Reliability Analysis of Ceramics Using the CERAM Computer Program

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ABSTRACT

The CERAM computer program for the structural reliability analysis of ceramic parts is presented. CERAM is based upon the Weibull's model and the more fundamental Multiaxial Elemental Strength Model placing emphasis upon the microstructural defects. Then, the analysis of thermal shocks failure for alumina disks provides an example of failure predictions using CERAM.

INTRODUCTION

Failure prediction with ceramics is a statistical-probabilistic problem, as a result of the presence of randomly distributed inherent microstructural defects. Thus, under predominantly tensile loading conditions these defects always cause fracture. As a consequence, ceramics exhibit an erratic fracture resistance, requiring failure probability computations for failure analysis and structural design.

There are several available models for failure probability evaluation with brittle materials. They can be essentially grouped into 2 main categories:
- the Weibull statistical model [1],
- and the more fundamental approaches which consider the fracture inducing flaws as physical entities [2-5].

In recent years, the Weibull's model has been primarily used for the routine analysis of failure strength scatters, predominantly in uniaxial stress states. This approach is the extension of failure probability equations to evaluate the fracture stress-failure probability relation.

Barnett et al. [6] and Freudenthal et al. [7] suggested a simple alternative to the Weibull formulation in multidimensional stress fields. The principal stresses are assumed to act independently. As a consequence the failure probability is calculated from the product of individual survival probabilities in the direction of tensile components.

The so-called Multiaxial Elemental Strength Model [8,9] derived by Lamon and Evans is based upon an Elemental Strength Approach to brittle fracture. The fracture inducing flaws are characterized by an elemental strength which represents the flaw extension stress. The flaw populations are then described using distributions in the elemental strengths (flaw density function). In various elemental strength treatments, the elemental strength is defined as the strength of a volume element containing the flaw ([2] and references therein). In his model, Batdorf considered the remote uniaxial normal fracture stress of a given crack [5].

The Multiaxial Elemental Strength Model is an approach to the determination of the flaw density function, for the most general stress state. The elemental strength was defined as a combination of the normal and shear stress components operating upon the flaws, through recent concepts of non-coplanar crack extension [10].

Based upon this Multiaxial Elemental Strength Model, the so-called CERAM computer program has been developed for failure prediction and reliability analysis with ceramic components having complex geometry and subject to complex stress states, in the presence of single or multiple and/or transient flaw populations [11]. CERAM also uses the statistical Weibull model (Barnett-Freudenthal approximation) for multiaxial loading. CERAM has been validated on a number of example problems.

The primary intent of this paper is to present the CERAM computer program. Then the program was used for examining the failure by thermal shocks of alumina disks. Failure predictions were based upon flaw characteristics determined from biaxial flexure of identical disks at room temperature.

THEORY: THE MULTIAXIAL ELEMENTAL STRENGTH MODEL

The Multiaxial Elemental Strength Model uses the following basic equation of failure probability [2, 7, 12]:

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where \( g(S) \, dS \) represents the number of flaws per unit volume with a strength between \( S \) and \( S + dS \). \( S \) is the remote stress of a volume element containing the flaw.

The criterion for flaw extension is based upon maximum in the strain energy release rate \( G_{\text{max}} \) in the direction of crack propagation [10]:

\[
G_{\text{max}} = \frac{(1 + \mu)(1 + x)}{4E} \left[ K_1^2 + 6 K_1^2 K_2^2 + K_2^4 \right]^{1/2} 
\]

where \( x = 3-6\mu \) under plane strain conditions, \( x = (3-\mu)/(1+\mu) \) under plane stress conditions, \( E \) is the Young's modulus, \( \mu \) is the Poisson's ratio, \( K_1 \) and \( K_2 \) are the mode I and mode II stress intensity factors.

An equivalent stress \( \sigma_E \) is then derived from equation (2) as the uniaxial tensile stress that would induce the same strain energy release rate \( G_{\text{max}} \) as the actual local stress field (\( \sigma_n^* \)):

\[
\sigma_n^* = \left[ \frac{\sigma_n^4 + 6 \tau^2 \sigma_n^2 + \tau^4}{\sigma_n^4} \right]^{1/4} 
\]

where \( \sigma_n \) and \( \tau \) are normal and shear stress components.

Crack extension occurs when \( \sigma_n^* \) reaches the critical value \( \sigma_{\text{cr}} \). The equivalent stress attains the equivalent strength \( \sigma_{\text{E}} \).

The flaw density function of equation (1) is then expressed in terms of the distribution \( g(S) \) of the equivalent strengths \( S \).

Failure probability is finally given by the following equations for surface - and volume - located flaws in 3-dimensional geometries:

\[
P_S = 1 - \exp \left( - \int \frac{dV}{V} \int g(S) \, dS \right) 
\]

\[
P_V = 1 - \exp \left( - \int \frac{dV}{V} \int g(S) \, dS \right) 
\]

where the subscripts \( S \) and \( V \) refer to surface and volume respectively.

The shape parameter \( m \) is the exponent of the equivalent strength distribution \( S_{\text{E}} \). \( \alpha_{\text{OM}} \) is a scale factor in the power function used for \( g(S) \).

CERAM includes the modules CERAM 2D and CERAM 3D for 2 - and 3 - dimensional analyses, and the corresponding PROB 2D and PROB 3D graphic codes for plotting maps of failure probability within components.

The theoretical models used in the program are:

1) The 2-parameter Weibull material strength distribution model for treating multiaxial failure, using the principle of independent action for polyaxial stress states. The following basic equations are used for failure probability computations for surface - and volume - located failures in 3-dimensional geometries:

\[
P_S = 1 - \exp \left( - \int \frac{dV}{V} \int dA \frac{g(S)}{\sigma_{\text{OWS}}} \right) 
\]

\[
P_V = 1 - \exp \left( - \int \frac{dV}{V} \int dA \frac{g(S)}{\sigma_{\text{OWV}}} \right) 
\]

The subscripts \( S \) and \( V \) refer to surface and volume. \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) are the tensile principal stresses, \( m \) is the Weibull modulus, \( \alpha_{\text{OWS}} \) the scale factor. \( A \) is the surface and \( V \) the volume of component.

2) The Multiaxial Elemental Strength Model. Failure probability computations are based on equations (4).

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All the programs run on VAX computers.

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The first set of input data for CERAM are the stresses computed within the finite elements. Boundary conditions and elastic and thermal properties are input data for the stress analysis.

A second set of input data defines the fracture inducing flaw populations located in the surface and in the volume of component. Up to seven distinct populations can be specified for each location. Each flaw population is characterized by a couple of statistical parameters, as a result of a strong sensitivity to these parameters, due to the power form of the failure probability equations. Calculations have shown that a reasonable 1% uncertainty on failure...
probability computations requires an uncertainty on the scale factors between 1% and 0.1%, for a shape parameter of 10 [13].

Determination of the statistical parameters requires the 3 following steps:

- strength data acquisition using mechanical tests
- fractographic examination of the specimens for fracture origin identification
- analysis of the strength distributions with regard to the involved flaw populations for determination of the relevant statistical parameters.

Strength data are usually measured using simple loading geometries such as 3-point or 4-point bending of bars or rods, biaxial flexure of disks, etc. Standards have not been recommended yet for ceramic testing with respect to structural reliability analysis purposes. Therefore mechanical tests should be designed with a view to extensive characterization of the fracture inducing flaw populations. Moreover, one should not rely on a single loading geometry. It is important to acquire data on all the fracture inducing flaw populations. The influence of certain populations may be hindered when typical stress states including high stress gradients are used, as observed on 3-point bending. As a result of the peak stresses operating on the surface, failure in 3-point bending is generally controlled by surface-located flaws. Application of a different stress state, with higher internal shearing effects [8,9] or an uniform stress-field may then reveal concurrent internal populations [14].

A single set of strength data is not sufficient for safe failure predictions. It is also important to evaluate the ceramic reproducibility by checking that the same populations are present whatever the batch, and in the components.

There are various methods available for the determination of statistical parameters from strength data. In reference [13], linear regression analysis, maximum likelihood estimation and mean strength based methods were compared with a CERAM based method in which the statistical parameters are derived from comparison of computations of strength-failure probability with experimental results. It was shown that the best fit to experimental data was obtained with failure predictions from scale factors determined using the CERAM based method. The lower scale factors were obtained with the maximum likelihood estimation and the mean strength based method, thus leading to the higher failure probabilities, whereas the higher scale factors were obtained with the linear regression analysis, thus leading to the lower failure probabilities. All the failure probabilities have been computed with CERAM.

PROGRAM CAPABILITIES

CERAM provides detailed failure probabilities within the mesh elements and the overall failure probability for each population and for the concurrent effect of multiple flaw populations. Various outputs are available:

- print-out of the detailed results
- probability contours.

Figure 1 shows an example of graphical output displayed on a Tektronix graphics screen, using the PROB 3D graphics module of CERAM. This map shows the failure probabilities within a ring of silicon carbide subjected to a diametral 4-point bending. Also displayed are various information including the time steps, the loading states, the dimensions, the location of flaw populations (volume, surface) and the fracture origin identification.

Fig. 1 - Fracture probabilities from volume located defects obtained for a SiC ring subject to 4-point loading. For symmetry reasons a fourth of the specimen is shown.

FP max = overall failure probability
FP max = maximum failure probability within the component.

failure probability model (Multiaxial Elemental Strength Model (Muest), Weibull).

CERAM has broad capabilities by allowing the user:
- to perform 2- or 3-dimensional analyses
- to evaluate failure from surface - and/or volume - located flaws
- to specify up to 7 different surface - or volume - located flaw populations
- to specify time-dependent flaw strength parameters to account for temperature or environmental effects
- and to analyze components made of one or several parts and of one or several materials.

EXAMPLES

Validation of the CERAM computer program used scaling of strengths to different specimen sizes and configurations. For this purpose, the flaw population characteristics (Weibull statistical parameters and the flaw strength parameters) were determined for a given ceramic from strength data measured on a given specimen size and configuration selected as reference. These flaw characteristics were subsequently incorporated in CERAM for computing the failure probability-strengths of different specimen sizes and/or configurations. Comparison of the results with experimental data and theoretical calculations then allowed evaluation of the CERAM computer program.

The validation of CERAM used various sets of experimental strength data, measured on various ceramics containing either single or multiple flaw populations and subjected to various loading conditions [1,13-19]. In particular in reference 13, 7 different loading geometries, two commercial ceramics and 715 specimens carefully prepared and tested were used.
The following factors which determine failure predictions were examined:
- the estimator for experimental failure probability,
- the uncertainty and the variability in the statistical parameters,
- the method used for determining the statistical parameters,
- the probabilistic model,
- the selection of batches,
- the finite element mesh,
- the results of stress analysis,
- the stress-to-failure (stress at fracture origin, peak stress, etc.).

This analysis showed that with reproducible ceramics and the appropriate factors, the failure probabilities predicted with CERAM were in good agreement with experimental failure data. In certain cases involving shearing effects, the Weibull model led to significant discrepancies between predictions and experimental results.

FAILURE PREDICTIONS FOR THERMAL SHOCKS

As an illustration, failure analysis using CERAM was made for alumina disks subject to thermal shocks [20]. As reported in reference [20], the samples were initially at high temperature. They were then individually subjected to a rapid quenching within the furnace using high velocity helium channeled onto the disk center. The resistance to thermal shocks was measured by the initial value of the critical temperature differential between the sample and the helium jet, at which fracture of the specimens was observed. 28 disk samples (5 cm in diameter by 0.25 cm thick) had been tested. The measured critical temperature differentials exhibited a significant scatter. They were handled as statistical data using the ranking statistics method. The fracture temperatures were ordered from lowest to highest. The i-th result in the set of samples was assigned a cumulative failure probability \( P_i \), calculated using the following estimator:

\[
P_i = \frac{i}{N+1}
\]

where \( N \) is the sample size.

The resultant failure probabilities were then plotted as a function of the fracture temperatures. Fractographic examination of specimens had shown that failure was dictated by a population of surface-located microstructural flaws.

Alumina properties for the temperature and stress analysis are given in Table 1. An axisymmetric 2D finite element analysis was carried out. A 313 element mesh was constructed (Figure 2). The temperature distributions were calculated using the program TOPAZ 2D. The thermal stress distributions were determined using the program NIKE 2D.

Convective heat transfers were considered through the upper surface of the disk quenched by the helium stream. The heat transfer coefficient \( h \) was assumed to be uniform over the diameter of the helium jet (1.52 mm) and to decrease inversely with distance \( r \) from the jet boundary to account for flow attenuation, according to the following relations:

\[
h = 0.256 \left( \frac{W}{cm^2 \text{ sec}} \right) \text{ for } r \leq 0.76 \text{ mm}
\]

\[
h = \frac{0.256}{r} \left( \frac{W}{cm^2 \text{ sec}} \right) \text{ for } r > 0.76 \text{ mm}
\]

Fig. 2 - Mesh used in the finite element calculation.

The flaw strength parameters for the surface-located fracture origins (Table 1) had been derived from strength data measured at room temperature on identical disks tested in biaxial flexure (ring-to-ring test) (Figure 3). Failure was also dominated by surface-located flaws. Subsurface fracture origins were identified in a small proportion of mechanical tests. The strength distribution was thus separated for the determination of pertinent statistical parameters, using the censored data method [21] to conform with the presence of 2 populations.

Fig. 3 - Strength distributions obtained for alumina disks using biaxial flexure tests.

The flaw strength parameters were then obtained using the CERAM based method. The experimental work [20] has shown that as expected this alumina ceramic exhibited a satisfactory reproducibility, as indicated by the comparison of the statistical parameters determined from the fracture temperatures and the
biaxial flexure strength data (see Table 1 and [20]). This reproducibility was attributed to the random selection of both batches, and to the absence of significant thermally induced effects.

### TABLE 1 - ALUMINA PROPERTIES USED FOR FINITE ELEMENT ANALYSIS AND FAILURE PROBABILITY COMPUTATIONS

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus (GPa)</td>
<td>390</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.22</td>
</tr>
<tr>
<td>Density (g/cm³)</td>
<td>3.9</td>
</tr>
<tr>
<td>Specific heat (J/g°C)</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>(400°C &lt; temp. &lt; 800°C)</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>(25°C &lt; temp. &lt; 400°C)</td>
</tr>
<tr>
<td>Thermal conductivity (W/m°C)</td>
<td>11.5</td>
</tr>
<tr>
<td>Thermal expansion coefficient (1/°C)</td>
<td>8.3 10⁻⁶</td>
</tr>
<tr>
<td>Flaw strength parameters*</td>
<td></td>
</tr>
<tr>
<td>Biaxial flexure: shape parameter</td>
<td>14</td>
</tr>
<tr>
<td>scale factor (MPa m¹/₁⁴)</td>
<td>73</td>
</tr>
<tr>
<td>Thermal shocks: shape parameter</td>
<td>13.2</td>
</tr>
</tbody>
</table>

* surface-located flaw population

Figure 4 shows the failure probability changes during thermal shocks. It can be seen that failure probability reaches a maximum after about 15 seconds. This result is in correlation with our experiments. Failure was evidenced by a typical burst which was easily heard a few seconds after the beginning of the thermal shocks. This result is also in agreement with the literature which already indicated that fracture under thermal shocks coincides with a maximum risk of rupture rather than with a maximum stress [22]. This may be attributed to the flaw induced fracture phenomenon under predominantly tensile loadings. Fracture requires that a critical defect be operated on by transient stress state, which is achieved when a sufficient amount of material is acted upon by a sufficient level of stresses.

![Image](image.png)

**Fig. 4** - Predictions of failure probability changes during thermal shocks performed at various initial temperature differentials.

Figure 5 shows that the failure probabilities computed with CERAM are in agreement with experimental results. However, it is obvious that the failure predictions with the Barnett-Freudenthal approximation are surprising when comparing with the general trend (see for example [23-26]) which shows that on simple loading geometries and stress states and in the presence of single flaw populations, the principle of independent action (PIA) gives non conservative failure probability. In the present paper, we used the scale factors obtained with the Multiaxial Elemental Strength Model even for failure predictions with the Barnett-Freudenthal approximation. The Multiaxial Elemental Strength scale factors are always lower than the PIA ones, which explains why our failure predictions with the PIA model were higher than expected.

However, it is reemphasized that failure predictions depend upon various factors, as previously mentioned. Among them the use of analytical or computer-based method, the variability of scale factors and the method used for their determination may have a significant influence. Thus, for example the use of linear regression analysis will give scale factors higher than those obtained with the CERAM-based method. As a consequence, higher failure probabilities will be predicted with CERAM when compared with analytical calculations. The discrepancy may be enhanced by other more or less controlled factors. Therefore, confidence in failure predictions should be evaluated with respect to the various factors which affect the results.

The close prediction of thermal shock failure from biaxial strength measurements strongly supports the notion of parity between thermal shock failure and mechanical failure in the absence of significant thermally induced effects. Finally, it shows that biaxial flexure of disks may be an efficient test for the simulation and prediction of the thermal shock failure of ceramics.

![Image](image.png)

**Fig. 5** - Comparison of the fracture temperatures of the alumina disks with the predictions using the CERAM computer code based on biaxial flexure of disks.
CONCLUSIONS

Our approach to the reliability analysis of ceramics places emphasis upon the flaw populations. This approach uses the Multiaxial Elemental Strength Model and the CERAM computer program for handling complex problems. The Multiaxial Elemental Strength Model is based on a non-coplanar strain energy release rate criterion for flaw extension. CERAM includes also the Weibull’s statistical model (Barnett-Freudenthal approximation).

CERAM allowed satisfactory failure predictions with various loading geometries and ceramics which have been reported elsewhere. In the present paper CERAM was satisfactorily used for the analysis of the failure of alumina disks under thermal shocks, based upon flaw strength parameters measured at room temperature.

The CERAM computer code offers broad capabilities to the user and in particular it allows the dynamic nature of flaw populations and multiple flaw populations to be considered. Structural reliability analysis is not simply a numerical problem. It requires that material testing for extensive flaw characterization be integrated in the computer based approach.

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