Extended Abstract†

Statistics of energy dissipation in the hypervelocity impact shock failure transition

Dennis Gradya*

aApplied Research Associates, 4300 San Mateo Blvd NE, Albuquerque, NM, 87110, USA

1. Introduction

In the hypervelocity impact event, shock waves subject material to failure transitions with the attendant dissipation of the imparted energy. Under shock compression, failure and dissipation entail intense compression, inelastic shear and compaction. Through shock interactions, states of dynamic tension are achieved and further failure dissipation involves fracture and fragmentation. The nature of failure of solids in the shock environment has encouraged considerable experimental effort through the past several decades. Such efforts have yielded results that suggest universality in the shock failure response over significant spans of shock intensity. Examples include the fourth-power relation between pressure and strain rate in solid-material compressive shock waves, and power-law relations capturing spall fracture strength and fragmentation size scale in dynamic tensile failure. Comparable power-laws also describe the shock compaction of distended solids. The present paper explores a statistical perspective of the underlying micro failure dynamics for the purpose of achieving better understanding of the macro failure trends noted above. A statistical correlation function description of the random micro velocity field is introduced. Through the attendant kinetic dissipation, the statistical fluctuation-dissipation principle is applied to the shock failure transition. From this statistical approach, power-law relations for compressive and tensile shock failure emerge that replicate the reported experimental behaviors.

The hypervelocity impact event entails the transfer of kinetic energy of the impacting body into internal energy and kinetic energy of the impacted material. Energies and energy rates are extreme. Subsequent to the impact shock event, the energy not remaining as kinetic energy resides as forms of thermal and athermal energy, a consequence of energy dissipation in the shock wave. The nature of energy dissipation in the shock wave is complex, and resides at the heart of much of shock wave physics.

Energy dissipation is heterogeneous, consummated through the collective dynamics of localized dissipative structures. Dissipative structures are unique to the material of concern and individually self-organize on the local scale in response to the dissipation and dissipation rates dictated by the encompassing shock wave. Dissipative structures can be, for example, thermal adiabatic shear bands in metals, shear and tensile fractures in brittle compounds, or planar compaction zones in porous matter. The formation and dynamics of dissipative structures are integral to material transitions caused by the shock, including onset of crystallographic and thermodynamic phase transformation, initiation of reaction in energetic materials, and the fracture and fragmentation of solid matter.

The dissipation of energy by the elemental dissipative structure is dichotomous. First, dissipation is frictional, occurring interior to the dissipative structure, contributing to the thermal phonon field and local temperature rise, as well as local athermal dissipation. Second, dissipation is kinetic, proceeding through acoustic phonon radiation from the dissipative structure.

† Full paper can found at https://www.sciencedirect.com/journal/international-journal-of-impact-engineering/special-issue/10MTGP5W4VJ

* Corresponding author. Tel.: 505-883-3636; fax: 505-872-0794.
E-mail address: dgrady@ara.com.
Radiation dissipation and frictional dissipation are comparable in magnitude, and governed by the same failure and dissipation properties of the material.

Within the framework of Boltzmann statistics, the statistics-based fluctuation-dissipation theorem has application to the shock-wave failure transition. Through application of the fluctuation-dissipation relation, the kinetic dissipation, namely the acoustic phonon field, is identified with the viscosity responsible for structuring the shock wave. Through application of the fluctuation-dissipation relation, the correlated kinetic dissipation necessary to achieve the shock failure transition is determined. From application of the fluctuation-dissipation relation emerge relations for the dependence of spall stress and fragment size on the intensity of the imparting shock wave, and the fourth-power relation for steady structured shock waves.

2. Dissipation dynamics and the shock failure transition

Through the use of statistical principles that are central to other areas of physics, a statistical framework for failure dynamics is constructed within the structured shock wave. Through this framework, expressions are developed for the viscosity appropriate to the spatial structuring of the shock wave and for critical state relations determining the shock-induced failure transition. Out of the development emerge relations for the experimental fourth-power behavior observed in structure shock measurements in solids and the rate dependence of spall strength and fragment size in the dynamic tensile failure of materials.

2.1. The Variant Velocity Field

At any point on the microscale within the material, as shock passage proceeds, particle velocity within a planar shock wave can be decomposed into the vector sum \( \mu + \bar{\mu} \), with \( \mu(t) \) the expected value of the velocity at any time over a sufficient span of a plane normal to the shock direction. The expected value \( \mu(t) \) is the continuum transport particle velocity in the structure shock wave. The expected value of the variant velocity over a comparable span of the plane is necessarily zero. The variance of the particle velocity, the expected value of the square of the variant velocity, \( \left\langle \bar{\mu}^2 \rightangle = \sigma^2(t) \), is nonzero.

2.2. Dissipation in the shock transition

Kinetic dissipation, \( E_k(t) = \left\langle \bar{\mu}^2 \rightangle /2 = \sigma^2 /2 \), associated with the variant velocity field, is the random acoustic phonon energy that is a consequence of the collective dissipative structure activity underway as failure proceeds through the shock transition. This kinetic dissipation will ultimately thermalize. Within the interim, however, the kinetic dissipation has an integral role to play in the correlation dynamics of the shock failure transition.

Concurrently, frictional deformation dissipation \( E_f \) occurs within the interior of elemental active dissipative structures. Depending on the material and mode of deformation in the shock transition, frictional dissipation is a collective term that can include fracture bond breakage, dislocation plasticity and distended matter compaction among others. Frictional dissipation within the elemental dissipative structure is characterized by a dissipation constant \( \Gamma \) with dimensions of energy per unit area. Familiar is that of fracture, where \( \Gamma \) is a measure of strain energy release as commonly specified through measurements of the material fracture toughness. Less familiar is application of the dissipation constant \( \Gamma \) to other forms of failure dissipation including plastic shear slip or distended matter compaction.

Dissipation dichotomy is central to the statistical physics governing the shock failure transition. Frictional dissipation and kinetic dissipation are both a consequence of the same underlying microstructural failure mechanics [1,2]. Both frictional dissipation and kinetic dissipation scale with the dissipation constant \( \Gamma \). Dissipation dichotomy was, perhaps, first noted in an analysis of dynamic fracture by Mott [1].

2.3. Correlation dynamics in the shock transition

Within the shock transition the variant velocity \( \bar{\mu} \) is a statistically random function of time and position. On any plane within the shock, with normal collinear to the planar shock direction, the variant velocity is statistically stationary; that is, no point on the plane is statistically preferred. At any time \( t \) within the structured shock wave, a temporal variant velocity correlation function on that plane, referenced to an arbitrary position \( x = 0 \) within that plane, is the expected value expression \( K(s,t) = \left\langle \bar{u}(0,t)\bar{u}(s,t) \right\rangle \). The temporal coordinate \( s \) is related to position coordinate \( x \) within the plane through the acoustic
impedance \( Z = \rho c \) through \( s = \rho x / Z \). The variant velocity correlation function \( K(s,t) \) is a measure of the temporal correlation of kinetic dissipation as dynamic failure ensues through the shock transition. Note that \( K(0,t) = \sigma^2(t) \).

Here, a powerful result emerging from the applications of Boltzmann statistics to complex systems [3] is applied to the shock failure transition. Namely, the integral of the variant velocity correlation function equates to a continuum self-diffusion property of the material,

\[
D(t) = \int_{0}^{\infty} \left\{ \tilde{u}(0,t) \tilde{u}(s,t) \right\} ds = \int_{0}^{\infty} K(s,t) ds .
\]  

Equation (1) is identified as the fluctuation-dissipation theorem [4] and, in this form, more commonly identified with the Green-Kubo equation [5]. The fluctuation-dissipation relation has wide application in relating continuum transport properties of a system to the underlying micro dynamics [4]. In the present treatment this expression relates the correlated kinetic acoustic dissipation fluctuations to a continuum phonon momentum diffusivity property \( D(t) \) within the advancing shock wave. As such, \( D(t) \) is identified with the kinematic acoustic phonon viscosity within the shock transition. The property \( D(t) \) is integral to the structuring of the time history of the wave through the shock transition.

The variant velocity correlation function plays a further role in the shock failure transition. The differential expression \( dm = Z ds \) is an element of areal mass. The failure transition within the shock, within a temporal correlation period \( \tau \), is the critical state at which the correlated kinetic dissipation \( E_s(\tau) dm \) achieves the requisite shock transition dissipation as expressed through the dissipation constant \( \Gamma \). Dissipation dichotomy through the shock transition is realized by equating the frictional dissipation constant \( \Gamma \) to the correlated kinetic dissipation through the shock transition period \( \tau \),

\[
\Gamma = \frac{1}{2} \int_{0}^{\infty} \left\{ \tilde{u}(0,\tau) \tilde{u}(s,\tau) \right\} Z ds = \frac{1}{2} \int_{0}^{\infty} K(s,\tau) Z ds .
\]  

Equation (2) yields an energy-time criterion for the shock transition.

A functional form for the velocity correlation function is not known. Selected statistical processes (e.g., Gaussian, Markovian), common to other random physical applications, dictate an exponential decay for the temporal correlation span [5]. Application of an exponential variant velocity correlation function, with temporal correlation span \( \tau \), is accepted here,

\[
K(s,\tau) = \left\{ \tilde{u}(0,\tau)^2 \right\} e^{-s/\tau^2} = \sigma^2(\tau) e^{-s/\tau^2} .
\]  

Integration of Equation (2) then yields,

\[
\Gamma = \frac{1}{2} \sigma^2 Z \tau .
\]  

As expressed, \( E_s(\tau) = \sigma^2(\tau)/2 \) is the specific kinetic dissipation necessary to the shock failure transition. As such, Equation (4) relating frictional and kinetic dissipation takes the elemental energy-time form,

\[
\Gamma/Z = E_s/\tau = \frac{1}{2} \sigma^2 \tau / 2 .
\]  

Equation (5), barring uncertainties of a proportionality constant of order unity, is an energy-time criterion descriptive of the shock failure transition. Equation (5) follows from considerations of the underlying micro-statistics failure and correlation dynamics that bring about the macro shock-failure properties observed in the shock-wave experiment. Such observations, among others, include the rated dependence of shock-induced spall strength, and the fourth-power dependence of steady structured shock waves in solids.

A comparison of Equation (2) with Equation (1) reveals the ratio \( \Gamma/Z \) equates to a critical state amplitude of the self-diffusion constant \( D(\tau) \) within the temporal span of the shock transition. This critical state constant is identified with the kinematic viscosity and dissipative action \( A \) associated with the shock failure transition [6]; namely, \( D(\tau) = A = \Gamma/Z \).

3. Closure

Page constraints subjected to the present extended abstract prohibits a fair discussion of the application of Equation (5) to the span of experimental shock transition data noted earlier. Briefly, dissipation dichotomy dictates that the kinetic dissipation
\(E_i(\tau)\) equate to one-half the total shock dissipation in a steady structured shock wave transition. For shock transitions sensibly described by linear shock versus particle velocity constants \(C_o\) and \(S\), shock dissipation to Hugoniot strain \(\dot{\varepsilon}\) is to first order \(SC_o e^{\gamma / \beta}\). Shock transition time is related to strain and strain rate through \(\tau = \varepsilon / \dot{\varepsilon}\). Constancy of energy-time in Equation (5) yields the universal fourth-power dependence of strain rate on shock amplitude observed in many solid matter materials [6]. Similarly, application of Equation (5) to steady structured shock waves in porous solids, accounting for the differences in shock dissipation, yields the more diverse power-law relations between strain rate and shock amplitudes observed in these materials [6].

Further, Equation (5) has application to fracture and fragmentation in the tensile spall failure of condensed matter. Relating kinetic dissipation \(E_i(\tau)\) to one-half the strain energy at the tensile strain amplitude of spall failure, and again relating the spall transition time to strain rate through \(\tau = \varepsilon / \dot{\varepsilon}\), expressions for the dependence on strain rate of the tensile spall strength and dissipation constant \(\Gamma\) can be identified with liquid surface energy, solid fracture toughness, or other energy properties characterizing spall cavitation of the material under dynamic tension.

Representative applications to compressive shock transition data are illustrated in Fig. (1). Pressure step versus strain rate structured shock wave data for a chemical compound \(Al2O3\), an unalloyed uranium metal and an HMX-binder mixture exhibit the fourth-power nature emerging from application of Equation (5). Alternatively, pressure step versus shock width plot of the same data are compared with polycrystal grain size. Further, structured wave measurements for sapphire and for single crystal HMX overlay the polycrystal data. These observations collectively suggest underlying dynamic plasticity accounts for viscous structuring of the shock waves through the shock transition. Dissipation constants, assessed for the three materials from Equation (5) and plotted against respective experimental HEL values, illustrate a systematic with this dynamic strength measure.

References
