A NODAL LOAD MODEL OF 3-D FINITE ELEMENT ANALYSIS FOR A CRANKSHAFT OF A TRUCK-MOUNTED COMPRESSOR

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ABSTRACT
In order to analyze the stresses and deformations of a crankshaft of a truck-mounted compressor by means of finite element method, the most important and difficult thing is how to determine the external force model on a crankshaft. The objective of this paper is to present a new load model of 3-D finite element analysis for a crankshaft of a truck-mounted compressor used in an oil field, i.e., the contacting-distribution forces are transferred into equivalent nodal forces. Firstly, the contacting-distribution forces between the crankshaft and the links are analyzed; secondly, formulations of equivalent node forces for the working conditions of 0° and 75° are deduced; finally, the crankshaft of a gas-engine driven truck-mounted compressor is taken as an illustration.

INTRODUCTION
Without question, the digital computer has had a significant impact on the engineering community. Larger and faster computers have made possible developments such as making the finite element method a practical and an invaluable tool in the area of structural analysis, especially in the area of complex structure such as a crankshaft of compressor. In order to get accurate results of the stresses and deformations of a crankshaft, the important thing, besides reasonably dividing the continuum into elements and imposing the boundary condition, is how to calculate the equivalent node forces. The reciprocation is produced by contacting forces between the crankshaft and the links. The forces and their directions are varying in different rotating angles of crankshaft. There are usually several cylinders which work in different stages at the same time in a compressor. The forces on the surfaces of the crankshaft are varying, so it is difficult to calculate the equivalent forces for finite element analysis. In this paper, a new nodal load model is presented, and as an example the model is used for a 3-D finite element analysis for the crankshaft of an oilfield high-pressure compressor. In the next section, the contacting force between crankshaft and links is presented. Thereafter, a new model of the effects of load to nodes is produced. This is followed by an example in which nodal load, stress and deformation, and the fatigue safety coefficient of the crankshaft of a gas-engine driven truck-mounted compressor used in oil field are calculated.

CALCULATING THE CONTACTING FORCES BETWEEN CRANKSHAFT AND LINK
The contacting forces between the crankshaft and the links are not well-distributed and need complex calculations in order to obtain accurate values, but in finite element analysis for engineering problem, generally we consider interaction loads between a bearing and a shaft as a cosine distribution with a 120° angle. The crankshaft in the compressor is supported on bearings. At sometime the cosine-distribution force on a crankshaft is shown in Fig. 1. The forces produced by link in a compressor are different in different rotating angles of the crankshaft. Assuming that crankshaft rotates to an angle of \( \alpha \), the compressive force produced by link arrives its maximum value \( q_\alpha \), and the half angle...
corresponding to the arc of the cosine distribution is \( \varphi_m \); usually \( 60^\circ \) (according to theoretical analysis and testing)\(^{[10]} \). The cosine-distribution forces appear as:

\[
q(\alpha, \varphi) = q_m(\alpha) \cos 1.5\varphi \quad \varphi \leq |\varphi_m|
\]  

(1)

where \( \varphi \) is the angle corresponding to an arc from the point analyzed and the point of \( q_m(\alpha) \). From Fig.1, we can obtain that

\[
p_i(\alpha) = \int_{-\varphi_m}^{\varphi_m} q(\alpha, \varphi) T R \cos \varphi d\varphi = T R q_m(\alpha) \int_{0}^{\pi} \cos 1.5\varphi \cos \varphi d\varphi
\]

\[
= 1.2 T R q_m(\alpha)
\]

Hence

\[
q_m(\alpha) = p_i(\alpha) / 1.2 T R
\]

(2)

Where

\[
p_i(\alpha) = \text{link forces} \quad (\text{N})
\]

\[
T = \text{link width} \quad (\text{mm})
\]

\[
R = \text{shaft radius} \quad (\text{mm})
\]

Substituting (2) into (1), the cosine distribution force on the surface of the shaft is obtained

\[
q(\alpha, \varphi) = \frac{p_i(\alpha)}{1.2 T R} \cos 1.5\varphi
\]

(3)

From (3), we can see that the force distributions are varying from one angle to another.

**DETERMINING THE MODEL OF THE EFFECTS OF LOAD TO NODES**

During finite element analysis, the cosine-distribution forces (Eq. (3)) cannot be used directly as an external load because the distribution forces do not generally act on the surface of an element, and even if they act on a surface of an element intentionally, it is difficult that a function of the loads of the four nodes of an element produces the distribution forces. Hence, it is necessary to transfer the cosine-distribution forces into nodal loads of element surface\(^{[9],[10]} \). In order to do that, the \( 360^\circ \) rotating angle of the crankshaft is divided into many steps (for example \( 15^\circ \) for each step). The equivalent nodal load for every step (\( \alpha = 15i \), \( i = 1, 2, 3, \ldots \), \( 24 \)) is calculated. The equivalent loads are vertical to the surfaces of the shaft because distribution forces are vertical to the surface. A typical crankshaft of a six-cylinder, three-stage, "w" arrangement compressor\(^{[7]} \) is illustrated in Fig. 2. The position of the links are symbolized as numbers \( 1^\circ \) to \( 6^\circ \). We take one of the six links as a research model (for example \( 3^\circ \)). The two external surfaces in direction of the link width are considered as surfaces of the element. The structure in direction of link width is divided into one element and the structure around the circular direction is divided into eight nodes as shown in Fig. 3. The numbers in the figure are the local nodal numbering. Assuming that the angle corresponding to the maximum distribution force \( q_m(\alpha) \) is \( 0^\circ \), following gives the calculating model of the equivalent load in two typical working conditions.

**The Nodal Load Model At The Working Condition of 0°**

The external force distributions of the 0° working condition are illustrated in Fig. 3. Take the coordinate axes as \( x \) and \( y \). The structure around the circular direction is divided into eight nodes, the local nodal numbering is shown in Fig. 3 that have been explained earlier. The surface of the element among nodes \( 1-1, 2-2, 3-3 \) are the line of width direction in link) is taken into account:

According to force equivalence we can obtain that:

in direction \( x \)

\[
2 P_{1-1} + 2 P_{2-2} \cos 45^\circ = \int_{0}^{\pi} q_m(\alpha) \cos 1.5\varphi R \cos \varphi T \sin \varphi
d\varphi = 0.0388 R T q_m(\alpha)
\]

(4)

in direction \( y \)

\[
2 P_{1-1} \times 0 + 2 P_{2-2} \sin 45^\circ = \int_{0}^{\pi} q_m(\alpha) \cos 1.5\varphi R \cos \varphi T \sin \varphi
d\varphi = 0.03254 R T q_m(\alpha)
\]

(5)

Where

\[
P_{n-n} = \text{the equivalent load of the nodal numbering "n"} \\
(n = 1, 2, 3, \ldots, 8)
\]

\[
D = \text{down position of a node}
\]
Solving the Eqs. (4),(5) gives

\[ P_{1-1} = 0.00313RTq_m(\alpha) \]
\[ P_{2-2D} = \frac{0.03254}{\sqrt{2}} RTq_m(\alpha) \]

Similarly, the surface of the element among nodes 2--2,3--3 is taken into account:

in direction x
\[ 2P_{2-2u} \cos 45^\circ + 2P_{3-3l} \times 0 = \int_0^\alpha q_m(\alpha) \cos 15^\circ \varphi \, d\varphi \]
\[ = 0.2004RTq_m(\alpha) \quad (6) \]

in direction y
\[ 2P_{2-2u} \sin 45^\circ + 2P_{3-3l} = \int_0^\alpha q_m(\alpha) \cos 15^\circ \varphi \, d\varphi \]
\[ = 0.5675RTq_m(\alpha) \quad (7) \]

Where
- \( u = \) up position of a node
- \( l = \) left position of a node

Solving the Eqs. (6),(7) gives

\[ P_{2-2u} = \frac{0.2004}{\sqrt{2}} RTq_m(\alpha) \]
\[ P_{3-3l} = 0.18355RTq_m(\alpha) \]

According to the symmetry of the element surfaces among 1--1,2--2 to among 5--5,4--4, and among 2--2,3--3 to among 4--4,3--3, we can obtain

\[ P_{3-3r} = P_{3-3l} = 0.18355RTq_m(\alpha) \]
\[ P_{4-4u} = P_{2-2u} = \frac{0.2004}{\sqrt{2}} RTq_m(\alpha) \]
\[ P_{4-4D} = P_{2-2D} = \frac{0.03254}{\sqrt{2}} RTq_m(\alpha) \]
\[ P_{5-5} = P_{1-1} = 0.00313RTq_m(\alpha) \]

Where \( r = \) right position of a node. Hence, the nodal loads are

\[ P_{1-1} = 0.00313RTq_m(\alpha) \]
\[ P_{2-2U} + P_{2-2D} = \frac{0.23294}{\sqrt{2}} RTq_m(\alpha) \]
\[ P_{3-3} + P_{3-3r} = 0.3671RTq_m(\alpha) \]
\[ P_{4-4} = \frac{0.23294}{\sqrt{2}} RTq_m(\alpha) \]
\[ P_{5-5} = P_{1-1} = 0.00313RTq_m(\alpha) \]

The Nodal Load Model At The Working Condition of 75°

At the working condition of 75°, the cosine-distribution forces rotate 75° clockwise along the surface of the crankshaft as illustrated in Fig. 4. Using the similar method to the former, we can obtain the loads of the nodes \( P_{3-3}, P_{4-4}, P_{5-5}, P_{6-6} \). By continuing to rotate the cosine distribution forces clockwise at some other angles along the surface of the crankshaft, we can obtain all the nodal loads.

Similarly, we continue to take links 1°, 2°, 4°, 5°, 6° shown in Fig. 2 into account, and pay attention to the fact that rotating angles of links 1°, 4° are ahead with 60°, rotating angle of links 2°, 5° are later with 60°, rotating angle of link 6° is synchronous compared with that of the link 3°.

After calculating the nodal loads produced by the six links, we should combine the loads at same node produced by different links together because both of the nearest two links produce the loads at the same node. The equivalent nodal loads having been combined together are divided into three components along x, y, z coordinates. Those components can be used for finite element analysis[9].

ANALYSIS OF A CRANKSHAFT OF A GAS-ENGINE DRIVEN COMPRESSOR

In this example, we use the load model for a crankshaft which is mounted in a truck-mounted gas-engine driven compressor used in oil field. The crankshaft is illustrated in Fig. 2. The nomenclature is given as follows:\[ ]
speed: 1000 rpm; stroke: 100 meter
number of cylinders: 6 inlet pressure: 1×10^6 pascal
arrangement type of the cylinders “W”
discharge pressure: 25×10^6 pascal
the radius of the shaft: 58.5 millimeter
width of the link: 45 millimeter

Calculating The Forces of Links
The load acted on the crankshaft resulted mainly from the forces of the links. The differences of angles among links 1°, 2°, 3° are 60°. The differences of the angles among links 4°, 5°, 6° are also 60°. Hence, the speeds, accelerations and inertia forces among links have the angle differences of 60° too. According to the theory on reciprocating compressors, we make a program to calculate the forces of links at the points of every 15° of crankshaft rotation. The forces for the typical four working conditions are put into Table 1.

Table 1 The forces of links of the typical four working condition

<table>
<thead>
<tr>
<th>angles (degree)</th>
<th>numbering of the cylinder</th>
<th>forces of links (N)</th>
<th>angles (degree)</th>
<th>numbering of the cylinder</th>
<th>forces of links (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>-24098.96</td>
<td>270</td>
<td>1</td>
<td>-19662.60</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>-37112.25</td>
<td>270</td>
<td>2</td>
<td>-21716.32</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>-28957.77</td>
<td>270</td>
<td>3</td>
<td>-29813.53</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>-8898.010</td>
<td>270</td>
<td>4</td>
<td>-30047.76</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>-17552.51</td>
<td>270</td>
<td>5</td>
<td>-8489.26</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>-19217.84</td>
<td>270</td>
<td>6</td>
<td>-12458.66</td>
</tr>
<tr>
<td>75</td>
<td>1</td>
<td>-32232.30</td>
<td>345</td>
<td>1</td>
<td>-22771.40</td>
</tr>
<tr>
<td>75</td>
<td>2</td>
<td>-20378.32</td>
<td>345</td>
<td>2</td>
<td>-38893.97</td>
</tr>
<tr>
<td>75</td>
<td>3</td>
<td>-9936.400</td>
<td>345</td>
<td>3</td>
<td>-30186.48</td>
</tr>
<tr>
<td>75</td>
<td>4</td>
<td>-18976.23</td>
<td>345</td>
<td>4</td>
<td>-6336.09</td>
</tr>
<tr>
<td>75</td>
<td>5</td>
<td>-21289.62</td>
<td>345</td>
<td>5</td>
<td>-15932.66</td>
</tr>
<tr>
<td>75</td>
<td>6</td>
<td>-25145.49</td>
<td>345</td>
<td>6</td>
<td>-18381.60</td>
</tr>
</tbody>
</table>

The Force Distribution On The Surfaces of The Crankshaft

Substituting the values in Table 1 into (3), the force distribution on the surfaces of the crankshaft are obtained. They are put into Table 2.

Table 2 The force distribution on the surfaces of the crankshaft

<table>
<thead>
<tr>
<th>angles (degree)</th>
<th>numbering of the cylinder</th>
<th>force distribution</th>
<th>angles (degree)</th>
<th>numbering of the cylinder</th>
<th>force distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>-7.629cos1.5φ</td>
<td>270</td>
<td>1</td>
<td>-6.224cos1.5φ</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>-11.767cos1.5φ</td>
<td>270</td>
<td>2</td>
<td>-6.875cos1.5φ</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>-9.167cos1.5φ</td>
<td>270</td>
<td>3</td>
<td>-9.438cos1.5φ</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>-2.817cos1.5φ</td>
<td>270</td>
<td>4</td>
<td>-9.512cos1.5φ</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>-5.556cos1.5φ</td>
<td>270</td>
<td>5</td>
<td>-2.687cos1.5φ</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>-6.084cos1.5φ</td>
<td>270</td>
<td>6</td>
<td>-3.944cos1.5φ</td>
</tr>
<tr>
<td>75</td>
<td>1</td>
<td>-10.203cos1.5φ</td>
<td>345</td>
<td>1</td>
<td>-7.208cos1.5φ</td>
</tr>
<tr>
<td>75</td>
<td>2</td>
<td>-6.451cos1.5φ</td>
<td>345</td>
<td>2</td>
<td>-12.312cos1.5φ</td>
</tr>
<tr>
<td>75</td>
<td>3</td>
<td>-3.145cos1.5φ</td>
<td>345</td>
<td>3</td>
<td>-9.556cos1.5φ</td>
</tr>
<tr>
<td>75</td>
<td>4</td>
<td>-6.007cos1.5φ</td>
<td>345</td>
<td>4</td>
<td>-2.006cos1.5φ</td>
</tr>
<tr>
<td>75</td>
<td>5</td>
<td>-6.739cos1.5φ</td>
<td>345</td>
<td>5</td>
<td>-5.044cos1.5φ</td>
</tr>
<tr>
<td>75</td>
<td>6</td>
<td>-7.960cos1.5φ</td>
<td>345</td>
<td>6</td>
<td>-5.819cos1.5φ</td>
</tr>
</tbody>
</table>
Equivalent Nodal Loads

We use the model of nodal forces put forward in this paper. The force distributions are transferred into equivalent nodal loads. As representatives, some values of the equivalent nodal loads at the working condition of 0° are put into Table 3. The positions of the nodes on the crankshaft are illustrated in Fig. 2. The numbers 35,36,62,63 are located on back face of the crankshaft. Others are located in the front.

### Table 3  Some equivalent nodal loads at the working condition of 0°

<table>
<thead>
<tr>
<th>nodal numbering</th>
<th>35</th>
<th>36</th>
<th>61</th>
<th>62</th>
<th>63</th>
<th>71</th>
</tr>
</thead>
<tbody>
<tr>
<td>the nodal loads in direction X (N)</td>
<td>341.4</td>
<td>5261.8</td>
<td>0</td>
<td>7762.3</td>
<td>13405.3</td>
<td>-75.7</td>
</tr>
<tr>
<td>the nodal loads in direction Y (N)</td>
<td>-341.4</td>
<td>0</td>
<td>-2395.4</td>
<td>-7762.3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>the nodal loads in direction X (N)</td>
<td>72</td>
<td>73</td>
<td>84</td>
<td>85</td>
<td>185</td>
<td>197</td>
</tr>
<tr>
<td>the nodal loads in direction Y (N)</td>
<td>-2810.9</td>
<td>0</td>
<td>-2810.9</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>the nodal loads in direction Y (N)</td>
<td>-2810.9</td>
<td>-11253.0</td>
<td>-2810.9</td>
<td>-8857.6</td>
<td>571.0</td>
<td>571.0</td>
</tr>
</tbody>
</table>

CONCLUSIONS

This paper puts forward a new model of calculating nodal loads in 3-D finite element analysis. The example presented here is a 3-D finite element analysis of a crankshaft of a gas-engine driven compressor used in an oil field. The maximum stress and deformation in the crankshaft are obtained. The fatigue safety coefficient is 1.74. It is easy to use this model for industries.

References


Figure 1: The cosine distribution forces
Figure 2: The structure of the crankshaft of a six-cylinder three-stage "W" arrangement truck-mounted compressor.

Figure 3: The external force distributions of the 0° working condition.

Figure 4: The external force distributions of the 75° working condition.