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# A predictor-corrector layerwise model for thick laminated composite plates and shells: Bending and vibration

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## Abstract

This paper describes a predictor-corrector theory based on a general higher-order layerwise model for the accurate prediction of the linear static and dynamic response of thick laminated composite plates and shells. The general polynomials introduced in the model account for the arbitrary variation of the transverse shear stresses across the thickness of each layer. The main purpose of the approach is to reduce the differences between the assumed variation of the transverse shear stresses provided by the constitutive equations and the computed variation of the same stresses from the equilibrium equations of elasticity. The present predictor-corrector layerwise model satisfies the continuity of the in-plane displacements and the transverse shear stresses at the interfaces. The numerical results for the bending and vibration of thick laminated composite plates and shells show that a high level of accuracy can be achieved with the same number of variables as that in Mindlin's theory.

## Introduction

Laminated plates and shells are important structural applications of fibrous composite materials in the areas of mechanical, aerospace, automobile, shipbuilding, and civil engineering, and in other branches of engineering. The most attractive properties of composite materials are their high strength-to-weight ratio and stiffness-to-weight ratio. The reliable use of advanced composite materials requires the accurate modelling of the behaviour of the composite structures under static and dynamic loading.

It is well known that the classical laminated theory, based on the Kirchhoff hypothesis of neglecting transverse shear strains and transverse normal strains, is inadequate in modelling the mechanical behaviour of thick laminated composite plates and shells. This is due to the high ratio of in-plane Young's modulus to transverse shear modulus of the most advanced composite materials. Consequently, transverse shear deformation plays an important role in the accurate prediction of the response characteristics of thick laminated composite plates and shells. Three-dimensional elasticity and quasi-three-dimensional models have been applied to the bending and vibration analysis of thick laminated plates and shells by, for example, Pagano (1969, 1970), Pagano and Hatfield (1972), Srinivas and Rao (1970), Noor (1973a, b), Noor and Rarig (1974), Ren (1987, 1989), Noor and Burton (1989, 1990a, b), Varadan and Bhaskar (1991), and Savoia and Reddy (1992). Obviously, it seems impractical to use three-dimensional elasticity and quasi-three-dimensional models to predict the response characteristics of general laminated plates and shells because of the complexity of the method or the huge computational requirements. However, these solutions can be used as the basis for assessing the accuracy and range of validity of two-dimensional approximate theories. As a result, a variety of two-dimensional shear deformation theories have been developed. These two-dimensional theories can be divided into two categories, namely, equivalent single-layer theories and layerwise theories.

Unlike equivalent single-layer models, layerwise models allow the in-plane displacements to vary in a piecewise

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manner through the thickness of the laminate and they naturally include the effect of transverse shear deformation. In contrast to the equivalent single-layer theories, the layerwise theories can reproduce the zig-zag behaviour of the in-plane displacements. This zig-zag behaviour is more pronounced for thick laminates where the transverse shear modulus changes abruptly through the thickness and can be seen in the exact elasticity solutions obtained by Pagano (1969, 1970), and Pagano and Hatfield (1972) for the bending of rectangular laminated plates, and by Ren (1987, 1989) for the bending of laminated shells.

Linear and cubic layerwise models were developed by Di Sciuva (1986) and Lee *et al.* (1990) which had the same number of variables as first-order shear deformation theories (FSDT) independent of the number of layers in the laminate. Both models satisfied the continuity conditions but only the cubic model could provide a more appropriate parabolic distribution of the transverse shear stresses with zero values at the free surfaces.

It was reported in Lee *et al.* (1990) that despite the good agreement of the maximum in-plane stresses with exact elasticity solutions for cylindrical bending of thick laminates, the computed in-plane displacements and stresses from the cubic model were not accurate enough at the interfaces. This was due to the use of a single parabolic function for the transverse shear stresses in the constitutive equations, which led to differences between the constitutive description and the computed variation of the same stresses from the equilibrium equations of elasticity.

The reduction of these differences for the bending and vibration of plates and shells is the basis of the predictor-corrector approach described in this paper.

## Theory

Consider the laminated composite plate of uniform thickness $h$ with $n$ orthotropic layers in Fig. 1.

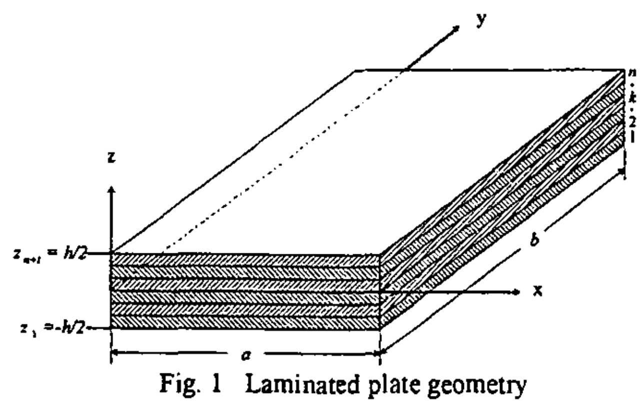

Fig. 1 Laminated plate geometry

In the present model, the layerwise displacement field for plates is written as

$$u^k = u_0^k - zw,_x + f_x^k(z)\phi_x^k$$
$$v^k = v_0^k - zw,_y + f_y^k(z)\phi_y^k \qquad (1)$$
$$w^k = w \qquad (k = 1,\ldots,n)$$

where the superscript $k$ refers to the $k$th layer and the other symbols have the usual meanings, and $u_0^k, v_0^k, \phi_x^k, \phi_y^k$ and $w$ are functions of $x$ and $y$. By imposing the continuity conditions of the transverse shear stresses and the in-plane displacements at the interfaces, the final displacement field can be expressed as

$$u^k = u_0^1 - zw,_x + p_k(z)\phi_x^1 + q_k(z)\phi_y^1$$
$$v^k = v_0^1 - zw,_y + r_k(z)\phi_x^1 + s_k(z)\phi_y^1 \qquad (2)$$
$$w^k = w \qquad (k = 1,\ldots,n)$$

where $p_k, q_k, r_k$ and $s_k$ depend on the functions $f_x^k$ and $f_y^k$ as well as the $z$-coordinates of the interfaces and the transverse shear moduli of the different layers. It is noted that there are only five variables (the same as in the FSDT), and the linear and cubic layerwise models can be recovered from the above displacement field by specifying the functions $f_x^k$ and $f_y^k$ in an appropriate manner. The governing equations and boundary conditions for bending corresponding to the final displacement field are derived using the Principle of Virtual Work. The governing equations are

$$\delta u_0^1 : \frac{\partial N_1}{\partial x} + \frac{\partial N_6}{\partial y} = 0$$
$$\delta v_0^1 : \frac{\partial N_6}{\partial x} + \frac{\partial N_2}{\partial y} = 0$$
$$\delta \phi_x^1 : \frac{\partial P_1}{\partial x} + \frac{\partial P_{61}}{\partial y} - R_1 = 0 \qquad (3)$$
$$\delta \phi_y^1 : \frac{\partial P_{62}}{\partial x} + \frac{\partial P_2}{\partial y} - R_2 = 0$$



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$$\delta w : \frac{\partial^2 M_1}{\partial x^2} + 2\frac{\partial^2 M_6}{\partial x \partial y} + \frac{\partial^2 M_2}{\partial y^2} + q = 0$$

and the boundary conditions for a smooth boundary are given by specifying

$$\begin{array}{lll} u_{0n}^1 & \text{or} & N_n \\ v_{0n}^1 & \text{or} & N_{ns} \\ w & \text{or} & \frac{\partial M_n}{\partial n} + 2\frac{\partial M_{ns}}{\partial s} \\ \frac{\partial w}{\partial n} & \text{or} & M_n \\ \phi_n^1 & \text{or} & P_n \\ \phi_{ns}^1 & \text{or} & P_{ns} \end{array} \qquad (4)$$

where $N_i$, $M_i$ and $P_i$ are the stress resultants associated with the layerwise theory. Also, the subscripts $n$ and $s$ refer to the normal and tangential directions, respectively, to the boundary of the plate.

The governing equations for bending are used in the predictor phase with $f_x^k = f_y^k = z$ to provide the initial solution from which a power series in $z$ of about five terms is obtained for the transverse shear stresses in each layer using Chebyshev expansion. In the corrector phase, this power series is equated to the corresponding one from the constitutive equations in order to determine the unknown coefficients in the functions $f_x^k$ and $f_y^k$. From these functions, a new set of governing equations is obtained which is different by a small amount from the old one. At this point, either a fresh solution or a modified solution can be found, with the latter made available through standard reanalysis techniques. A similar methodology can be used for vibration. The governing equations and boundary conditions are, of course, obtained from Hamilton's Principle.

For laminated composite shells, consider the orthogonal curvilinear coordinate system $(\alpha, \beta, \zeta)$ in which $\alpha$ and $\beta$ are the two curvilinear coordinates on the mid-surface where $\zeta$=0, and $\zeta$ is a rectilinear coordinate measured along the normal to the mid-surface, as shown in Fig. 2. The radii of curvature in the $\alpha$ and $\beta$ directions are denoted by $R_\alpha$ and $R_\beta$, and the surface metric coefficients of the mid-surface are $A_\alpha$ and $A_\beta$.

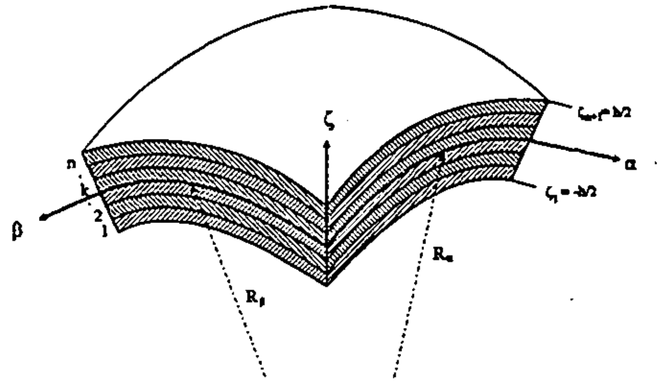

Fig. 2 Laminated shell geometry

Here, the layerwise displacement field for shells is taken as

$$\begin{aligned} u_\alpha^k &= (1+R_\alpha^{-1}\zeta)u_0^k - \zeta A_\alpha^{-1} w_{,\alpha} + f_\alpha^k(\zeta)\phi_\alpha^k \\ v_\beta^k &= (1+R_\beta^{-1}\zeta)v_0^k - \zeta A_\beta^{-1} w_{,\beta} + f_\beta^k(\zeta)\phi_\beta^k \\ w^k &= w \qquad (k=1,\dots,n) \end{aligned} \qquad (5)$$

and the final displacement field which satisfies the continuity conditions of the transverse shear stresses and the in-plane displacements at the interfaces is

$$\begin{aligned} u_\alpha^k &= (1+R_\alpha^{-1}\zeta)u_0^1 - \zeta A_\alpha^{-1} w_{,\alpha} + p_k(\zeta)\phi_\alpha^1 + q_k(\zeta)\phi_\beta^1 \\ v_\beta^k &= (1+R_\beta^{-1}\zeta)v_0^1 - \zeta A_\beta^{-1} w_{,\beta} + r_k(\zeta)\phi_\alpha^1 + s_k(\zeta)\phi_\beta^1 \\ w^k &= w \qquad (k=1,\dots,n) \end{aligned} \qquad (6)$$

Again, by using the Principle of Virtual Work and Hamilton's Principle, the governing equations and boundary conditions for bending and vibration can be obtained. The details are given in Cao (1995) and the predictor-corrector approach is basically similar to that for plates.

## Numerical Results

Figures 3 and 4 show the excellent agreement of the analytical results from the present predictor-corrector approach with the exact elasticity results of Pagano (1970) for his bidirectional bending problem of an antisymmetric two-layer (0°/90°) cross-ply square laminate ($a/h$ = 4) with layers of equal thickness under simply-supported conditions and sinusoidal transverse load. The material constants for

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each orthotropic layer are taken to be $E_L$=25$E_T$, $G_{LT}$=0.5$E_T$, $G_{TT}$=0.2$E_T$, $\nu_{LT} = \nu_{TT}$=0.25 where the subscripts L and T refer to the directions parallel and perpendicular to the fibre orientation, respectively. The non-dimensionalized stresses $\bar{\sigma}_1$ and $\bar{\sigma}_5$ are defined in the usual manner as follows:

$$\bar{\sigma}_1 = \frac{\sigma_1}{q_0 R^2}, \qquad \bar{\sigma}_5 = \frac{\sigma_5}{q_0 R} \tag{7}$$

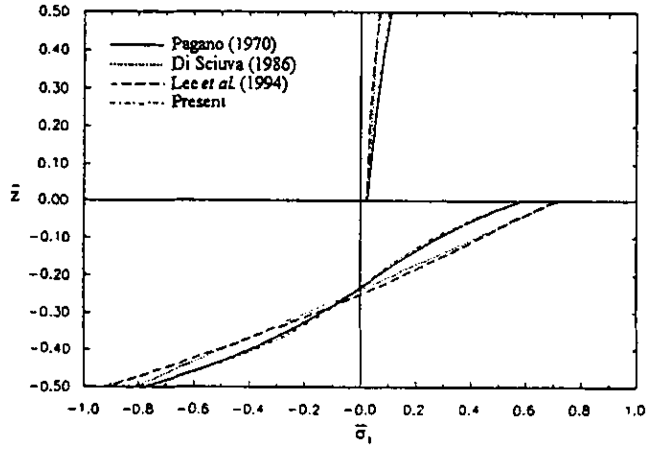


Fig. 3 Distribution of in-plane stress $\bar{\sigma}_1(a/2, a/2, \bar{z})$ in a (0°/90°) square laminate with $a/h$ = 4 under sinusoidal loading

For the purpose of comparison with other layerwise or zig-zag theories which have the same number of variables, the results from Di Sciuva's (1986) linear theory and Lee *et al.*'s (1990) cubic theory are also presented. The results from the linear theory have been calculated independently (without any shear correction factor) by using $f_x^k = f_y^k = z$ in Eq. (1)) whereas the results from the cubic theory can be found by following the analysis given in Lee *et al.* (1994). It is seen that the results from the present theory follow the curves from exact elasticity much more closely than the others. This is most obvious in Fig. 4 for the transverse shear stress $\bar{\sigma}_5$ which shows substantial differences between exact elasticity and the other two theories. The maximum value of $\bar{\sigma}_5$ occurs near $\bar{z} = -0.2$. At this location, the value of $\bar{\sigma}_5$ from the present theory is off by about 2% while those from the other two theories are in error by as much as 14%.

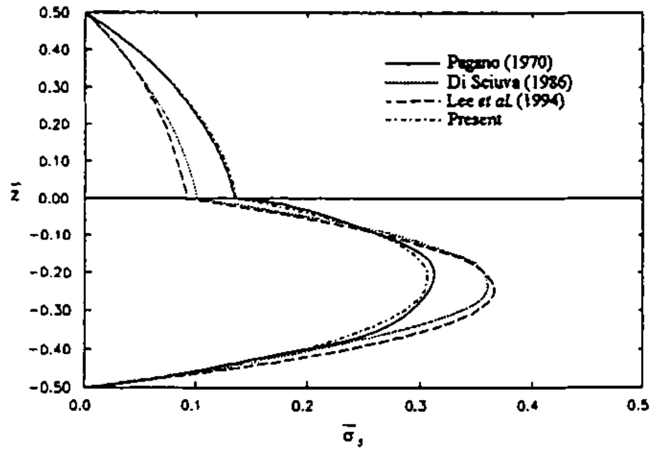


Fig. 4 Distribution of transverse shear stress $\bar{\sigma}_1(a/2, a/2, \bar{z})$ in a (0°/90°) square laminate with $a/h$ = 4 under sinusoidal loading

The effect of the length-to-thickness ratio $a/h$ on the accuracy of the fundamental frequencies from the present approach is shown in Table 1 for a simply-supported antisymmetric ten-layer cross-ply square laminate with the material properties given in the table. The length-to-thickness ratio is varied from 2.5 to 100.

Table 1 Effect of thickness ratio on the fundamental frequency $\bar{\omega} = \omega\sqrt{\rho h^2 / E_T}$ of simply-supported ten-layer antisymmetric square laminate with $E_L/E_T$=15, $G_{LT}/E_T$=0.5, $G_{TT}/E_T$=0.35, $\nu_{LT}$=0.3 and $\nu_{TT}$=0.49.

| | | $\omega^2/\omega^2_{exact}$ | | |
|---|---|---|---|---|
| $a/h$ | $\bar{\omega}_{exact}$ | FSDT | Noor | Present |
| 2.5 | 0.91280 | 1.168 | 0.9875 | 0.99853 |
| 10/3 | 0.62976 | 1.143 | 0.9978 | 0.99847 |
| 5 | 0.35171 | 1.099 | 1.005 | 0.99855 |
| 10 | 0.10968 | 1.037 | 1.005 | 0.99872 |
| 20 | 0.02961 | 1.011 | 1.002 | 0.99924 |
| 100 | 0.00128 | 1.000 | 1.000 | 1.00018 |

A comparison of the present results with those from the three-dimensional elasticity solutions (Noor and Burton, 1989), first-order shear deformation theory (Noor and Burton, 1989) and the predictor-corrector procedure of Noor and Burton (1989) once again shows that the proposed predictor-corrector model is in excellent agreement with the exact three-dimensional elasticity results and is generally more accurate than the other theories.



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The accuracy of the present predictor-corrector approach for the bending of shells is also indicated in Fig. 5 which shows the variation of the non-dimensionalized maximum deflection $\overline{w}$ with the radius-to-thickness ratio $R/h$ of an antisymmetric two-layer (0°/90°) cross-ply simply-supported cylindrical panel undergoing cylindrical bending in the presence of a sinusoidal transverse load. The two layers are of equal thickness with the fibres parallel to the $\theta$ and $x$ directions in the top and bottom layers, respectively. The laminated shell is taken to be infinitely long, with radius $R$=10 and included angle $\varphi = \pi/3$. The material constants for each orthotropic layer are again taken to be $E_L$=25$E_T$, $G_{LT}$=0.5$E_T$, $G_{TT}$=0.2$E_T$, $\nu_{LT} = \nu_{TT}$ =0.25.

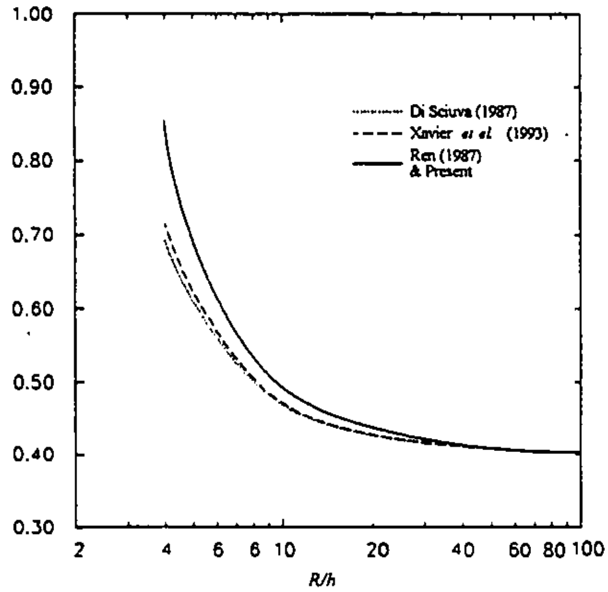


Fig. 5 Maximum deflection as a function of $R/h$ in cylindrical bending of a (0°/90°) cylindrical laminated shell under sinusoidal loading

It is clear from Fig. 5 that there is hardly any difference between the exact elasticity curve from Ren (1987) and the present one in contrast with the other layerwise results obtained independently using Di Sciuva's (1987) and Xavier *et al.*'s (1993) theories.

To verify further the accuracy and effectiveness of the present predictor-corrector procedure, parametric studies were performed in order to show the effect of variation in the material and geometric properties of the laminated composite cylinders on their free vibration response. The material properties of the individual layers used in the following analysis are as follows: $G_{LT}$=0.6$E_T$, $G_{TT}$=0.5$E_T$, $\nu_{LT} = \nu_{TT}$ =0.5. The three-dimensional elasticity solutions provided by Noor *et al.* (1990), Noor and Burton (1990a) and Noor and Rarig (1974) are taken as the standard for comparison.

The effect of lamina orthotropy $E_L/E_T$ of the individual layers on the natural frequencies with $m$=1 and $n$=2 obtained for simply-supported cylinders with different lamination schemes is indicated in Fig. 6 for the antisymmetric two-layer (0°/90°) cross-ply laminated composite cylinders with $L/R$=1 and $R/h$=5. It is observed from Fig. 6 that the natural frequencies obtained by the present theory are very close to the three-dimensional solutions for these composite cylinders for the range of lamina orthotropy considered. The accuracy of the present results is insensitive to the lamina orthotropy of the individual layers whereas the errors in the results from Di Sciuva's (1987) and Xavier *et al.*'s (1993) theories increase with the degree of orthotropy.

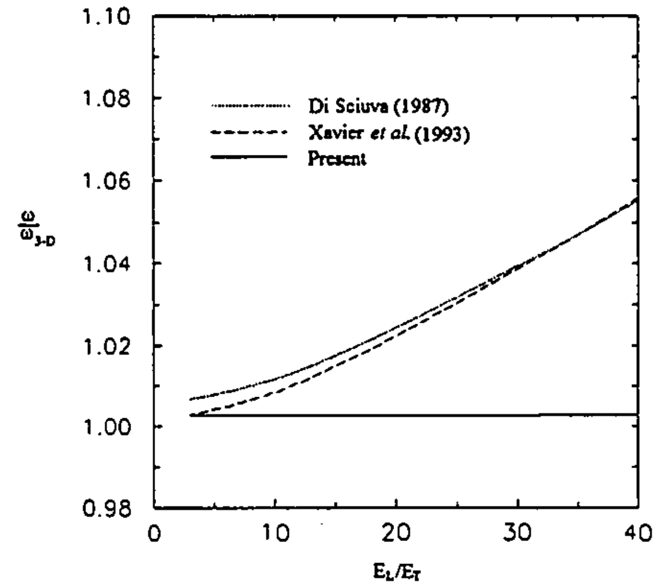


Fig. 6 Effect of lamina orthotropy on accuracy of natural frequencies for (0°/90°) composite cylinders with $L/R$=1, $R/h$=5, and $m$=1, $n$=2.

The effect of the radius-to-thickness ratio $R/h$ on the natural frequencies from the various layerwise theories is plotted in Fig. 7 for the two-layer cylinders with $L/R$=1, $E_L/E_T$=30, and $m$=1, $n$=2. It is found that the present theory provides accurate natural frequencies over the range of $R/h$ from 2.5 to 20. For instance, the present results are in error by only a marginal 1% for a very thick cylinder with $R/h$=2.5 and $n$=2 while the results from Di Sciuva's (1987) and Xavier *et al.*'s (1993) theories are off by as much as 11%.



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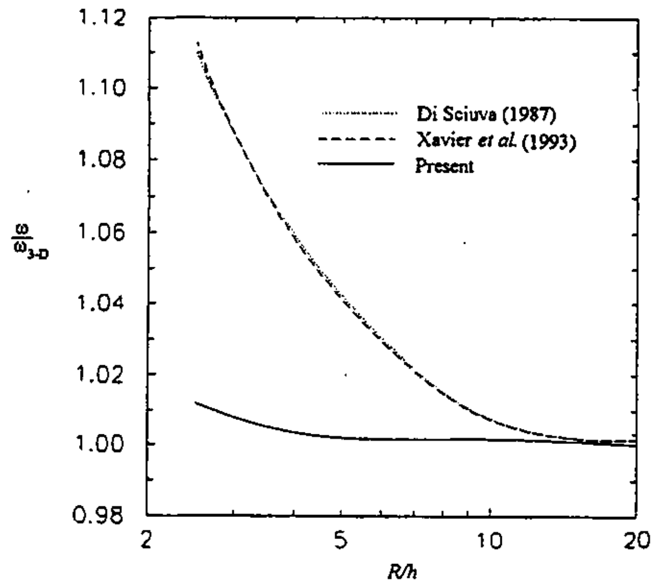


Fig. 7 Effect of radius-to-thickness ratio on accuracy of natural frequencies for (0°/90°) composite cylinders with $L/R$=1, $E_L/E_T$=30, and $m$=1, $n$=2.

The effect of the length-to-radius ratio $L/R$ on the natural frequencies is indicated in Fig. 8 for the two-layer composite cylinders with $R/h$=5, $E_L/E_T$=30, and $m$=1, $n$=2. For the two-layer short cylinders with $L/R$=0.5, the natural frequencies predicted by Di Sciuva's (1987) and Xavier *et al.*'s (1993) theories are significantly higher than the three-dimensional solutions by as much as 11%. In contrast, the errors in the present results are less than 2%.

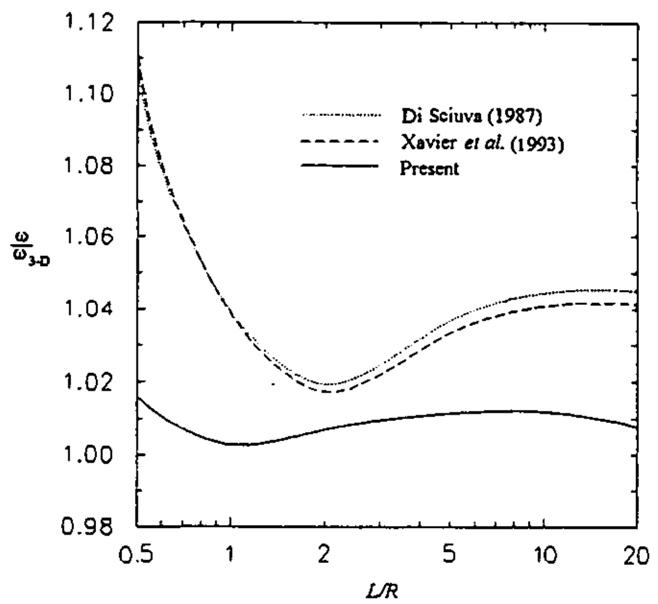


Fig. 8 Effect of length-to-thickness ratio on accuracy of natural frequencies for (0°/90°) composite cylinders with $R/h$=5, $E_L/E_T$=30, and $m$=1, $n$=2.

## Conclusions

This paper has demonstrated the effectiveness of a predictor-corrector approach based on the use of a general higher-order layerwise displacement model for thick laminated plates and shells. In the predictor phase, the linear layerwise model was employed for the purpose of obtaining a good estimate of the transverse shear stress distributions throught the use of the equilibrium equations of elasticity. Then, in the corrector phase, the estimate is used to deduce the new polynomial functions for the layerwise displacement field.

The examples on the bending and vibration of thick laminated plates and shells showed clearly the accuracy of the present predictor-corrector approach over other layerwise models with the same number of variables. The numerical results were found to be in excellent agreement with available three-dimensional elasticity solutions. The results also confirmed that the inaccuracies in the linear and cubic layerwise models were due entirely to the differences between the assumed shape of the transverse shear stresses provided by the constitutive equations and the calculated shapes of the same stresses from the equilibrium equations of elasticity.

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