IMPLEMENTING NEW UNCERTAINTY GUIDELINES AND STANDARDS IN TURBINE TESTING

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ABSTRACT

The use and application of uncertainty analysis in engineering has evolved considerably over the past decade. The methods formulated in the 1970's and 1980's that were incorporated into ANSI/ASME Standards and that were used during the AGARD-sponsored Uniform Engine Test Programme (UETP) have been superseded by more rigorous approaches. In this paper, the uncertainty methodology used in the UETP is compared with the newer approaches that will need to be implemented in future turbine test programs.

INTRODUCTION

The use and application of uncertainty analysis in engineering has evolved considerably since Kline and McClintock's classic paper in 1953. Developments in the field have been especially rapid and significant over the past decade, with the methods formulated by Abernethy and co-workers (Abernethy and Thompson, 1973; Abernethy et al., 1985) that were incorporated into ANSI/ASME Standards (ANSI/ASME, 1984, 1986) and that were used during the Uniform Engine Test Programme (AGARD, 1989a, 1989b) being superseded by a more rigorous approach (ISO, 1993). Publication in late 1993 by the International Organization for Standardization (ISO) of the Guide to the Expression of Uncertainty in Measurement in the name of ISO and six other international organizations has, in everything but name only, established a new international experimental uncertainty standard.

The approach in the ISO Guide deals with "Type A" and "Type B" categories of uncertainties, not the more traditional engineering categories of bias and precision uncertainties, and is of sufficient complexity that its application in normal engineering practice is unlikely. This issue has been addressed by AGARD Working Group 15 on Quality Assessment for Wind Tunnel Testing (AGARD, 1994), by the Standards Subcommittee of the AIAA Ground Test Technical Committee (AIAA, 1995), and by the ASME Committee PTC 19.1 that is revising the ANSI/ASME Standard (ANSI/ASME, 1986). The documents produced by two of these groups and in preparation by the ASME Committee present and discuss the additional assumptions necessary to achieve a less complex "large sample" methodology that is consistent with the ISO Guide, that is applicable to the vast majority of engineering testing, including most single-sample tests, and that retains the use of the traditional engineering concepts of bias and precision uncertainties.

In this paper, the uncertainty approach used in the Uniform Engine Test Programme (UETP) is compared with the newer approaches that will need to be implemented in future turbine test programs. All approaches (including that of ISO) are discussed using the bias/precision categorization of uncertainties.

UNCERTAINTY ANALYSIS APPROACHES

The word accuracy is generally used to indicate the relative closeness of agreement between an experimentally-determined value of a quantity and its true value. Error is the difference between the experimentally-determined value and the truth, thus as error decreases accuracy is said to increase. Only in rare instances is the true value of a quantity known. Thus, one is forced to estimate error, and that estimate is called an uncertainty, U. Uncertainty estimates are made at some confidence level - a 95% confidence estimate, for example, means that the true value of the quantity is expected to be within the ±U interval about the experimentally-determined value 95 times out of 100.

Total error can be considered to be composed of two components: a precision (random) component ε and a bias (systematic) component β. An error is classified as precision if it contributes to the scatter of the data; otherwise, it is a bias error. It is assumed that corrections have been made for all systematic errors whose values are known. The remaining bias errors are thus equally as likely to be positive as negative.

As an estimator of β, a bias limit B is defined. A 95% confidence estimate is interpreted as the experimenter being 95% confident that the true value of the bias error, if known, would fall within ±B. A useful approach to estimating the magnitude of a bias error is to
assume that the bias error for a given case is a single realization drawn from some statistical parent distribution of possible bias errors. For example, suppose a thermistor manufacturer specifies that 95% of samples of a given model are within ±1.0 C of a reference resistance-temperature (R-T) calibration curve supplied with the thermistor. One might assume that the bias errors (the differences between the actual, but unknown, R-T curves of the various thermistors and the reference curve) belong to a Gaussian parent distribution with a standard deviation $b = 0.5$ C. Then the interval defined by $±B = ±2b$ would include about 95% of the possible bias errors that could be realized from the parent distribution. (The bias limit is sometimes called the "systematic uncertainty.")

The estimate of the precision error for a variable is the sample standard deviation, or the estimate of the error associated with the repeatability of a particular measurement. Unlike the bias error, the precision error varies from measurement to measurement. As the number of measurements, $N_i$, of a particular variable, $X_i$, tends to infinity, the distribution of these possible errors becomes Gaussian. The sample standard deviation for each variable is determined as

$$S_i = \left[ \frac{1}{N_i(N_i-1)} \sum_{k=1}^{N_i} (X_{ik} - \bar{X}_i)^2 \right]^{1/2}$$  \hspace{1cm} (1)$$

where

$$\bar{X}_i = \frac{1}{N_i} \sum_{k=1}^{N_i} X_{ik}$$  \hspace{1cm} (2)$$

For the case of single readings ($N_i = 1$), previous information must be used to calculate $S_i$ (Steele et al., 1993).

In nearly all experiments, the measured values of different variables are combined using a data reduction equation (DRE) to form some desired result. An example from the UETP is the determination of specific fuel consumption when the basic measured quantities are scale force, fuel flow, and engine inlet pressure and temperature. A general representation of a data reduction equation is

$$r = r(X_1, X_2, \ldots, X_j)$$  \hspace{1cm} (3)$$

where $r$ is the experimental result determined from $J$ measured variables $X_j$. Each of the measured variables contains bias errors and precision errors. These errors in the measured values then propagate through the data reduction equation, thereby generating the bias and precision errors in the experimental result, $r$.

The ISO Approach

Defining $u_c^2$ as an estimate of the variance of the distribution of total errors in the result, $b^2$ as an estimate of the variance of a bias error distribution, and $S^2$ as an estimate of the variance of a precision error distribution, we can write (Coleman and Steele, 1995)

$$u_c^2 = \sum_{i=1}^{J} b_i^2 + 2 \sum_{i=1}^{J} \sum_{k=1}^{J} \theta_{ik} b_{ik} + \sum_{i=1}^{J} s_i^2 + 2 \sum_{i=1}^{J} \sum_{k=1}^{J} \theta_{ik} s_{ik}$$ \hspace{1cm} (4)$$

where $b_{ik}$ is an estimate of the covariance of the bias errors in $X_i$ and the bias errors in $X_k$, $s_{ik}$ is an estimate of the covariance of the precision errors in $X_i$ and the precision errors in $X_k$, and

$$\theta_{ik} = \frac{\partial r}{\partial X_i}$$ \hspace{1cm} (5)$$

In keeping with the nomenclature of the ISO Guide, $u_c$ is called the combined standard uncertainty.

No assumptions about type(s) of error distributions are made to obtain the preceding equation for $u_c$. To obtain an uncertainty $U_r$ (termed the expanded uncertainty in the ISO Guide) at some specified confidence level (95%, 99%, etc), the combined standard uncertainty $u_c$ must be multiplied by a coverage factor, $K$,

$$U_r = Ku_c$$ \hspace{1cm} (6)$$

It is in choosing $K$ that assumptions about the type(s) of the error distributions must be made.

An argument is presented in the ISO Guide that the error distribution of the result, $r$, may often be considered Gaussian because of the Central Limit Theorem, even if the error distributions of the $X_i$ are not normal. In fact, the same argument can be made for approximate normality of the error distributions of the $X_i$ since the errors typically are composed of a combination of errors from a number of elemental sources.

If it is assumed that the error distribution of the result, $r$, is normal, then the value of $K$ for $C\%$ coverage corresponds to the $C\%$ confidence level $t$-value from the $t$ distribution such that

$$U_r^2 = \sum_{i=1}^{J} b_i^2 (t b_i)^2 + 2 \sum_{i=1}^{J} \sum_{k=1}^{J} \theta_{ik} (t b_{ik})^2 + \sum_{i=1}^{J} s_i^2 (t s_i)^2 + 2 \sum_{i=1}^{J} \sum_{k=1}^{J} \theta_{ik} (t s_{ik})^2$$ \hspace{1cm} (7)$$

The effective number of degrees of freedom $v_r$ for determining the $t$-value is given (approximately) by the so-called Welch-Satterthwaite formula as

$$v_r = \frac{\left[ \sum_{i=1}^{J} (b_i^2 S_i^2 + s_i^2 t^2) \right]^2}{\sum_{i=1}^{J} \left( \sum_{i=1}^{J} (b_i^2 S_i^2 + s_i^2 t^2) \right)}$$ \hspace{1cm} (8)$$

where the $v_{di}$ are the number of degrees of freedom associated with the $S_i$ and the $v_{bi}$ are the number of degrees of freedom to associate with the $b_i$.

If an $S_i$ has been determined from $N_i$ readings of $X_i$ taken over an
appropriate interval, the number of degrees of freedom is given by

$$v_N = N - 1$$  \hspace{1cm} (9)$$

For the number of degrees of freedom $v_{hi}$ to associate with a non-
statistical estimate of $b_i$, it is suggested in the ISO Guide that one
might use the approximation

$$v_{hi} = \frac{1}{2} \left( \frac{\Delta b_i}{b_i} \right)^{-2}$$  \hspace{1cm} (10)$$

where the quantity in parenthesis is the relative uncertainty of $b_i$. For
example, if one thought that the estimate of $b_i$ was reliable to within
± 25%, then

$$v_{hi} = \frac{1}{2} (0.25)^{-2} = 8$$  \hspace{1cm} (11)$$

The **Large Sample Approach**

The approach in the ISO Guide is of sufficient complexity that its
application in normal engineering practice is unlikely. The current
uncertainty analysis documents produced by AGARD Working Group
15 on Quality Assessment for Wind Tunnel Testing (AGARD, 1994),
by the Standards Subcommittee of the AIAA Ground Test Technical
Committee (AIAA, 1995), and in preparation by the ASME Committee
PTC 19.1 that is revising the ANSI/ASME Standard
(ANS/ASME, 1986) present and discuss the additional assumptions
necessary to achieve a less complex "large sample" methodology. This
methodology is consistent with the ISO Guide, is applicable to the vast
majority of engineering testing, including most single-sample tests, and
retains the use of the traditional engineering concepts of bias and
precision uncertainties. This approach is discussed below.

Considering the 95% confidence t table, one can observe that for
$v_{hi} \geq 9$ the values of t are within about 13% of the traditionally-taken
"large sample" t-value of 2. This difference is relatively insignificant
compared with the uncertainties inherent in estimating the $S_i$, $b_i$, and
$v_{hi}$ necessary for evaluating Eqs. (7) and (8). Therefore, for most
engineering applications it is recommended that Gaussian error
distributions and $v_{hi} \geq 9$ be assumed so that $t = 2$ always for 95%
confidence. (This could be called the "large sample assumption.")
This assumption eliminates the need for evaluation of $v_{hi}$ using the
Welch-Satterthwaite formula and thus the need to estimate all of the
$v_{si}$ and the $v_{bi}$.

Consideration of the Welch-Satterthwaite formula (Eq. (8)) shows that,
because of the exponent of 4 in each term in the denominator, $v_f$
most influenced by the number of degrees of freedom of the largest
of the $\theta S_i$ or $\theta b_i$ terms. If, for example, $\theta S_3$ is dominant, then $v_f$
$= v_{S3} \geq 9$ for $N_3 \geq 10$ (recalling Eq. (9)). If, on the other hand,
$\theta b_3$ is dominant, then $v_f = v_{b3} \geq 9$ when the relative uncertainty in $b_i$
is about 24 percent or less (recalling Eq. (10)). Therefore invoking the "large sample assumption" essentially means that if a $\theta S_i$ is
dominant then its $N_i \geq 10$ or if a $\theta b_i$ is dominant then the relative uncertainty in that $b_i$ is about 24 percent or less. If there is no single
dominant term, but there are M different $\theta S_i$ and $\theta b_i$ that all have the
same magnitude and same number of degrees of freedom $v_{si}$, then

$$v_f = M v_{si}$$  \hspace{1cm} (12)$$

If $M = 3$, for example, $v_{si}$ would only have to be 3 or greater for $v_f$
to be equal to or greater than 9. Therefore, $t$ can often legitimately be
taken as 2 for estimating the uncertainty in a result determined from
several measured variables even when the numbers of degrees of
freedom associated with the measured variables are very small.

If the "large sample assumption" is made so that $t = 2$, then the
95% confidence expression for $U_f$ becomes

$$U_f^2 = \sum_{i=1}^j \theta_i^2 B_i^2 + 2 \sum_{i=1}^j \sum_{k=i+1}^j \theta_i \theta_k B_i B_k$$  \hspace{1cm} (13)$$

Recalling the definition of bias limit, the ($2b_i$) factors are equal to the
95% confidence bias limits $B_i$. If no further assumptions are made we
can rewrite Eq. (13) as

$$U_f^2 = \sum_{i=1}^j \theta_i^2 B_i^2 + 2 \sum_{i=1}^j \sum_{k=i+1}^j \theta_i \theta_k B_i B_k$$  \hspace{1cm} (14)$$

where $B_{ki}$ is the 95% confidence estimator of the covariance of the
bias errors in $X_i$ and $X_k$.

If we define the bias limit (systematic uncertainty) of the result as

$$B_i^2 = \sum_{i=1}^j \theta_i^2 B_i^2 + 2 \sum_{i=1}^j \sum_{k=i+1}^j \theta_i \theta_k B_i B_k$$  \hspace{1cm} (15)$$

and the precision uncertainty of the result as

$$(2S)^2 = \sum_{i=1}^j \theta_i^2 (2S)^2 + 2 \sum_{i=1}^j \sum_{k=i+1}^j \theta_i \theta_k (2S)^2$$  \hspace{1cm} (16)$$

then Eq. (14) can be written as

$$U_f^2 = B_i^2 + (2S)^2$$  \hspace{1cm} (17)$$

and Eqs. (15) and (16) can be viewed as propagation equations for the
bias uncertainties and precision uncertainties, respectively, of the
variables. As shown by Steele et al. (1994) using Monte Carlo
simulations, the appropriate large sample 99% confidence uncertainty is obtained by replacing the coverage factor of 2.0 in Eq. (13) by a coverage factor of 2.6.

The $B_k$ terms in Eqs. (14) and (15) must be considered when some of the bias limits for the measured variables are correlated. Correlated bias limits are those that are not independent of each other, typically a result of different measured variables sharing some identical elemental bias error sources. The methodology for properly handling correlated bias limits is given by Brown et al. (1996).

The Abernethy et al. Approach

An approach that was widely used in the 1970's and 80's was the $U_{RSS}$: $U_{ADD}$ technique formulated by Abernethy and co-workers (Abernethy and Thompson, 1973; Abernethy et al., 1985) which was incorporated into two ANSI/ASME Standards (ANSI/ASME, 1984, 1986). The $U_{ADD}$ formulation was used during the Uniform Engine Test Programme (AGARD, 1989a, 1989b). According to Abernethy et al.,

$$U_{RSS} = (B_r^2 + (tS_r^2)^2)^{1/2} \tag{18}$$

for a 95% confidence estimate and

$$U_{ADD} = B_r + (tS_r) \tag{19}$$

for a 99% confidence estimate, where $B_r$ is given by

$$B_r = \left( \sum_{i=1}^J (\delta_i^2 / \beta_i^2) \right)^{1/2} \tag{20}$$

and where $B_r$ and the $B_i$'s are 95% confidence bias limit estimates. $S_r$ is given by

$$S_r = \left( \sum_{i=1}^J (\delta_i^2 / \beta_i^2) \right)^{1/2} \tag{21}$$

and the $t$ in Eqs. (18) and (19) is the 95% confidence $t$-value from the $t$ distribution for $v_r$ degrees of freedom given by

$$v_r = \frac{(0, S_r^4)}{\sum_{i=1}^J \left( \eta_i S_r^2 / \beta_i \right)} \tag{22}$$

**COMPARISON OF UNCERTAINTY ANALYSIS APPROACHES**

Consideration of Eqs. (18)-(22) in the context of the ISO approach, which can be rigorously derived (Coleman and Steele, 1995), shows that they cannot be justified. The $U_{ADD}$ approach has always been advanced on the basis of *ad hoc* arguments and with results from a few Monte Carlo simulations, but (as argued in the ISO Guide) for a 99% confidence level, the $t$-value appropriate for 99% confidence should be used as the value of $K$ in Eq. (6) to obtain a 99% confidence estimate for an assumed Gaussian distribution. This was also shown by Steele et al. (1994), who concluded on the basis of a number of Monte Carlo simulations that the "$U_{ADD}$ model for 99% coverage is erratic and very sensitive to the relative magnitudes of the random and systematic uncertainty contributions.

The determination of the degrees of freedom in order to determine $t$ depends on the number of degrees of freedom in both the precision and bias uncertainty estimates (ISO method), while the Abernethy approach does not consider the influence of the number of degrees of freedom in the bias uncertainty estimates. The Abernethy et al. approaches also do not account for the possibility of correlated bias error effects (taken into account in the $B_k$ covariance terms in Eq. (15)) or correlated precision error effects (taken into account in the $S_k$ terms in Eq. (16)). The importance of correlated bias error effects has been shown by Coleman et al. (1995) and Coleman and Steele (1989), and the importance of considering correlated precision error effects has been demonstrated by Hudson et al. (1995).

**CONCLUSION**

The method (Abernethy and Thompson, 1973) that has been used for determination of experimental uncertainties by the turbine testing community has been superseded by a more comprehensive approach that can be justified on a rigorous basis. The new approach (ISO, 1993; AGARD, 1994; AIAA, 1995) should be adopted for estimating uncertainties in future turbine testing programs.

**REFERENCES**


American National Standards Institute/American Society of Mechanical Engineers (1986), Measurement Uncertainty, PTC 19.1-1985 Part 1, ASME.


