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NEW NON-DIMENSIONAL PARAMETERS IN FLUID MECHANICS AND THEIR APPLICATION TO TURBINE FLOWMETER DATA ANALYSIS

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ABSTRACT

Dimensional analysis has been used in experimental fluid mechanics for over a hundred years. Controllable and uncontrollable variables in an experiment can be efficiently organized into nondimensional groups or parameters. Such nondimensional parameters are used for geometric scaling, and for developing dynamic similitude in experimental processes. Commonly used nondimensional parameters in fluid mechanics include Reynold's No., Mach No., Froude No., Weber No., Strouhal No., etc. Most modern text books and technical papers discuss the use of Buckingham Pi Theorem for developing the nondimensionalization process. An often ignored and somewhat older technique is the Rayleigh Method. Both the Pi Theorem and the Rayleigh Method are founded on the Principle of Dimensional Homogeneity, and require some experience in the grouping of physical variables. The present paper uses the Rayleigh method to develop two new nondimensional parameters. A discussion is presented about the use of the parameters in the application of turbine flowmeter calibration and test data analysis. It is shown that data analysis for turbine flowmeters is considerably simplified by the use of the new parameters.

1 INTRODUCTION

The basic principle of the operation of turbine flowmeters in fluid flow measurement has been in use for over 400 years (Baker, 1991). The early uses were more for the measurement of speed and distance-log than for fluid volumetric flow-rate. A typical present-day flowmeter consists of a bladed rotor supported by bearings in a housing with a magnetic (mag), or modulated carrier (RF) pick-off to generate pulses corresponding to the rotation of the rotor in the fluid stream. The frequency of the pulse-stream is proportional to the flow velocity, within some limitations of operating conditions. Thus, when installed in a pipe, the volumetric flowrate can be determined from the measurement of the output frequency. The volume flowrate can be converted to mass flowrate if the fluid density is known. Turbine flowmeters are widely used in the industries of aerospace, oil and gas, automobile, etc. because of their reasonable cost, good accuracy, operational simplicity and durability in field usage.

Turbine flowmeters are calibrated to determine the frequency-flowrate relationship. This relationship is a function of the temperature and pressure of the fluid which govern fluid properties and the geometric dimensions of the meter. In general, the operating conditions are different from the calibration-conditions, thus requiring the development of a correlation technique. The correlation model is used to process the calibration data, as well as the meter output frequency in operation, to determine the volume flowrate at operating conditions. Several models are used for the characterization of the frequency-flowrate relationship. The most comprehensive model is the Strouhal-Roshko characterization (Mattingly, 1992). Despite recent attempts to standardize this model (SAE, 1997), it is the least frequently used because of the inherent complexity in translating it into an algorithm for automated data acquisition systems (ADAS). This led to the current effort to develop a new correlation model.

The following sections give a brief overview of the existing correlation models with their limitations, and describe the development of the new model resulting in two new non-dimensional parameters. Sample case studies using the new parameters are presented with suggested use of the parameters in other areas of applied fluid mechanics.

2 EXISTING CORRELATION MODELS

Several correlation models are used to characterize the frequency-flowrate relationship for a turbine flowmeter. It is an acceptable practice to consider the fundamental dimensions in applied mechanics as Mass (M), Length (L), and Time (T). Assuming incompressible operating conditions and negligible mechanical and electromagnetic forces (viz. negligible bearing

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and magnetic drag) for a flowmeter, the controllable parameters are the volumetric flowrate (Q, with the associated dimension of L^3T^{-1}), the fluid density (ρ, with the dimension of ML^{-3}), and the fluid absolute viscosity (μ, with the dimension of $ML^{-1}T^{-1}$). The uncontrollable parameters are the meter bore-diameter (D, with the dimension of L), and the output frequency (f, the response function and a characteristic of the meter, with the dimension of T^{-1}). It may then be stated (Hochreiter, 1957) that a correlation model can be developed on the basis of the functional relationship

$$Q = fn\,(f, D, \rho, \mu) \qquad (1)$$

A methodically developed model for dynamic similitude of the characteristics of a meter must include these five dimensional parameters. The goal of the correlation model is to develop the characteristics to determine the unknown variable Q from a measurement of the response variable f. It may be noted that conditions (viz. temperature and pressure) that affect ρ also affect μ. Hence, it is a common practice to combine μ and ρ into the ratio of the two, the kinematic viscosity, ν, with the dimension of $L^2\,T^{-1}$. Both ρ and μ, and hence ν, are strong functions of temperature, but are weak functions of pressure. Following is a brief description of the existing correlation models.

2.1 The Constant K-factor Model

The ratio of f and Q is often referred to as the meter K-factor (with the dimension of L^{-3}). It is identical to the ratio of the number of pulses counted over a period of time to the volume flow through the meter over the same time. This model assumes that the meter K-factor is constant over the entire range of the meter in a specific application. This range is also commonly referred to as the linear range of a flowmeter at a specific fluid viscosity. While it may be possible to carefully fine-tune a flowmeter for such linear operation over a limited range, the model suffers from serious limitations. As is evident from Equation (1), the other three parameters, viz., D, ρ and μ (alternatively, D and ν) must remain constant over the range of operation. Hence, the calibration must be done at the same ν as the application fluid (preferably using the same fluid), and the operating temperature and pressure must be held constant.

Figure 1 shows the typical K-factor characteristics of a 0.4-40 Liters per Minute (LPM) turbine flowmeter with RF pickoff. The K-factor is plotted against f at two different ν's, and the plots are generated from calibration data referenced to 15.5°C. The upper curve is at a ν of 1.1 cs and the lower one is at 16.7 cs. The significant difference between the two curves indicates that the constant K-factor model is valid only at one fixed viscosity. In addition, at the lower ν of 1.1 cs, the linear range of the flowmeter is approximately 80 to 2700 Hz, with a tolerance band of ±1.5 percent. The corresponding turn-down ratio is 34:1. However, at the higher viscosity of 16.7 cs, the linear range is narrowed to approximately 795 to 2600 Hz (i.e., a turn-

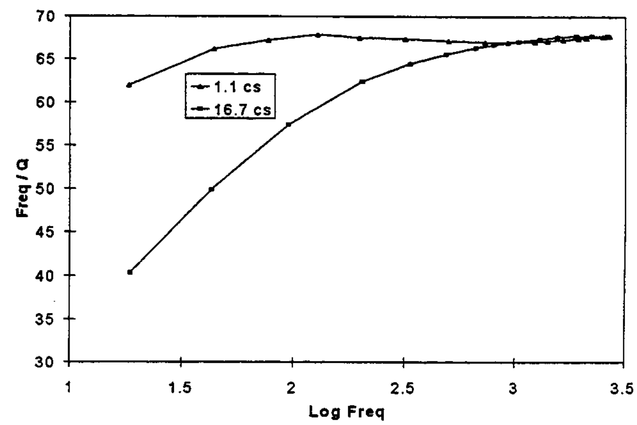

Figure 1 Constant K-factor Model

down ratio of 3:1), keeping the same tolerance band of ±1.5 percent. It is thus extremely important, for this model to be applied, to calibrate the meter at the viscosity (ν) of application, determine the linear range with an acceptable tolerance band of uncertainty, and ensure that the conditions of operation do not result in a change in the viscosity of the fluid. It is a simple model to use in an ADAS, when the resulting measurement uncertainties can be tolerated. However, dynamic similitude is lost in this dimensional model and, when the limitations are not adhered to, the errors in the measured flowrate can climb to several percentage points.

2.2 The Universal Viscosity Curve (UVC) Model

This model characterizes the K-factor as a function of the ratio f/ν (with the associated dimension of L^{-2}). A characteristic curve is created by plotting the K-factor in the y-axis and the ratio f/ν in the x-axis, thus taking into account the effect of kinematic viscosity. Since both the K-factor and f/ν are dimensional quantities, this is a dimensional correlation model. However, as is evident from Equation (1), along with the assumption of incompressibility, only D has to be held constant for the model to be valid. Typically, the flowmeter is calibrated in fluids of two or more ν's covering the expected operational range of ν. The characteristic curves (K-factor vs f/ν) are then plotted from the calibration data, and the curves are blended manually to create a single characteristic curve, i.e., the UVC.

Figure 2 shows the first step towards generating the UVC characteristics for the same flowmeter with the same calibration data as in Section 2.1. The effect of viscosity is now taken into account by using it as a factor in the X-axis. The blended UVC is generated by eliminating the portion of the low viscosity curve that *falls away* from the continuum, and is shown in Figure 3. This leads to some risk of compromising accuracy when using this model at low-viscosity low-flow conditions, which is the part of the curve dropped off while generating the UVC. Even though dynamic similitude is lost, the model works when there is no significant change in D, and it fails when D changes, usually due to changes in pressure and temperature, both of

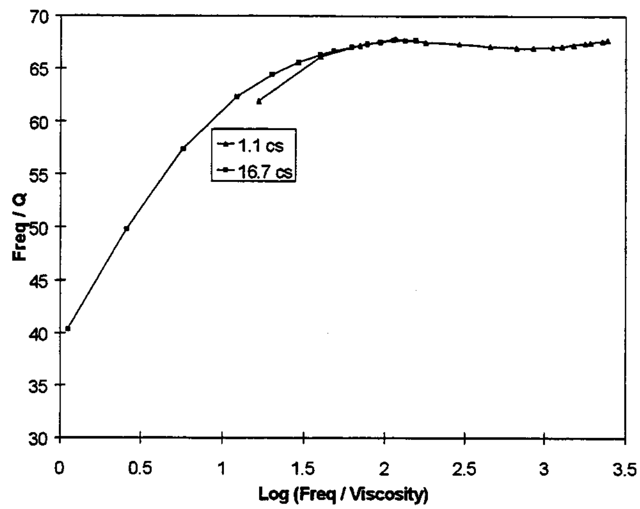

Figure 2 UVC Model Step 1

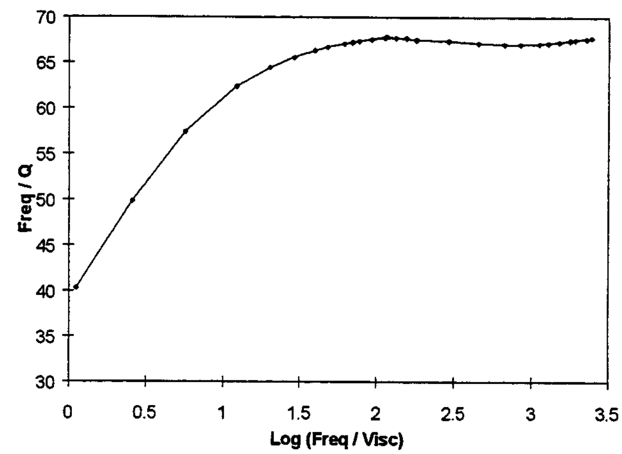

Figure 3 Universal Viscosity Curve (UVC)

which have relatively small effects on D. The model involves manual intervention to blend the curves. The blended UVC is nonlinear in nature resulting in the added complexity of digitizing the curve to create a look-up table in an ADAS. In general, the nature of the curve has been found to be different even within the same family and size of flowmeters. In addition, if the range of viscosity of the application fluid is large, it requires several calibrations at intervals of viscosity to generate the blended UVC.

2.3 The Strouhal-Roshko (St-Ro) Characterization Model

The basic parametric relationships of this nondimensional model were first developed by V. P. Head in 1950 and documented in an unpublished Research Report at Fisher & Porter Company. This is the most comprehensive correlation model and has been widely reported in the available literature (Hochreiter, 1957; Shafer, 1961; Craft, 1992; Mattingly, 1992; Ruffner and Olivier, 1994). This was also included in a recently published Aerospace Recommended Practice (SAE, 1997) in an effort to standardize fuel flow calculation procedures while using turbine flowmeters. The model was developed by using Buckingham Pi Theorem (Buckingham, 1915). A detailed discussion of this theorem is available in most modern text books in Fluid Mechanics in the section on non-dimensionalization. Applying the theorem to the five parameters of Equation (1), two nondimensional parameters may be obtained,

Strouhal Number $$St = f * D^3 / Q \quad (2)$$

Roshko Number $$Ro = f * D^2 / \nu \quad (3)$$

The relationship between the two numbers gives the flowmeter characteristic as

$$St = fn\,(Ro) \quad (4)$$

Since the St-Ro characterization involves all the five parameters of Equation (1), it provides dynamic similitude through the nondimensionalization process. It may be noted that the only difference between this model and the UVC model discussed in Section 2.2 is in the introduction of the D-terms in Equations (2) and (3). Typically, the flowmeter is calibrated in fluids of two or more ν's covering the expected operational range of ν. The measured bore diameter, adjusted for the calibration conditions, is recorded. It may be noted that the measured D at the reference conditions, D_{ref} acts as a scaling factor, with appropriate exponents, for both St and Ro. Hence, D_{ref} can be set to unity without sacrificing the nondimensional characteristics of the correlation model. The St-Ro characterization is plotted for each calibration and the curves are blended manually, or using some analytical curve fitting technique like the cubic-spline algorithm.

Figure 4 shows the St-Ro characterization using the earlier calibration data. Since the calibration was performed at a temperature close to the reference temperature, the difference between Figure 2 and Figure 4 is minimal. It may be noticed in Figure 4, that the curves at the two viscosities tend to form a continuum except for the low-viscosity low-flow area which tends to *fall away*. This is the region of the meter operation where the mechanical forces are no longer negligible compared to the fluid forces. It is difficult to characterize this region because the magnitude of the mechanical forces depends on the lubricity of the fluid and the mechanical condition of the bearings, in addition to the fluid viscosity. To avoid this region and stay on the continuum of the characteristic curve, the concept of f_{min} was introduced (Olivier 1995). Based on the observation of calibration data, f_{min} was defined as the minimum frequency above which the flowmeter stayed on the continuum, and below which the meter should not be used. However, f_{min} depends on the viscosity of the fluid, and it has

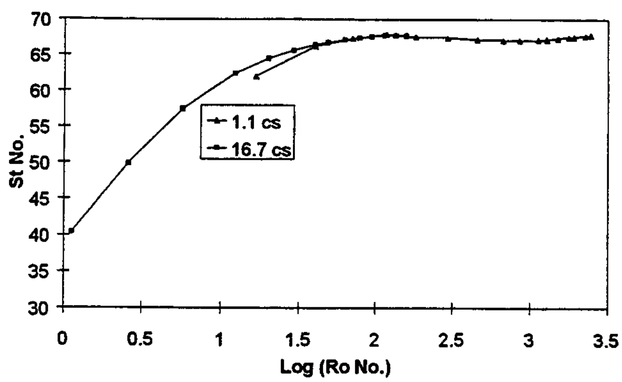

Figure 4 St-Ro Characterization

to be determined for each viscosity - by calibrating the meter at a higher viscosity, plotting the St-Ro curves and then manually determining the point of *falling away*. This technique limits the operational range of the meter by running it above the lower cut-off frequency, f_{min} thus keeping the St-Ro characteristic on the continuum. The resulting curve is similar to the UVC, but is restricted from falling into the low flow departure area. Multiple calibrations are required over the range of viscosity expected during the application, including at least one viscosity above the range for the determination of f_{min}. Also, like the UVC, the curve has been found to be different even within the same family and size of flowmeters. Each such uniquely nonlinear curve needs digitization to generate a look-up table for applications using an ADAS.

3 NEW CORRELATION MODELS

The St-Ro model, as discussed in Section 2.3 above, was developed using the Pi Theorem. An often ignored and somewhat older nondimensionalization technique is the Rayleigh Method (Rayleigh, 1899). Applying this method, Equation (1) may be restated as,

$$Q = K_1 * f^a * D^b * \rho^c * \mu^d \tag{5}$$

where K_1 is a dimensionless constant, and a, b, c and d are exponents of the variables. Substituting the dimensions of the various parameters, the dimensional equation is obtained as,

$$M^0 L^3 T^{-1} = M^{(c+d)} L^{(b-3c-d)} T^{-(a+d)} \tag{6}$$

Now, using the Principle of Dimensional Homogeneity, both sides of Equation (6) may be balanced to yield,

$$\left.\begin{aligned} c + d &= 0 \\ b - 3c - d &= 3 \\ a + d &= 1 \end{aligned}\right\} \tag{7}$$

Equation (7) is a set of three equations with four unknowns. Hence, it may be solved for three of the unknowns in terms of the fourth. Thus, four correlation models can be developed. However, when a, b, and c are expressed in terms of d, and, when a, b, and d are expressed in terms of c, the resulting nondimensional groups are similar to the St-Ro model described in Section 2.3. Hence, the St-Ro model falls out as a sub-set of the four models resulting from the Rayleigh approach. This leaves us with two additional possible models, which have not been reported yet.

3.1 New Correlation Model 1

The terms of Equation (7) may be rearranged to express a, c, and d in terms of b, and substituted in Equation (5). After simplifying, the following expression may be obtained,

$$(Q * f^{0.5} / \nu^{1.5}) = K_1 * \{f * D^2 / \nu\}^{b/2} \tag{8}$$

The nondimensional group on the right side of Equation (8) is Ro as defined in Equation (3). The group on the left side is a new nondimensional group, which may be called Islam Number (Is),

$$\text{Islam Number} \qquad Is = Q * f^{0.5} / \nu^{1.5} \tag{9}$$

Thus, Equation (8) may be restated as,

$$Is = K1 * Ro^{b/2} \tag{10}$$

This, then is a new correlation model involving all the five parameters of Equation (1), resulting in a functional relationship between Is and Ro. A careful study of Equation (10) reveals that in the logarithmic domain, the Is-Ro characterization may be expected to be linear, or close to linear, depending on the variation of the exponent b. An application of this model to the same calibration data for the 0.4-40 LPM flowmeter is discussed in Section 4.

3.2 New Correlation Model 2

As shown in Section 3.1 above, the terms of Equation (7) may also be rearranged to express b, c and d in terms of a. Substitution of the results in Equation (5), and simplification yield,

$$Q / (D * \nu) = K_1 * \{f * D^2 / \nu\}^a \tag{11}$$

The nondimensional group on the right side is the familiar Ro. The group on the left side of Equation (11) is yet another new

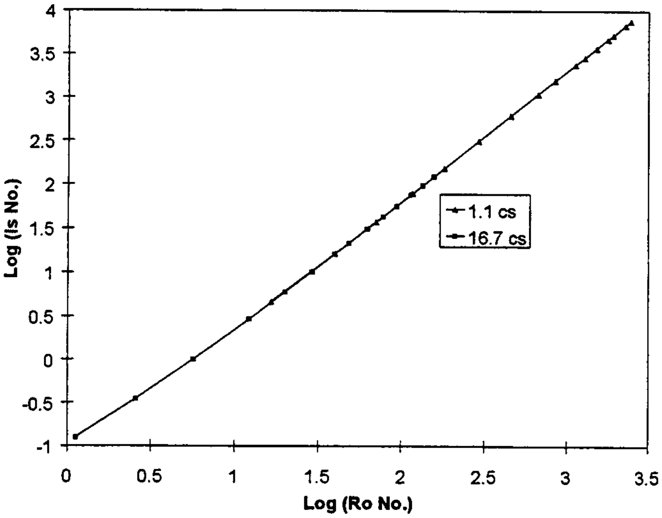

Figure 5 Is-Ro Characterization

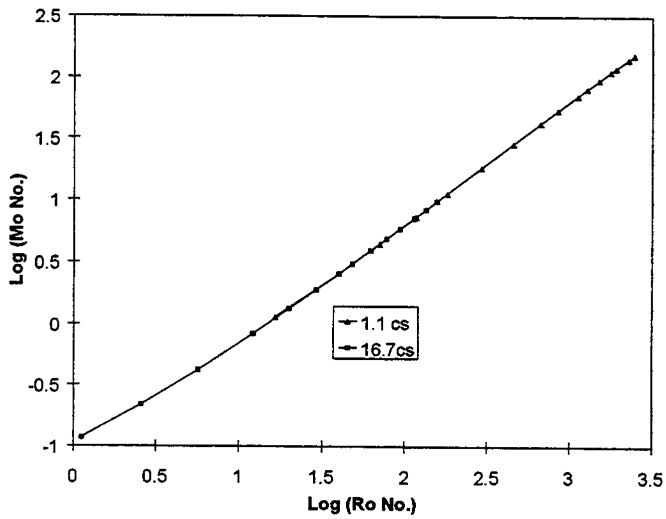

Figure 6 Mo-Ro Characterization

group which may be called Mozumdar Number (Mo),

$$\text{Mozumdar Number} \quad Mo = Q / (D * \nu) \tag{12}$$

Hence, Equation (11) can be rewritten as,

$$Mo = K1 * Ro^{a} \tag{13}$$

This is a second new correlation model involving all the five parameters of Equation (1). The functional relationship between Mo and Ro may be expected to be linear, or close to being such, in the logarithmic domain depending on the variation of the exponent, a. An application of this model to the same calibration data is discussed in the following section.

4 APPLICATION OF NEW CORRELATION MODELS

The same calibration data set from the 0.4-40 LPM turbine flowmeter is used to generate Figure 5, which shows the Is-Ro characterization in the logarithmic domain as given in Section 3.1. As expected, it is close to a straight line. The minor departure from a straight line may be compensated for by using a simple polynomial algorithm of higher order. When data from two or more viscosities are combined into a single set, a fifth order polynomial has been found to yield good results.

Proceeding as before and reprocessing the data for the Mo-Ro correlation, the model shown in Figure 6 is obtained. As expected, in the logarithmic domain, this characterization is also close to a linear representation of the functional relationship between the parameters. In addition, for combined data from two or more viscosities, a fifth order polynomial has been found to yield good results.

In applications where a ±1.5 to ±2 percent curve-fit uncertainty can be accepted, either of the Is-Ro, and the Mo-Ro correlations may be used as linear first order models, i.e., a straight line fit may be used. Where a higher order of accuracy is desired, the fifth order polynomial may be used. It has also been found that both the models hold for families of flowmeters of different sizes. In addition, where the variation in fluid viscosity is large, it is possible to establish the correlations with fewer calibrations at different viscosities.

5 SUMMARY & CONCLUSIONS

Two new correlation models have been developed for turbine flowmeter data analysis. This was achieved by adopting a systematic approach using the Rayleigh Method along with the Principle of Dimensional Homogeneity resulting in two new nondimensional parameters.

Nondimensional parameters may be created by the combination, or by the manipulation of existing nondimensional parameters. For example, Roshko Number can be arrived at by the multiplication of Strouhal Number and Reynolds Number (Craft, 1992; Mattingly, 1992; Ruffner and Olivier, 1994). Similarly, multiplying the numerator and the denominator of an existing nondimensional number by two parameters of identical dimensions can also lead to a new nondimensional parameter. Evidently, the different approaches to dimensional analysis are not comprehensive. Most fluid dynamicists believe that the Buckingham Pi Theorem is the most effective means of dimensional analysis and hence, it has become the method of choice. However, the Pi theorem requires considerable insight into the physics of a process along with experience in grouping controllable and uncontrollable variables in an application. There is great confusion about the rules concerning the number of dimensional parameters while applying the Pi theorem (Granger 1985). Though generally avoided, the Rayleigh method is easier to apply than the Pi theorem. In the present application, the St-Ro characterization, originally developed using the Pi theorem, came out as a subset of the correlations from the approach using the Rayleigh method. However, the *vice versa* is not true, i.e. neither the Is-Ro, nor the Mo-Ro correlation could be arrived at using the Pi theorem. It may be noted that the creation of nondimensional parameters does not lead to the development of effective correlation models in experimental fluid mechanics. The development of correlation models requires a methodical and systematic approach.

The new correlation models have been successfully applied to several flowmeters of different sizes. Both the models overcome many of the limitations of the existing models. It may be recognized, that while the new models offer significant benefits over the existing models, they do not, by themselves, change the measurement uncertainties. As in the St-Ro characterization, the new Is-Ro and the Mo-Ro models capture all the parameters from Equation (1), within the assumptions of incompressibility and the negligibility of mechanical forces compared to fluid forces. In specific, the new models reduce the required number of calibrations over the operating range of the application fluid, and simplify the algorithm for ADAS thus providing the potential to reduce curve-fit errors.

The newly formed Is-number is a non-dimensional combination of three physical parameters, viz., fluid kinematic viscosity, frequency and volume flowrate. Thus, Is-number may be used in other applications where these physical parameters come into play, e.g., in Karman vortex street or other vortex shedding phenomena. Similarly, Mo-mumber is also a non-dimensional combination of three physical parameters, viz., fluid kinematic viscosity, volume flowrate and a characteristic linear dimension like diameter, or length. Hence, Mo-number may also be used in other applications where these parameters play a role in governing the flow phenomena.

In conclusion, it may be noted that both the new correlation models lead to considerable simplifications in the analysis of turbine flowmeter data. The authors have not recommended one model over the other as this may depend on operational simplicity in a specific application. However, both the models can be effectively used with relatively simple polynomial curve-fit algorithms in automated data acquisition systems.

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