only way to avoid undesired discretization errors in a boundary-fitted coordinate system.

**Boundary Conditions**

The steady or unsteady flow calculation for a cascade demands the implementation of different boundary conditions which determine the solution. These boundary conditions are: the kinematic flow condition (vanishing normal velocity at the blade's surfaces), inlet and outlet boundary conditions (either prescribed flow values for steady flow or nonreflecting boundary conditions for unsteady flow at the in- and outlet boundary of the computational grid), and periodic boundary conditions on certain parts of the outer boundaries of the computational domain (prescribed periodicity of the flow due to the geometric repeat condition of the cascade).

On the blade's surface, the normal relative velocity is set to zero

\[ (\mathbf{v} - \mathbf{v}_k) \cdot \mathbf{n} = 0 \quad \text{with} \quad \mathbf{v}_k = [x_{\xi}, y_{\eta}]_{\text{wall}} \]  

(17)

where \( \mathbf{v}, \mathbf{v}_k \) and \( \mathbf{n} \) are the Cartesian velocity vector, the prescribed kinematic velocity vector of the blade's surface, and the normal vector on the body, respectively. Inserting equation (17) into the normal momentum equation (see, [19], for example), after some elementary transformations we obtain
\[ \begin{split} 
( x_i^2 + y_j^2 ) \frac{\partial p}{\partial \eta} &= ( x_i x_m + y_j y_n ) \frac{\partial p}{\partial \xi} \\
\frac{\partial}{\partial \eta} \left\{ \frac{\nu}{( x_i^2 + y_j^2 )} \left( y_j x_r + x_j y_r \right) + \frac{2 \nu}{\sqrt{x_i^2 + y_j^2}} \left( y_j x_r + x_j y_r \right) \right\} 
\end{split} \] (18)

where \( \nu \) denotes the tangential relative velocity on the blade's surface.

The density \( \rho \) and the tangential relative velocity \( \nu \) at the body's surface, both needed to compute the normal pressure derivative, are calculated by extrapolation from the interior field. If the grid lines are orthogonal at the body, the first term on the right-hand side of (18) vanishes, and hence the pressure at the body can be determined directly by (18) without the solution of a linear system of equations.

The implementation of inlet and outlet boundary conditions is accomplished by the method of Chakravarthy [20], who proposed a quasi-one-dimensional approach. The implementation of boundary conditions (prescription of certain flow-quantities such as total enthalpy, pressure, etc.) are then given by prescribing fixed values for the vector \( \partial B / \partial \dot{Q} \). Nonreflecting boundary conditions (suppression of incoming waves) are obtained by the condition \( \partial B / \partial \dot{Q} = \sigma \), which is equivalent to \( \partial \eta / \partial \dot{\sigma} = 0 \).

It is easy to show (see [11]) that (21) can be rewritten as

\[ \frac{\partial \dot{Q}}{\partial \tau} = D \left[ \frac{\partial \hat{F}}{\partial \xi} + \frac{\partial \hat{G}}{\partial \eta} \right] \] (22)

(22) is the system of modified Euler equations valid on the in- and outlet boundaries, differing from (8) in the multiplication of the flux terms with an influence matrix \( D \) effecting the implementation of boundary conditions. What remains is the application of periodic boundary conditions. Since their implementation strongly depends on the computational grid used and the prescribed oscillation mode of the cascade, the next chapter will discuss this.

GRID GENERATION

The grid generation needed to compute the solution of the Euler algorithm in a boundary-fitted coordinate system is obtained by a method presented by Carstens [21]. The procedure used is an elliptic grid generation code based on the solution of two coupled Poisson equations. A basic feature of the present method is the possibility to control the line spacing and intersection angle of the grid lines at the physical boundaries; in this case, for example, at the blade's surface and at the channel boundaries.

The type of grid used for the cascade flow calculation is a line-periodic H-grid, i.e. each grid point on the lower channel boundary has its counterpart on the upper channel boundary in the direction of the cascade axis (Figure 1). Although this type of grid is characterized by rapidly changing metrics near the stagnation point and rather skewed grid meshes near the trailing edge of the blade, its advantage is that an implicit code is easily introduced on the grid and the periodic boundary conditions can be imposed in an implicit manner.

For unsteady flow, due to oscillatory motion of the blades, the grid point displacement must be carefully organized. As the blades are in relative motion to each other, the total grid has to be deformed steadily to enable it to conform to the new position of the vibrating blades after each time step. An important feature of any unsteady grid generation code is control of the grid point speed in the interior field. In order to accomplish this requirement, the time-dependent grid in every blade channel is computed by harmonic interpolation (with respect to time) of a set of steady-state grids with different blade amplitudes. This technique guarantees a smooth grid at every time step and the desired control of grid point speed.

The number of blade channels in which the flow has to be computed depends on the oscillation mode of the cascade. The type of blade modes considered here are tuned modes where the blades are oscillating with the same fre-
frequency, same amplitude and a constant interblade phase angle. Tuned modes always result in a pitchwise spatially periodic flow, where the spatial periodic length is determined by the interblade phase angle. For example, if an interblade phase angle of ± 180° is prescribed, the flow values are repeated every time when proceeding two blade pitches along the cascade axis. Hence, two blade channels are sufficient for computation. This case is depicted in Figure 1, where the blades perform bending vibrations perpendicular to their chord and have just reached their extreme position. Analogously, the flow has to be calculated in four blade channels for a phase angle of ± 90° and so on. It should be mentioned that it is also possible to restrict the flow computation to only one blade channel for this type of motion (see Kau and Gerolymos). The spatial periodicity condition is then not imposed instantaneously as in the present method, but with a time lag corresponding to the interblade phase angle. Although this technique has the special merits of a reduced computational domain, its disadvantages are that the flow values on the periodic channel boundaries have to be stored in space and time (storage problems) and that many cycles of motion must be computed to drive the solution to periodic convergence in time (slow convergence).

TEST FACILITY AND CASCADE GEOMETRY

The tests on the two cascades presented below were performed in the non-rotating annular cascade tunnel at the Swiss Federal Institute of Technology, Lausanne (Bölls [22], Bölls et al. [23], Figure 2). The flow angle in the test section can be regulated from 0° to ± 75° (measured from axial direction) and the inlet Mach number can be varied from 0.3 to 1.6. The flow conditions are measured with aerodynamic probes and pressure taps. Schlieren optics and laser holography are used for flow visualization.

For time-dependent measurements, the blades in the test cascade are mounted on elastic springs and are driven into a vibration mode by means of a vibration control system [Kirschner et al. [24], Schlaffi [25]]. Each blade is suspended on a spring-mass system. The spring is designed to reproduce the first three eigenfrequencies as well as the vibration direction of the first bending mode, as determined from the blades in the full-scale turbine. The blades are forced into vibration by means of individual electromagnetic exciters, each with its own feed-back loop.

The two test cascades that have been investigated experimentally and theoretically are typical examples of their species. Both are turbine cascades generated by a cylindrical cut at nearly fifty percent of the blade's height.

The first cascade (Figure 3) originates from the rotor of a test steam turbine and is identical with the fourth standard configuration of the IUTAM 'Workshop on Aeroelasticity in Turbomachines', see [14]. This cascade represents a typical section of modern free-standing turbine blades and is characterized by profiles with relatively high thickness and camber. At the nominal operating point — which is regarded here — the maximum Mach number on the suction side remains subsonic, while the outflow Mach number reaches 0.90. Unsteady pressures were measured with 11 pressure transducers, 6 on the suction surface and 5 on the pressure surface of the blades.

The second test cascade (Figure 4) is made up of a modern highly loaded gas turbine profile designed for use in the last rotor blade row of a machine. In contrast to the steam turbine cascade, the blades have a smaller thickness and a steady curvature on the suction surface. The cascade has been investigated at an off-design incidence angle of 33° with an outflow Mach number of 0.99. At this flow condition a separation bubble is observed on the suction side near the leading edge. After the flow has been accelerated to supersonic values on the suction surface, a shock appears which seems to be rather sensitive to small changes in the flow conditions. The equipment for unsteady pressure measurement consists of 18 pressure transducers on the suction surface and 9 on the pressure surface. For technical reasons the installation in one single block is not possible; therefore, the transducers are distributed over three neighbouring blades. As some of them are used for control measurements at the same blade position, the results of only 14 transducers on the suction surface and 8 on the pressure surface are taken for comparison with theory.

PRESENTATION AND DISCUSSION OF RESULTS

Results for unsteady cascade flow were calculated for two different turbine cascades, namely a steam turbine cascade in high subsonic flow at design conditions and a gas turbine cascade in transonic flow at off-design conditions. The geometrical data of these two configurations have been listed in the previous section (Figures 3 and 4). In the following, the test cascades will be named Cascade 1 (steam turbine) and Cascade 2 (gas turbine), respectively.

Unsteady pressure distributions were measured in both cascades for tuned bending modes, i.e. heaving oscillations with the same frequency and same amplitude, but with a constant controlled interblade phase angle. The vibration direction, measured by the angle δ against the profile chord, is depicted in Figures 3 and 4. For both cascades, Cascade 1 performed oscillations with a frequency of 150 Hz and a dimensionless amplitude (referred to chord length L) of y₀/L = 0.003, whereas the corresponding data for Cascade 2 were 212 Hz and y₀/L = 0.004. Defining the reduced frequency by

\[ \omega^* = \frac{2\pi f (L/2)}{V_2} \]

with

- \( f \) = frequency
- \( L \) = chord length of the blade
- \( V_2 \) = cascade outlet velocity,

we obtain \( \omega^* = 0.1153 \) for Cascade 1 and \( \omega^* = 0.1545 \) for Cascade 2, where \( V_2 \) was determined from the measured static outlet pressure. The interblade phase angle \( \Theta \) in both cascade configurations is defined such that it is positive if the phase of the pressure side blade advances to the phase of the regarded blade, i.e. a positive \( \Theta \) causes a traveling wave running to the left side in Figures 3 and 4, whereas a negative \( \Theta \) produces a traveling wave in the opposite direction. All theoretical results for steady and unsteady flow were computed with the H-grid of Figure 1 (grid size: 91 points in \( \xi \)-direction, 27 points in \( \eta \)-direction).

First of all, the computed results for steady flow must be validated, because they are the starting point for the unsteady flow calculations. The steady pressure coefficient (index s) for both cascades is defined with respect to the inlet values of total and static pressure.
with
\[ p_1 = \text{static inlet pressure} \]
\[ p_{t_1} = \text{total inlet pressure} \]

Figure 5 shows the comparison between experimental and theoretical results of Cascade 1 for the design incidence angle \( \beta_1 = 44^\circ \). The agreement between theory and experiment is good, since only small deviations near the stagnation point and on the aft portion of the suction side are noticeable. The wiggles in the theoretical pressure distribution at the trailing edge are due to the fact that an inviscid solution produces a strong nonphysical expansion followed by a recompression at a thick rounded trailing edge.

The experimental and theoretical results for Cascade 2 are depicted in Figure 6. The tests were performed at an off-design inlet flow angle of \( \beta_1 = 33^\circ \), which differs widely from the nominal inlet flow angle of \( \beta_1 = 10.5^\circ \). In contrast to Cascade 1, significant differences are observed between measured and predicted results. The reason is the incorrectly computed pressure on the suction surface, especially at the leading edge and in the shock region. From surface paint flow visualization it can be concluded that a separation bubble exists from the leading edge to approximately 30 percent of chord length on the suction side. After the separation bubble, the surface flow is accelerated to supersonic values. At approximately 75 percent of chord length the flow is recompressed by a normal shock. As expected, an inviscid flow computation leads to strong deviations in the separation region. Instead of an increasing pressure, the Euler code produces a strong overexpansion just after the leading edge. This is followed by a short smooth recompression, whereafter the flow is accelerated with a smaller increase of velocity as observed in the experiment. Consequently, the computed shock is far behind the measured one. Figure 6 shows a calculated shock position at approximately 90 percent of chord length.

The calculation of the unsteady flow quantities for tuned modes has always been performed with the same technique: at rest in the computed steady flow, the blades are started with the prescribed oscillation mode. The calculation is stopped when good periodic convergence of the unsteady pressure, lift, and moment coefficients is achieved, i.e. when the peak values of these quantities for two successive periods of blade motion do not differ from each other more than 0.1 %. Since the reduced frequencies in all test cases used have a relatively low value, three or four cycles of blade motion were enough to drive the solution to the desired convergence. Applying a Fourier analysis to the time-dependent pressure, lift and moment coefficients for the last calculated period, the first harmonics of these quantities were obtained and can be compared with the corresponding measured values. Denoting the dimensionless amplitude of a harmonic blade oscillation with \( y_0/L \), the unsteady pressure coefficient (index us) is then defined as the complex number
\[ c_{p_{us}} = \frac{\tilde{p}}{(y_0/L)(p_{t_1} - p_1)} e^{i\varphi} \]

\[ \tilde{p} = \text{pressure amplitude} \]
\[ \varphi = \text{phase angle between pressure and blade motion} \]

Figures 7 and 8 show the comparison between the computed and measured unsteady pressure distributions for Cascade 1 at the two interblade phase angles \( \Theta = 180^\circ \) and \( \Theta = -90^\circ \), respectively. The results of the first harmonics are presented in a module-phase diagram. Although a certain overprediction of the suction peak near the leading edge is observed in the theoretical results, good agreement with the experimental data is obtained for the phase lead or lag of the unsteady pressure with respect to the blade motion. The integrated unsteady pressure, the lift coefficient, can be decomposed into two parts: the in-phase and the out-of-phase part, where the latter (the so-called "aerodynamic damping") determines the aerodynamic stability of the cascade. Figure 9 shows the aerodynamic damping as function of the interblade phase angle. For this test case, experimental data are available in steps of \( \Delta \Theta = 90^\circ \), whereas the numerical results were computed in steps of \( \Delta \Theta = 45^\circ \).

Figure 9 demonstrates the strong influence of the interblade phase angle on the aerodynamic stability of tuned bending modes. Here we can observe a high amount of aerodynamic damping for phase angles in the domain of \( +90^\circ \) and a significant excitation for \( \Theta \)-values in the area of \( -90^\circ \). Although the absolute values of the computed results are different from the experimental ones, the transition from damping to excitation is in agreement with the theory.

For Cascade 2 unsteady pressure distributions were measured for ten equidistant interblade phase angles, namely for \( 0^\circ, \pm 36^\circ, \text{etc.} \) Figures 10 and 11 show two representative experimental and theoretical pressure distributions at \( \Theta = 180^\circ \) and \( \Theta = -72^\circ \). In both cases the expected differences between measured and predicted values on the suction surface are obvious. Regarding the front part of the suction side, the computed unsteady pressure shows strong discrepancies in comparison with the experimental data, which may be explained by the different unsteady responses to an overexpansion (Euler solution) and a steady separation bubble (experiment). For \( \Theta = 180^\circ \) the pressure peak caused by the shock is predicted well with respect to magnitude, but not with respect to location (see steady pressure). The corresponding computed phase relation between pressure and motion is in satisfactory agreement with the measured data except for the shock region. In the second case (\( \Theta = -72^\circ \)), the unsteady response to the shock turns out more moderately; the pressure peak at one transducer seems to not be in agreement with the other measured data. Again, the pressure phase on both sides of the blade is predicted with some differences, but shows the correct trend.

Finally, the aerodynamic damping coefficient as function of the interblade phase angle is presented in Figure 12. Surprisingly, the agreement between computed and measured data is much better than expected. The angle of maximum excitation at \( \Theta = -36^\circ \) is predicted well, whereas the angle of maximum damping is shifted from \( \Theta = 180^\circ \) (experiment) to \( \Theta = 180^\circ \) (theory). The absolute value of aerodynamic damping is generally overpredicted, a fact that was already observed in Cascade 1.
SUMMARY AND CONCLUSIONS
A comparison between experimental and theoretical results for oscillating turbine cascades has been presented here. The experimental investigations were performed in an annular test facility where unsteady blade pressures were measured in two different test cascades, one operating at design, the other at off-design conditions. The corresponding theoretical results were computed with a 2D Euler code, whose special features are the use of flux vector splitting in connection with a time-dependent grid generation. Unsteady pressure and damping coefficients have been presented for tuned bending modes at moderate reduced frequencies.

The comparison between experimental data and the corresponding computed values is encouraging for Cascade 1, where the tests were made at the nominal inlet flow angle. The calculated unsteady pressure is in reasonable agreement with the measured one, for magnitude as well as for phase. For this reason, the domains of stable and unstable interblade phase angles are predicted well by the computed results.

Strong discrepancies are observed when comparing the experimental and computed unsteady pressure for Cascade 2. Here the tests were made at an off-design incidence angle that exceeds the nominal inlet angle by more than 20 degrees. A plausible explanation for the differences between measured and predicted values is the incapability of an Euler code to model the flow separation at the leading edge, which strongly influences the complete suction surface flow on the blades. The calculated aerodynamic damping coefficient is in reasonable agreement with the measured one, but only further investigations with viscous flow solvers could help clarify the discrepancies mentioned above.

REFERENCES
Fig. 1. Moving H-grid for tuned bending oscillations with an interblade phase angle of 180°

Fig. 2. Test facility for annular cascades

Fig. 3. Geometry of first test cascade

Fig. 4. Geometry of second test cascade
**Fig. 5.** Theoretical and experimental steady pressure distribution, Cascade 1

Ma_1=0.28, Ma_2=0.90
- present theory
- exp., pressure side
+ exp., suction side

**Fig. 6.** Theoretical and experimental steady pressure distribution, Cascade 2

Ma_1=0.40, Ma_2=0.99
- present theory
- exp., pressure side
+ exp., suction side

**Fig. 7.** Theoretical and experimental unsteady pressure distribution, Cascade 1, bending motion, Θ = 180°

Ma_1=0.28, Ma_2=0.90
- theory
- exp., suction side
- exp., pressure side

**Fig. 8.** Theoretical and experimental unsteady pressure distribution, Cascade 1, bending motion, Θ = -90°

Ma_1=0.28, Ma_2=0.90
- theory
- exp., suction side
- exp., pressure side
Fig. 9. Aerodynamic damping versus interblade phase angle, Cascade 1, bending motion

Fig. 10. Theoretical and experimental unsteady pressure distribution, Cascade 2, bending motion, $\Theta = 180^\circ$

Fig. 11. Theoretical and experimental unsteady pressure distribution, Cascade 2, bending motion, $\Theta = -72^\circ$

Fig. 12. Aerodynamic damping versus interblade phase angle, Cascade 2, bending motion