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MEASUREMENT OF TRANSFER MATRICES AND SOURCE TERMS OF PREMIXED FLAMES

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ABSTRACT

An experimental method to determine the thermoacoustic properties of a gas turbine combustor using a lean-premixed low emission swirl stabilized burner is presented. To model thermoacoustic oscillations, a combustion system can be described as a network of acoustic elements, representing for example fuel and air supply, burner and flame, combustor, cooling channels, suitable terminations, etc. For most of these elements, simple analytical models provide an adequate description of their thermoacoustic properties. However, the complex response of burner and flame (involving a three-dimensional flow field, recirculation zones, flow instabilities and heat release) to acoustic perturbations has - at least in a first step - to be determined by experiment. In our approach, we describe the burner as an active acoustical two-port, where the state variables pressure and velocity at the inlet and the outlet of the two port are coupled via a four element transfer matrix. This approach is similar to the "black box" theory in communication engineering. To determine all four transfer matrix coefficients, two test states, wbich are independent in the state vectors, have to be created. This is achieved by using acoustic excitation by loudspeakers upstream and downstream of the burner, respectively. In addition, the burner migbt act as an acoustic source, emitting acoustic waves due to an unsteady combustion process. The source characteristics were determined by using a third test state, which again must be independent from the two other state vectors. In application to a full size gas turbine burner, the method's accuracy was tested in a first step without combustion and the results were compared to an analytical model for the burner's acoustic properties. Then the method was used to determine the burner transfer matrix with combustion. An experimental swirl stabilized premixed gas-turbine burner was used for this purpose. The treatment of burners as acoustic two-ports with feedback including a source term and the experimental determination of the burner transfer matrix is novel.

NOMENCLATURE

A cross sectional area
L_{red} reduced length
M Mach number
S scattering matrix
T transfer matrix

c speed of sound
f Riemann invariant traveling in downstream direction
g Riemann invariant traveling in upstream direction
k wave number
p pressure fluctuations
v velocity fluctuations
x axial position

ω angular frequency
$_u$ upstream
$_d$ downstream

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INTRODUCTION

Thermoacoustic combustion instabilities are a cause for concern in many combustion applications such as household burners, gas turbines and rocket engines. Combustion instabilities can not only increase emissions of noise or pollutants such as unburnt hydrocarbons or oxides of nitrogen but also lead to very high pressure pulsations, resulting in structural damage of the combustor. If they occur, combustion instabilities may interfere with the operation of the combustion system or limit at least its operational range. Pressure amplitudes are i.e. in the order of 1% of the mean operating pressure but can also be as large as 10% of the mean operating pressure as observed in solid propellant rocket motors (Raun *et al.*, 1993). As the oscillation amplitudes scale linearly with the mean pressure and operating pressure ranges up to 35 MPa, the resulting pressure oscillation can reach considerable amplitudes, on the order of atmospheric pressure. For stationary gas-turbines the drive for lower emissions of oxides of nitrogen has lead during the last decade to the wide spread of lean premix burners and convectively cooled combustion chambers. These technological changes have resulted in a reduced stability of flame anchoring and lower acoustic damping. Consequently, modern gas turbines are more susceptible to combustion driven oscillations and the importance of thermoacoustic phenomena in gas turbine combustors has increased sharply.

Thermoacoustic instabilities involve a feedback cycle comprising fluctuations in acoustic pressure, velocity, heat release rate and the acoustic characteristics of the combustor and air supply, which act as resonators. Lord Rayleigh (1878) deduced a fundamental stability criterion for thermoacoustic oscillations: non steady heat release can enhance acoustic oscillations if the heat release rate peaks at the moment of greatest compression. Although this criterion provides merely a necessary but not a sufficient condition for instability, as it does not take into account the stabilizing influence of dissipation of acoustic energy, it expresses essential physical insight into the problem of thermoacoustic instability. The Rayleigh criterion also illustrates that the stability analysis of a combustion system requires detailed knowledge of the phase relationship between pressure oscillations and the various processes controlling the rate of heat release.

Thermoacoustic systems can be represented conveniently as a network of acoustic elements which correspond to various components of the system, e.g. fuel and air supply, burner and flame, combustor, cooling channels, suitable terminations, etc. (Keller, 1995; Polifke, Paschereit and Sattelmayer, 1997). This approach has often been used in the acoustic analysis of ducts and mufflers (Munjal, 1986). The elements in such a network can be described as acoustic multi-ports. For many of these elements, simple analytical models, based on linearization of the governing equations, provide an adequate description of their thermoacoustic properties. However, the complex response of burner and flame to acoustic perturbations has - at least in a first step - to be determined by experiment. In our approach the "black box" theory used in communications engineering has been extended to treat problems of combustion generated sound. This method has been successfully applied to describe fluid machines as acoustical multi ports (e.g. Cremer 1971, Bodén and Åbom 1995, Lavrentjev and Åbom 1996). We describe the burner and the flame as an active acoustical two-port, where the state variables pressure and velocity at the inlet and the outlet are coupled via a four element transfer matrix (Paschereit and Polifke, 1997).

The interaction between large scale coherent structures which are related to flow instabilities, the heat release and the combustor acoustics was shown to be a driving mechanism leading to unstable combustion (Shadow and Gutmark 1992, McManus et al. 1993, Paschereit et al. 1998a, 1998b, 1998c, 1998d). Theoretical flame models have been derived to describe the relationship of unsteady heat release, acoustics and flow perturbations, e.g., in afterburners (Bloxsidge et al., 1987), flat flames in open tubes (McIntosh, 1990) and in simple experimental premix burners (Janus and Richards, 1996). However, the highly three-dimensional flow field of a swirl stabilized burner and the interaction between heat release and the flow field still requires experimental determination.

The suggested approach of experimental determination of the burner transfer function in this paper can be used to shed light into flame/acoustic interaction and will allow the development of an empirical flame model for swirl stabilized premixed combustion.

NOISE GENERATION IN COMBUSTORS

An overview illustrating some basic mechanisms of thermoacoustic instability is given in Figure 1. The definition of a simple combustion system consisting of a hood, the swirl stabilized burner and the combustion chamber is given in Figure 2. Burner flow turbulence and flow instabilities (which are manifested in the shedding of vortices) influence the momentary rate of fuel consumption at the flame front. As a consequence, fluctuations in heat release occur that result in fluctuations in pressure and velocity. The downstream traveling waves will be reflected by the acoustic boundary conditions of the combustor exit and lead to pressure and velocity oscillations at the burner exit. This feedback cycle will again cause fluctuations in heat release. Furthermore, the fluctuations in pressure and velocity may result in inhomogeneities in the equivalence ratio, which are convected by the mean flow towards the flame. As

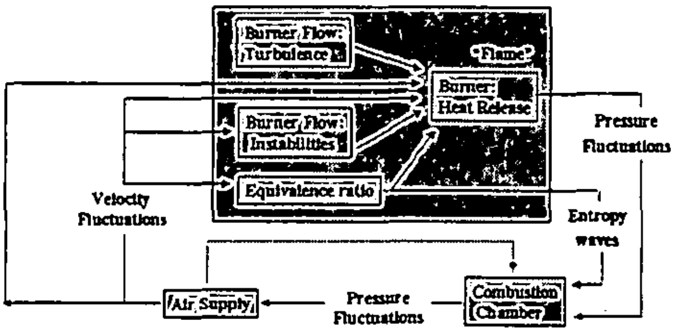

Figure 1. Noise Generation in Combustors.

these inhomogeneities are consumed by the flame, they will influence the momentary heat release rate and therefore, also the rate of volume production, which couples via the impedance at the burner exit with the system acoustics. In addition to this direct interaction, there is a second mechanism that couples equivalence ratio fluctuations with the system acoustics: the adiabatic flame temperature of richer (leaner) "pockets" of mixture is higher (lower) than average. Thus, equivalence ratio fluctuations will lead to fluctuations in the hot gas temperature downstream of the flame, i.e. so called entropy waves. The entropy waves are convected downstream by the mean flow and couple at the choked combustor exit with the system acoustics. Entropy waves are considered in our simulation of the combustor but will not be discussed in this paper.

MODELING OF THERMOACOUSTIC SYSTEMS

The Network Representation of Thermoacoustic Systems

A thermoacoustic system can be represented as a network of acoustic elements, which correspond to various components of the system, e.g., ducts, area changes, nozzles and diffusers and the burner with flame. The network representation of a generic gas turbine combustor is shown in Figure 2.

Under the assumption of harmonic time disturbances $\exp(i\omega t)$, the unknowns of the system are the amplitudes of acoustic variables at the end points of these elements. The equations of linear acoustics are then used to generate coupling relations - i.e. fluctuations of velocity v and pressure p - across every element. As a simple example, consider a duct of length L. The propagation of acoustic waves traveling in up- and downstream direction yields the

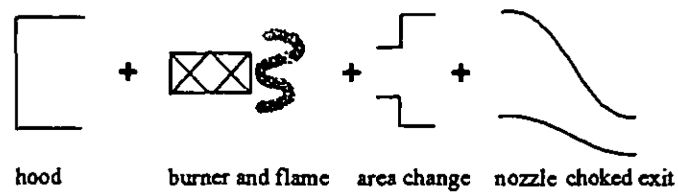

Figure 2. Example of modeling the combustor as a network of acoustic elements.

relation

$$\begin{pmatrix} p_d \\ v_d \end{pmatrix} = \begin{pmatrix} \cos(kL) & iZ_0 \sin(kL) \\ \frac{1}{Z_0} i \sin(kL) & \cos(kL) \end{pmatrix} \begin{pmatrix} p_u \\ v_u \end{pmatrix}, \qquad (1)$$

between pressure p and velocity v at the up- and downstream end (subscripted u and d, respectively). Here, $Z_0 = \rho c$ is the characteristic impedance and k is the wave number, equal to the ratio of angular frequency ω and sound speed c in the case of plane waves without mean flow and dissipative effects. The matrix in equation (1) is called the transfer matrix T of the element.

Each of the elements in Figure 2 relates acoustic pressure and velocity (or incoming and outgoing Riemann invariants, which represent the up- and downstream traveling waves) on both sides in a simple algebraic way as shown in Figure 3. Physically it does not matter if an element relates the Riemann-invariants or the acoustic pressure and velocity, since they can be related to each other using equations 2 and 3

$$\frac{p(\omega)}{\rho c} = f + g, \qquad (2)$$

$$v(\omega) = f - g. \qquad (3)$$

It is merely a matter of convention as to which of these representations is used.

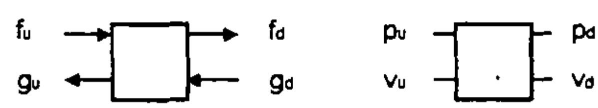

Figure 3. Acoustic elements relating Riemann-invariants or pressure and velocity fluctuations on both sides of the element.

The representation of the transfer matrix of a simple tube in terms of Riemann invariants is

$$\begin{pmatrix} f_d \\ g_d \end{pmatrix} = \begin{pmatrix} e^{-ikL} & 0 \\ 0 & e^{ikL} \end{pmatrix} \begin{pmatrix} f_u \\ g_u \end{pmatrix}, \qquad (4)$$

Equivalently, the Riemann invariants can be related by a scattering matrix

$$\begin{pmatrix} f_d \\ g_u \end{pmatrix} = \begin{pmatrix} e^{-ikL} & 0 \\ 0 & e^{-ikL} \end{pmatrix} \begin{pmatrix} f_u \\ g_d \end{pmatrix}, \tag{5}$$

which relates the incoming signal (f_u, g_d) to the outgoing signal (f_d, g_u).

The thermoacoustic description of a complete combustion system of equations $\mathbf{S} \cdot \mathbf{u} = \mathbf{d}$ is obtained by combining the coefficients of all the transfer matrices of all elements into one System matrix $\mathbf{S}$ and all unknowns in a vector $\mathbf{u}$. The vector $\mathbf{d}$ represents the driving forces in the system.

The Flame Transfer Function

Noise generation problems in enclosures like duct systems are of special interest because of the interaction (feedback) of the wave field in the enclosure with the source. This becomes especially true in thermoacoustic problems, where a self excitation mechanism resulting from the interaction of the sound field in the enclosure and the burner as a sound source may lead to high pulsation amplitudes. To model noise generation problems, especially those involving a feedback cycle as in thermoacoustics, it is essential to characterize the noise sources. Their strength and interaction with the acoustic wave field have to be known.

Many noise sources can be calculated from first principles. However, the highly three-dimensional flow field of a swirl stabilized burner and the interaction between heat release and the flow field still requires experimental determination. One possibility which is especially useful in the low frequency regime is to describe a burner as an active acoustic two-port. In this approach the burner is treated as a "black box". Questions about the internal character of the system can then be put aside. However, the data obtained for the burner gives valuable information for the formulation of premix flame models.

In the acoustic one-port model the burner and the upstream conditions are treated as a black box (Fig. 4). For a measured source impedance Z_s and source strength v_d the upstream conditions are not allowed to change because they will influence these quantities. For practical applications this implies that the upstream acoustic boundary conditions being dictated by the plenum in which the burner is mounted as well as everything upstream of the plenum has to remain unchanged (as long as they are acoustically coupled). Only then, can the measured results be used to model the combustor.

The shortcoming of the one-port treatment of the burner can be overcome by describing *only* the burner as a black box allowing "two ports", designated as input and

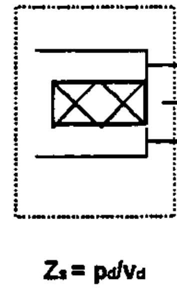

Figure 4. A burner modeled as an acoustic one-port.

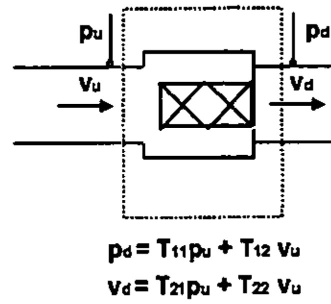

Figure 5. A burner modeled as an acoustic two-port.

output (Fig 5). Even more ports are allowed, if, e.g., fluctuations in the fuel supply rate have a reasonable influence on the acoustic behavior of the burner. In the frequency domain the two-port can be described by the equation

$$\begin{pmatrix} p_d \\ v_d \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} p_u \\ v_u \end{pmatrix}. \tag{6}$$

Here the subscripts u and d indicate upstream and downstream pressures p and velocities v respectively.

In the transfer matrix above the noise generation of the flame has not been considered. In order to include a source term the representation in terms of Riemann invarients is more convenient. The principle is shown in figure 6. In

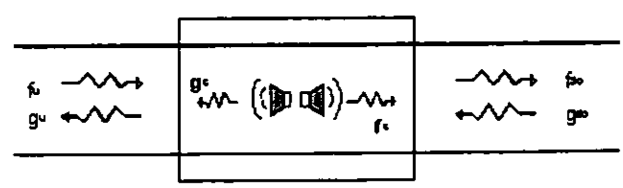

Figure 6. Representation of the flame as a sound source.

addition to merely coupling the Riemann invariants across the burner (and flame), the burner itself may act as a noise source generating waves f_s and g_s. These waves which are emitted by the burner are uncorrelated with the soundfield in the combustor, i.e., they do not depend on the incoming waves f_u and g_d and are thus not included in the transfer matrix T. This yields the following relation

$$\begin{pmatrix} f_d \\ g_u \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} f_u \\ g_d \end{pmatrix} + \begin{pmatrix} f_s \\ g_s \end{pmatrix}. \tag{7}$$

The matrix in the above representation is the scattering matrix S as described by equation 5 in the previous section.

Equation 7 is the complete description of an active two-port with a source term. For each combination of incoming signals (f_u, g_d) it provides the outgoing signal (f_d, g_u). Note that for $f_u = g_d = 0$ the outgoing signal is only the source (f_s, g_s). The problem here is how to determine the six unknowns (four elements of the scattering matrix and two source terms) in equation 7 by simply measuring acoustic pressures? A detailed answer on how to separate the different terms will be given in the next section, however, since the system consists of only two equations, three independent test states are required to solve for the unknowns. Different methods to create these different test states are described in Paschereit and Polifke (1998). Here, the test states were created by forcing upstream of the burner, then downstream and successively at both sides of the burner at the same time.

EXPERIMENTAL METHODS

Experimental Set-Up

The combustion facility is shown in Figure 7. The atmospheric test rig consists of a plenum chamber upstream of the swirl-inducing burner and a combustion chamber downstream of the burner. The plenum chamber contains perforated plates to reduce the turbulence level of the flow. The circular combustion chamber consists of an air cooled double wall quartz glass to provide full visual access to the flame. The exhaust system is an air cooled tube with the same cross-section as the combustion chamber to avoid acoustic reflections at area discontinuities. The acoustic boundary conditions of the exhaust system could be adjusted from almost anechoic (reflection coefficient $|r| < 0.15$) to open end reflection. An experimental swirl stabilized burner was used in the experiments which were performed using two operational modes - premixed combustion and partially premixed combustion (quasi diffusion flame). In the premixed mode, natural gas was injected upstream of the swirling air to premix the fuel with the air. The flame was stabilized in a recirculation region near the burner outlet. During non-premixed combustion a pilot flame was utilized and the fuel was injected into the recirculation region resulting in a quasi diffusion flame. Controlled excitation of the burner flow was accomplished by a circumferential array of four loudspeakers equally spaced in polar angle. One set of loudspeakers was placed at a distance of $x/D = 4.2$ upstream of the dump plane and the second set at $x/D = 9.6$ downstream of the dump plane. The nominal power of the 330 mm speakers was 120 W. Only axisymmetric excitation with the upstream loudspeakers was used and the loudspeakers were operated at zero phase difference.

Pressure fluctuations were measured using water-cooled 1/4" condenser microphones.

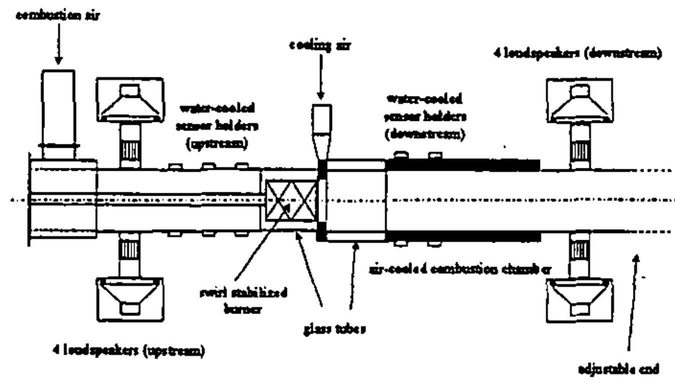

Figure 7. Experimental arrangement of the combustor.

Measurement of acoustic quantities in ducts with mean flow

To determine the thermoacoustic characteristics of the burner/flame the up- and downstream propagating waves (Riemann invarients) have to be determined. At least two microphones are required for this type of measurement. Using two microphones at two different axial positions x_1 and x_2 in a tube the Riemann invarients f and g at cross section x_1 are given by

$$\begin{pmatrix} f(x_1) \\ g(x_1) \end{pmatrix} = \frac{1}{\Phi^- - \Phi^+} \begin{pmatrix} \Phi^- & -1 \\ -\Phi^+ & 1 \end{pmatrix} \begin{pmatrix} p(x_1) \\ p(x_2) \end{pmatrix}, \qquad (8)$$

where $\Phi^\pm = \exp(-ik_\pm(x_2 - x_1))$ and $k_\pm = \frac{\omega/c}{M \pm 1}$. Once, the Riemann invarients are known at location x_1 they can be stepped into any location x_i, using the relations

$$\begin{aligned} f(x_i) &= f(x_1)e^{-ik_+(x_i - x_1)}, \\ g(x_i) &= g(x_1)e^{-ik_-(x_i - x_1)}. \end{aligned} \qquad (9)$$

In our approach an extension of the two-microphone method, the multi microphone method, is used to improve the accuracy of the measured data. Using two pressure signals it is possible to measure exactly the two quantities, i.e., the incident and the reflected wave components. By making more pressure measurements the number of equations is larger than the number of unknowns, thus the problem is overdetermined. The calculated incident $f(x_i)$ and reflected $g(x_i)$ wave components are fitted to the measured quantities

$p(x_i)$ by using the non-linear Levenberg-Marquardt method to minimize the χ^2 quantity in the frequency domain

$$\chi^2 = \sum_{i=0}^{N-1} [(f(x_i) + g(x_i)) - p(x_i)]^2, \tag{10}$$

where N corresponds to the number of microphones used for the multi microphone method.

Determination of the flame transfer function

Assume that the response of the flame depends only on the acoustic field incident upon the burner the flame transfer function can be determined by using, e.g., the two source method. Here, acoustic forcing upstream and downstream of the flame provides two different acoustic test states to furnish a system of four linear independent equations with a unique solution for the coefficients of T:

$$\begin{pmatrix} p_u^{(1)} \\ u_u^{(1)} \\ p_u^{(2)} \\ u_u^{(2)} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} & 0 & 0 \\ T_{21} & T_{22} & 0 & 0 \\ 0 & 0 & T_{11} & T_{12} \\ 0 & 0 & T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} p_d^{(1)} \\ u_d^{(1)} \\ p_d^{(2)} \\ u_d^{(2)} \end{pmatrix}, \tag{11}$$

where the superscripts refer to the two test states (1) and (2).

The method described above is only applicable if the acoustic field incident upon the burner was only the result of the loudspeakers. However, the turbulent flow through the burner and flow instabilities traveling into the flame may generate sound waves, which are incoherent with the forcing signal, so that the burner itself acts as a sound source. Thus the method to measure the flame transfer function works only when the signal brought in by the loudspeakers exceeds the noise generated by flow and combustion. A method has been developed which deals with this problem. Here, the pressure signal picked up by the microphone is considered to be the sum of three different contributons (Figure 8):

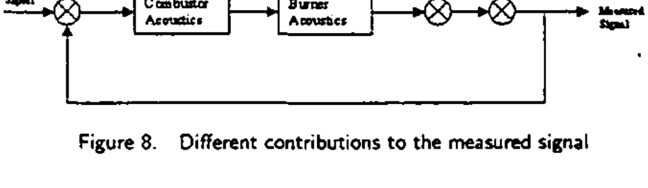

Figure 8. Different contributions to the measured signal

- correlated signal: this contribution is correlated with the excitation signal. These are the signals needed to determine the transfer matrix. This signal contains the passive contribution of the burner/flame to an acoustic excitation. Compared to a simple acoustic element such as the duct described in the previous section, the burner/flame might also actively respond to the excitation. A feedback mechanism between, e. g., acoustic waves and flow instabilities (or equivalence ratio fluctuations) exists which might trigger a periodic combustion as the vortices are convected into the flame. Consequently the transfer matrix also contains an active part which is coupled to the sound field.
- coherent uncorrelated signal: this is sound generated by the flame with a fixed frequency and phase. It is uncorrelated with the forcing signal but is coherent between multiple axially distributed microphones and not random. This contribution is referred to as the source term, and consists of sound generated by the flame or the flow field in a periodic - not random - way. This contribution is, e. g., generated by naturally occurring flow instabilities forming vortices which in turn influence the heat release of the flame. Equivalence ratio fluctuations leading to (periodic) unsteady heat release as discussed in Figure 1 may be an additional mechanism leading to sound generation by the flame
- random noise: this contribution is uncorrelated with the forcing signal and has no periodic component. It can be attributed to the turbulent flow and subsequent combustion noise. The length scale of this contribution is much smaller than the axial spacing of the microphones. As a consequence there is no coherence between multiple axial distributed microphones

In order to measure the transfer matrix of the flame and a possible source term a decomposition of the different contributing signals has to be performed. To determine the transfer function the only part of the signal needed is the one which is correlated with the forcing signal. All other contributions are part of the source term.

The uncorrelated coherent signal and the random noise, which is generated by turbulence and combustion, can be eliminated from the measured signal by a cross correlation between the measured signals and the forcing signal. This measurement gives the scattering matrix which contains no source term. The source term can then be determined by an additional measurement without any excitation and a subsequent substraction of measured signals and calculated signals using the "source free" scattering matrix which was described above.

A more reliable method is to use three different acoustic test states to solve equation 7 for the six unknowns. This is the method which was applied here. By subtracting two different test states (1) and (2) from each other the source term can be eliminated, so that

$$\begin{pmatrix} f_d^{(1)} - f_d^{(2)} \\ g_u^{(1)} - g_u^{(2)} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} f_u^{(1)} - f_u^{(2)} \\ g_d^{(1)} - g_d^{(2)} \end{pmatrix}. \quad (12)$$

Dividing both sides by $f_u^{(1)} - f_u^{(2)}$ and repeating the operation with test state (2) and (3) leads to the following set of four equations so that the system can be solved for the four unknowns:

$$\begin{aligned} \frac{f_d^{(1)} - f_d^{(2)}}{f_u^{(1)} - f_u^{(2)}} &= S_{11} + S_{12}\frac{g_d^{(1)} - g_d^{(2)}}{f_u^{(1)} - f_u^{(2)}}, \\ \frac{g_u^{(1)} - g_u^{(2)}}{f_u^{(1)} - f_u^{(2)}} &= S_{21} + S_{22}\frac{g_d^{(1)} - g_d^{(2)}}{f_u^{(1)} - f_u^{(2)}}, \\ \frac{f_d^{(2)} - f_d^{(3)}}{f_u^{(2)} - f_u^{(3)}} &= S_{11} + S_{12}\frac{g_d^{(2)} - g_d^{(3)}}{f_u^{(2)} - f_u^{(3)}}, \\ \frac{g_u^{(2)} - g_u^{(3)}}{f_u^{(2)} - f_u^{(3)}} &= S_{21} + S_{22}\frac{g_d^{(2)} - g_d^{(3)}}{f_u^{(2)} - f_u^{(3)}}. \end{aligned} \quad (13)$$

Note, that this methods works as well if more than three test states are available. The equations can then be solved by applying the method of least squares. Once the scattering matrix is found, the source term can be determined by

$$\begin{pmatrix} f_s \\ g_s \end{pmatrix} = \begin{pmatrix} f_d \\ g_u \end{pmatrix} - \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} f_u \\ g_d \end{pmatrix}. \quad (14)$$

From the source free scattering matrix the transfer matrix can be determined by a simple matrix transformation.

RESULTS AND DISCUSSION

The transfer matrix without combustion

In a first step the transfer matrix was measured without combustion and without flow using a pure tone excitation. A model based on the non-steady Bernoulli equation for compact elements has been developed (Paschereit and Polifke 1998)and led to the following theoretical coefficients of the transfer matrix

$$T = \begin{pmatrix} 1 & \rho_u c_u \left[M_u \left(1 - \zeta - \left(\frac{A_u}{A_d}\right)^2 \right) - i\frac{\omega}{c} L_{red} \right] \\ 0 & \frac{A_u}{A_d} \end{pmatrix}, \quad (15)$$

where p and u are set as variables. The term A_u/A_d is the ratio between the areas upstream and downstream of the burner, ζ the pressure loss coefficient of the burner, L_{red} a reduced length accounting for inertia effects of the accelerated air column within the burner, ρ_u the density upstream of the burner and c_u the speed of sound upstream of the burner. Good agreement between this theoretical model was found for low frequencies and confirmed the validity of the measurement technique. As acoustic radiation and scattering effects might become important for higher frequencies and have not been considered in the model a deviation between measured and analytical results for the higher frequencies could be observed.

The Transfer Matrix with Combustion

To determine the transfer matrix with combustion a total of seven microphones was used. Three microphones were located at different axial locations upstream of the burner and four microphones were located downstream of the burner. To obtain a low background noise level the combustion chamber was equipped with an anechoic end. The multiple source (forcing) method was used to create three different test states, i.e., forcing was applied either upstream or downstream of the burner or at both sides of the burner simultaneously.

The coherence between the forcing signal and the measured microphone signals is an indicator for the data quality. The coherence between the forcing signal and the closest mi-

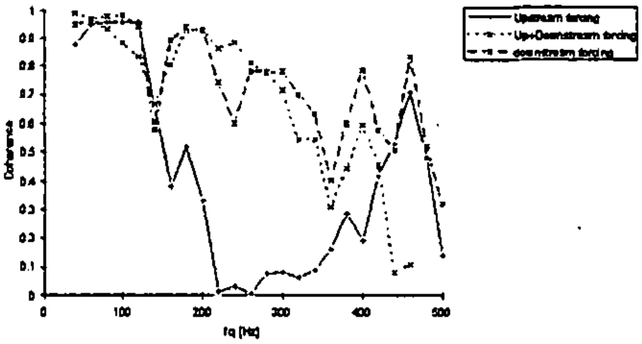

Figure 9. Coherence function $\Gamma_{\overline{xy}}$ between excitation signal and a microphone downstream of the burner for three different test states.

crophone to the burner is displayed in Figure 9 for the three different test states. Except for the forcing upstream of the burner the quality of the measured signal is good as indicated by a coherence value larger than 0.5. When forcing upstream the acoustic waves are reflected by the impedance of the burner resulting in a low level of the measured signal in the combustion chamber. In addition, low values only at specific frequencies, as in Figure 9 are indicative of microphones placed in pressure nodes. This will be reflected in the coherence signal between the forcing signal and the specific microphone. However, for the determination of the transfer matrix a low coherence in one of the used microphones is not a problem as multiple microphones are used to determine the wave fields in the combustor. Using this technique the measurement was less sensitive to errors and a low coherence in one of the microphone signals, as displayed in Figure 9, did not cause the method to be inaccurate.

In a first step the transfer matrix was calculated without separating the source term using equation 11. The coefficients are displayed in Figure 10 in a frequency range of $40 < f < 400$ Hz. The coefficient T_{11} is close to unity for frequencies $f < 250$ Hz. The spike at $f = 120$ Hz is believed to be related to the source term contribution, which has not been separated from the transfer matrix displayed in Figure 10. For $f > 250$ Hz an increase to 2 can be observed. T_{12} shows almost linear behavior for frequencies $f > 250$ Hz as predicted by the model without flame (equation 15). However in the low frequency range a deviation between the model and the measurement can be observed. T_{12} does not decrease with decreasing frequency as expected from the model but remains constant and shows even a slight increase. This behavior might be attributed to a source term. A similar behavior can be observed for T_{22}.

It was thus of further interest to determine the source term of the burner and the flame. Again, to measure this quantity a third test state is required which was provided by acoustic excitation upstream and downstream of the burner. The source term, consisting of f_s and g_s calculated from equations 13 and 14, is plotted as a function of the frequency in Figure 11. The source term contains a significant contribution in the low frequency region $40 < f < 160$ Hz with a maximum at $f = 80$ Hz. For frequencies above 160 Hz the source term is very small, e.g., the burner does not act as a source at these high frequencies. The generation of noise in the low frequency region is related to the burner flow turbulence leading to fluctuations in heat release and thus to acoustic waves. For comparison the wave components f_u and g_d incident on the burner are displayed in Figure 12. For distinct frequencies the source term is of the same order of magnitude as the incident sound field on the hurner indicating a significant source character of the burner/flame.

Having measured the "source free" scattering matrix the "source free" transfer matrix could then be determined by a simple matrix transition and is displayed in Figure 13. The transfer matrix without source term differs in the low frequency region from the transfer matrix which includes the source term. In the frequency region $40 < f < 160$ Hz the flame generates noise which is reflected in the source term.

The data for the source free transfer matrix shows a smaller peak at $f = 120$ Hz but looks less smooth when compared to the transfer matrix which includes the source term. Both transfer matrices have been determined from three linear independent test states. As only two test states were required to calculate the transfer matrix which includes the source the system was overdetermined and thus its sensitivity to measurement errors reduced. The calculation of the source free transfer matrix, on the other hand, required all three test states and was thus more sensitive to measurement errors. The quality of the data can be improved by measuring more than the required three test states.

CONCLUSIONS

The response of burner/flame to acoustic perturbations plays a crucial role when modeling thermoacoustic systems. In our approach we describe the burner and the flame as an active acoustic two-port ("black box") in which the state variables pressure and velocity are coupled via a four element transfer matrix. The four coefficients of the transfer matrix required two linear independent test states which were achieved by acoustic forcing upstream and downstream of the burner, respectively. The measurement technique has been verified without combustion by comparing it to a simple model with good agreement.

The measurement technique was then applied to an experimental swirl stabilized premixed burner with combustion. Here, the turbulent flow through the burner and flow instabilities traveling into the flame may generate sound waves, which are incoherent with the forcing signal, so that the burner itself acts as a sound source. To understand the interaction between sound and heat release the source character of the burner has to be separated from the plain transfer function. In the case of the burner transfer function formulation with source term six unknowns have to be determined: the four coefficients of the transfer matrix T and two coefficients g_s and f_s describing the source character of the flame. The Riemann invarients g_s and f_s represent here waves which are emitted by the burner acting as a source. To solve for all six unknowns a third linear independent test state is required which was achieved by acoustic excitation upstream and downstream of the burner/flame simultaneously.

The measured source term indicated that the burner/flame acts as a sound source mainly in the low frequency region $40 < f < 160$ Hz. For frequencies above 160 Hz the source term is very small, e. g., the burner does not act as a sound source at these higher frequencies.

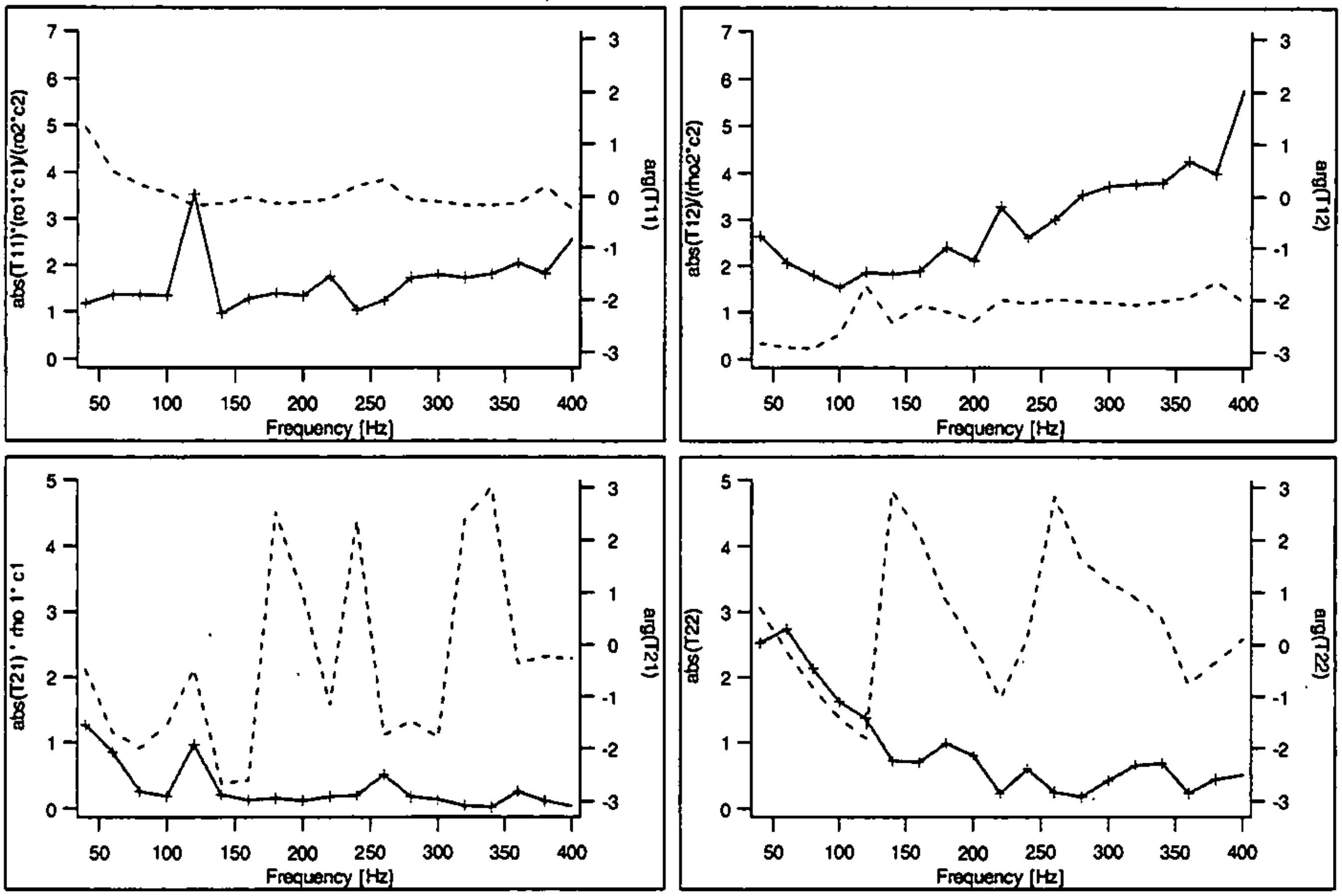

Figure 10. The transfer matrix of the burner/flame including the source term. Solid line: absolute value; dashed line: phase

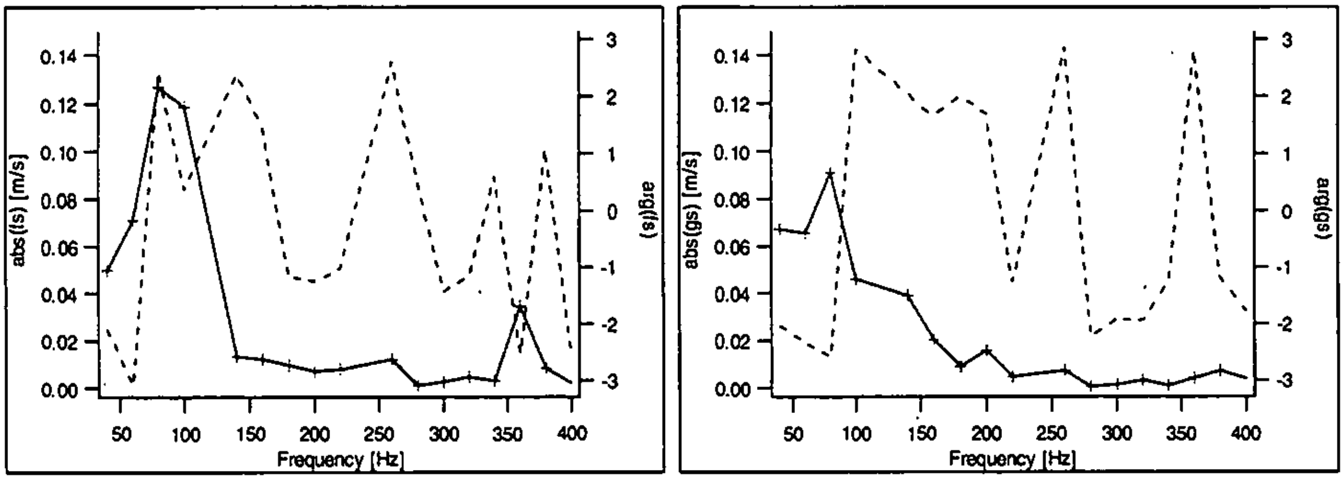

Figure 11. The source term of the flame. Solid line: absolute value; dashed line: phase

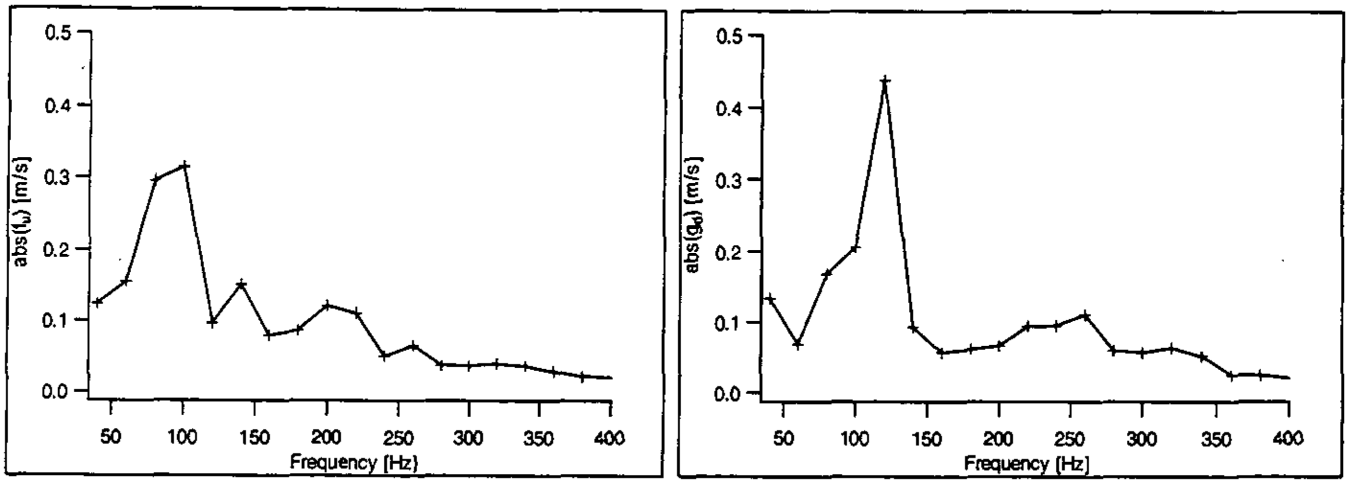

Figure 12. Incident wave components f_u (upstream of the burner) and g_d (downstream of the burner). Forcing was from upstream and downstream of the burner simultaneously.

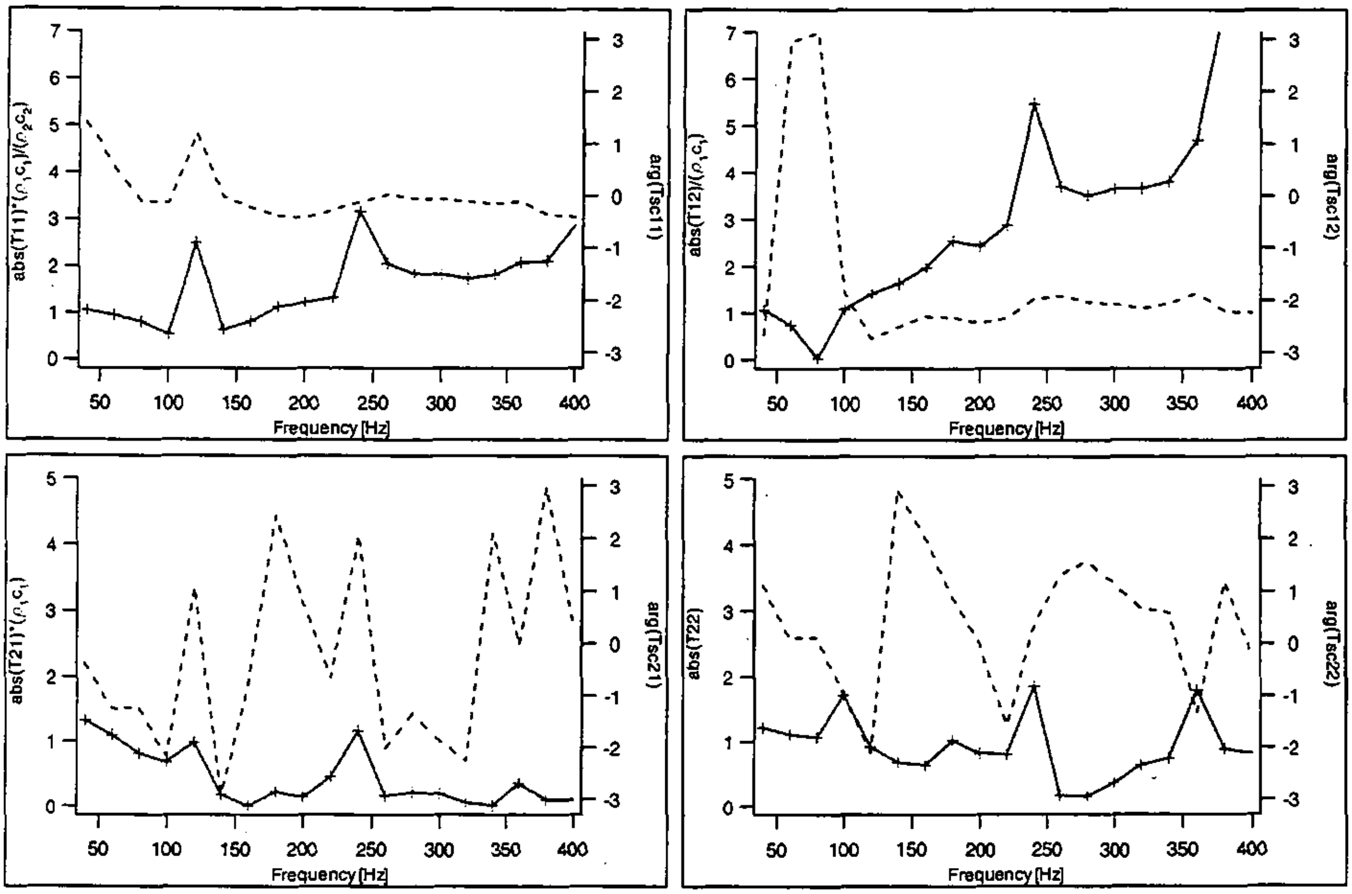

Figure 13. The transfer matrix of the flame without source term. Solid line: absolute value; dashed line: phase

The measured flame transfer function and source term data will be used for the formulation of premix flame models and the stability analysis of thermoacoustic systems.

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