EFFECTS OF CRITERION FUNCTIONS ON INTERMITTENCY IN HEATED TRANSITIONAL BOUNDARY LAYERS WITH AND WITHOUT STREAMWISE ACCELERATION

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ABSTRACT

Attempting to understand the mechanisms of momentum and thermal transports in transitional boundary layers, has resulted in the use of conditional sampling to separate the flow into turbulent and non-turbulent portions. The choice of a proper criterion function to discriminate between the two flow conditions is critical. A detailed experimental investigation was performed to determine the effects of different criterion functions on the determination of intermittency for application in heated transitional boundary layers with and without streamwise acceleration. Nine separate criterion functions were investigated for the baseline case without pressure gradient and three cases with streamwise pressure gradient. Inherent differences were found to exist between each criterion function's turbulence recognition capabilities. The results indicate that using a criterion function based on Reynolds shear stress, \( \left( \frac{\partial u'}{\partial y} \right)^2 \), for turbulent/non-turbulent discrimination in a heated transitional boundary layer is superior to a single velocity or temperature scheme. Peak values in intermittency for the early to mid-transitional region were found to occur away from the wall at approximately \( y/\delta = 0.3 \) for all cases. To match the universal intermittency distribution of Dhawan and Narasimha (1958), the minimum values of intermittency at \( y/\delta = 0.1 \) should be used as the representative "near-wall" values.

NOMENCLATURE

- \( C_f \) - skin friction coefficient, \( \tau_w/(\rho U_{\infty}^2/2) \)
- \( C_p \) - pressure coefficient, \( \frac{P - P_{\text{ref}}}{\frac{1}{2} \rho U_{\infty}^2} \)
- \( K \) - pressure gradient parameter, \( \frac{\nu}{U_{\infty}^2} \frac{dU_{\infty}}{dx} \)
- \( P \) - static pressure
- \( Re_x \) - Reynolds number, \( \frac{U_{\infty} x}{v} \)
- \( t \) - instantaneous fluctuation in temperature
- \( t' \) - rms value of temperature fluctuation
- \( T \) - instantaneous temperature
- \( u,v,w \) - instantaneous velocity fluctuations in streamwise, cross-stream, and spanwise directions
- \( u',v',w' \) - rms values of velocity fluctuations
- \( u^* \) - friction velocity, \( \sqrt{\tau_w}/\rho \)
- \( U,V \) - instantaneous velocities
- \( \bar{U} \) - mean velocity
- \( U^+ \) - \( \frac{U}{u^*} \)
- \( \bar{u} \) - mean Reynolds shear stress
- \( \bar{u}' \) - mean Reynolds streamwise heat flux
- \( \bar{v} \) - mean Reynolds cross-stream heat flux
- \( x \) - coordinate in streamwise direction
- \( y \) - coordinate normal to the surface
- \( Y^+ \) - \( y u^*/\nu \)

Greek

- \( \Gamma \) - intermittency factor
- \( \delta \) - boundary layer thickness at 0.995 \( U_{\infty} \)
- \( \delta^* \) - displacement thickness
- \( \varepsilon \) - turbulent dissipation rate
- \( \nu \) - kinematic viscosity
- \( \xi \) - length in transition region, \( x \Gamma = 0.75 \cdot \chi \Gamma = 0.25 \)

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\( \rho \) - density  
\( \tau \) - time  
\( \tau_w \) - shear stress on the wall

Subscripts  
\( \infty \) - free-stream value  
\( nt \) - non-turbulent  
\( ref \) - reference location at \( x = 20 \) cm  
\( s \) - onset of transition  
\( t \) - turbulent  
\( w \) - at the wall

INTRODUCTION

Boundary layer transition from laminar to turbulent flow has been recognized as an important feature in the through-flow of a gas turbine (Graham, 1979 & 1984; Mayle, 1991). Heat transfer in a turbulent boundary layer with a moderate Prandtl number is typically treated as a passive process controlled by the turbulent momentum transport. For a gas turbine blade, where as much as 50-80% of the turbine blade surface is covered with flow undergoing laminar-turbulent transition (Turner, 1971), this relation between momentum and thermal transport has not been verified. In addition, turbine blades are exposed to diverse pressure gradients that may compound these transport differences. Recognizing and understanding the fundamental mechanisms involved in transitional convective heat transfer are keys to improving the heat transfer modeling and enhancing the accuracy of thermal load predictions on gas turbine blades.

Attempting to understand the mechanisms in transitional momentum and thermal transports has resulted in the use of conditional sampling to separate the flow into turbulent and non-turbulent portions. Conditional sampling techniques used in turbulent boundary layer and shear flows are many; however their application to heated transitional flow is not well developed. The choice of a proper criterion function to discriminate between the two flow conditions is critical. The use of temperature as a passive contaminant to discriminate between the turbulent and non-turbulent portions as done in transitional boundary layers since discrepancies between the momentum and thermal transport in a transitional boundary layer exist. Blair (1982 & 1992), Sharma (1987), and Volino and Simon (1991) determined that the length of transition for accelerating flows is longer for the thermal than the momentum boundary layer. Sharma recommended the use of a separate intermittency factor for the thermal boundary layer under these conditions. In light of these observations, a need exists to determine the sensitivity of the intermittency factor in the heated transitional boundary layer to the choice of criterion function.

Emmons' (1951) statistical theory of transition introduced the concept of an intermittency factor for calculation in the transitional boundary layer. Emmons proposed that the turbulent patches could be treated as fully turbulent flow and the non-turbulent patches as laminar flow. The bulk flow properties could then be reconstructed as \( X = (1-\Gamma)X_{nt} + \Gamma X_t \). For example, the skin friction coefficient in the transitional region could be found from the intermittency factor and the appropriate combinations of the turbulent and laminar values; \( \Gamma = (1-\Gamma)C_{fnt} + \Gamma C_{ft} \). By knowing the intermittency factor at any streamwise location, the bulk properties of the transitional boundary layer could be determined. Treating the turbulent portion of transitional flow as a fully turbulent flow and the non-turbulent portion as a laminar flow is a widely used engineering practice (Arnal, 1984; Narasimha, 1985, and Mayle, 1991). This practice has been recently challenged by Kuan and Wang (1990). Their challenge was not on the concept of intermittency but on the adequacy of treating the turbulent portion as a fully turbulent flow and the non-turbulent portion as a laminar flow. They concluded that both turbulent and non-turbulent portion of the transitional flow are different from their counterpart in the fully turbulent and laminar flow.

The determination of the turbulent and non-turbulent portions of the transition region and their subsequent separation relies on the technique of conditional sampling. This technique was discussed in detail by Hedley and Keffer (1974) and Muck (1980). It is comprised of three main stages: selection of a criterion function, determination of a threshold level, and generation of an intermittency function.

Turbulent flow is a three-dimensional rotational flow characterized by the dissipation of mechanical energy into heat through a cascade of eddies of diminishing sizes. The criterion function should be ideally representative of this turbulence and offer a good contrast between the turbulent and non-turbulent portions. However, detection of an energy cascade requires spectral analysis and renders an instantaneous decision for or against turbulence difficult. Fluctuating vorticity was used by Corrsin and Kistler (1955) but this requires the use of a complex probe capable of spatial differentiation and is considered by most too difficult to implement, especially in a transitional boundary layer.

Chen and Blackwelder (1976), Muck (1980), and Antonia (1981) considered the use of a passive scalar such as temperature to be superior to velocity or vorticity as a criterion function. However, Muck (1980) pointed out that the question remains whether the thermal interface coincides with the turbulent (vorticity) interface. For a fully turbulent heated boundary layer, turbulent/non-turbulent discrimination occurs primarily in the outer boundary layer where the turbulent fluid is rotational and the non-turbulent fluid is irrotational. The temperature in the rotational portion remains constant and is lower than the temperature in the turbulent regions. The classic temperature discrimination scheme uses the temperature directly and identifies "hot" fluid as turbulent and "cold" fluid as non-turbulent. The validity of this classic scheme needs to be reexamined in the transitional boundary layer where the vorticity dynamics are different. The dynamics of the vortices on the rotational/irrotational interface of the fully turbulent boundary are different than the vortex tubes on the boundary between the turbulent and non-turbulent portion in a transitional boundary layer. The temperature in the irrotational portion of the turbulent outer boundary layer maintains a constant lower temperature than the rotational portion. However, the temperature of the non-turbulent portion of the transitional boundary layer is not necessarily lower than the temperature in the turbulent portion. In the transitional boundary layer, the temperature profile, similar to the velocity profile, will alternate between a laminar-like profile and a turbulent-like profile. In addition, the "calming region" at the trailing edge of a turbulent patch imposes...
difficulty for discriminating the turbulent/non-turbulent portions since both fluctuating magnitudes and mean values are changing. No such calming region is observed in the interface between a turbulent boundary layer and free-stream.

Difficulties also arise in using velocity fluctuations. Velocity fluctuations are not unique to the turbulent fluid and may be due to amplified oscillations of the free-stream disturbances or Tollmien-Schlichting waves. As a result, some procedure must be used to desensitize it. Also, spurious dropouts (short regions where the criterion function falls below the threshold level indicating non-turbulent flow) occur within a turbulent burst and some form of smoothing is required. Smoothing may take the form of a running average to eliminate the spurious dropouts or the use of a holding time where any excursions shorter than the holding time are still considered turbulent. Hedley and Keffer (1974) recommended

\[
\left( \frac{\partial^2 u}{\partial y^2} \right)^2 + \left( \frac{\partial^2 v}{\partial y^2} \right)^2 \quad \text{or} \quad \left( \frac{\partial u}{\partial t} \right)^2 + \left( \frac{\partial v}{\partial t} \right)^2
\]

use as the discrimination scheme stating that the Reynolds shear stress has a lack of definition at the interface leading edge. Antonia (1972) used \( \left( \frac{\partial u}{\partial y} \right)^2 \) and reported a sharp drop in the Reynolds shear stress at the interface conflicting with the results of Hedley and Keffer. Muck (1980) investigated several discrimination schemes and concluded using \( \left| \frac{\partial u}{\partial y} \right| \) or \( \left| \frac{\partial^2 \nu}{\partial x^2} \right| \) worked best and was closest to the temperature scheme.

The threshold value is the minimum value of the criterion function set just above the background noise and non-turbulent fluctuations. Several different methods for choosing a threshold level have been proposed. Corrsin and Kistler (1955) plotted the cumulative distribution functions of the intermittency as a function of threshold value. The point of maximum curvature was then used to select the threshold value. This method worked well when the intermittency was low but was unreliable for high values of intermittency (Muck, 1980). Hedley and Keffer (1974) raised the threshold level and determined the average time duration of all the non-turbulent zones of the discrimination scheme until a constant value was reached. This process was repeated for all streamwise and cross-stream locations. A similar threshold value was observed to occur when the beginning of the constant time average duration was reached. This single value was used for all locations. However, this method is very sensitive to spurious dropouts and the smoothing procedure. Antonia (1972) set the threshold equal to a fraction (0.3) of the overall average of the function. With all these different schemes being tried what remains clear is that there is no rational method of choosing a threshold value. It is typically adjusted by trial and error until the results conform with the individual researchers expectations (Muck, 1980).

The above mentioned investigations were performed in fully turbulent boundary layers. These same ideas are usually extended into transitional boundary layers but it remains to be verified because the flow and thermal structures in the transitional boundary layer are different from those in the turbulent boundary layer as explained previously. The work presented in this paper is a systematic investigation performed to determine the effects of different criterion functions on intermittency in a heated transitional boundary layer, to establish an adequate conditional sampling technique to separate the flow into the appropriate turbulent/non-turbulent portions, and to specifically investigate the difference between velocity and thermal intermittency if it exists. Experiments were first performed in a transitional boundary layer flow over a flat surface without a steamwise pressure gradient and followed by three cases of accelerated boundary layer at three different K values.

**Experimental Program**

**Test Facility**

The test facility used in this research program consisted of a 2-D, open circuit, blowing type wind tunnel. The maximum air speed was 35 m/s, uniform within 0.7% and steady within 1% over a 20-hour period. An inlet airflow filter box was covered with a layer of Rayon-viscous felt capable of filtering out particles larger than 5 μm. The free-stream air temperature was controlled by the heat exchanger and the air conditioning system in the laboratory and could be maintained within 0.5 °C over a period of 20 hours and uniformly within 0.1 °C. A suction fan and low-pressure plenum were installed at the leading edge to provide suction. A detailed description of the wind tunnel is provided by Kuan (1987) and Kuan and Wang (1990).

To provide the two-dimensional flow required in this investigation, the test section was designed with a large aspect ratio of 6. The test section was 0.15 m wide, 2.4 m long, and 0.92 m high consisting of a heated test wall, an outer observation wall, a top wall cover, and a bottom wall table. A composite construction was utilized for the rectangular 2.4 m x 0.92 m heated test wall. The back surface was covered with 25.4 cm of R30 fiberglass insulation to minimize backplane conduction losses. The heating pad consisted of a heater foil sandwiched between glass cloth and silicon rubber sheets. A 1.56 mm thick aluminum sheet was Vulcanized to the front surface of the heater pad to ensure uniformity of the heat flux. A 1.56 mm polycarbonate sheet was placed on the front surface to provide a smooth test surface on which the air flows and measurements were taken. One hundred eighty-five 3-mil E-type thermocouples were embedded beneath the test surface and were strategically placed along the test surface to capture the evolution of the wall heat transfer during the transitional flow process.

Fourteen measuring holes of 2.54 cm diameter were drilled along the centerline axis and eight measuring holes of equal diameter were drilled along the off-centerline in the cross-span direction. The first centerline measuring hole (station 1) is located 20 cm from the leading edge with the remaining measuring holes placed every 15 cm (labeled sequentially station 2 through station 14). Plexiglas plugs, flush with the inner surface, were used to plug the holes when measurements were not being taken. Slots cut into the table and the top wall provide for adjustment of the outer wall in order to vary the pressure gradient in the test section. A schematic of the thermocouple layout and the location of the profile measurement locations is shown in Figure 1. The detailed description of the test section and heated test wall was documented by Wang, Keller, and Zhou (1992) and Zhou (1993).
Figure 1: Thermocouple layout on heated test wall.

**Geometry Of The Test Section**

For the baseline case, with no acceleration, the outer wall of the test section was adjusted to account for the growth of the boundary layer and to maintain a near-zero pressure distribution inside the test section with a variation of pressure coefficient, \( C_p \), within 1 percent as shown in Figure 2.

Three different favorable pressure gradients were utilized in this investigation. A constant pressure gradient parameter, \( K \), was maintained during each case. One of the advantages of using a constant \( K \) over other pressure gradient parameters is that a constant \( K \) can be directly related to the geometry of the test section. By linearly decreasing the wall separation between the inlet and exit, a relatively constant \( K \) value can be obtained. For each acceleration case, the width of the test section inlet was maintained at 1524 cm and the downstream width was arranged to decrease linearly to the exit plane. An exit width of 14.6 cm was used for the lowest acceleration case of \( K = 0.07 \times 10^{-6} \) while an exit width of 8.9 cm was used for the highest acceleration case of \( K = 0.25 \times 10^{-6} \). The free-stream velocity distribution and pressure coefficient for each case is shown in Figure 2. It should be noted that a constant \( K \) flow is inherently different from a Falkner-Skan flow, which has a constant \( \Lambda \left( = \frac{\delta^2}{\nu} \frac{dU}{dx} \right) \) value. For a bounded passage flow, as in the turbine, a use of \( K \)-value to characterize the flow acceleration is more appropriate than the use of \( \Lambda \) even for situations with the boundary layer thickness much smaller in comparison with the passage width. Detailed explanations concerning the physical meaning of flow and thermal features of accelerated boundary layers with constant \( K \) values and the differences between a constant \( K \) and a constant \( \Lambda \) flow were made by Zhou and Wang (1992).

**Three-Wire Sensor**

A specially designed miniature 3-wire probe was used to measure all the boundary layer velocity and temperature data. An 'X' array, consisting of 1.0 mm long and 2.5 \( \mu \)m diameter Wollaston type platinum coated tungsten wires were utilized for the velocity sensors. An active sensing length of 0.5 mm was etched in the center. The 'X' wires were placed orthogonal to each other with a spacing of 0.35 mm. The temperature sensor is a 0.35 mm long unplated platinum wire 1.2 \( \mu \)m in diameter placed normal to the mean flow direction in a plane parallel to the plane of the cross wire and spaced 0.35 mm from the 'X' array. This orientation for the temperature wire was chosen to eliminate any streamwise temperature gradients. Due to the difficulty in maintaining the accurate sensor arrangement during fabrication when bending the three pairs of prongs, as for a typical boundary-layer type probe, the prongs were kept straight; the probe stem was bent at an angle of 10° from the probe axis. This angle was chosen to ensure that both of the 'X' wires touched the wall simultaneously without interference between the probe stem and the wall (see Figure 3). A complete description of the probe design and qualification, specifically in a heated transitional boundary layer, can be found in Shome (1991) and Wang, Shome, and Zhou (1993).

Figure 2: Free-stream velocity and corresponding \( C_p \) values for each case.

Figure 3: 3-wire boundary layer sensor.
Measurements and Instrumentation

The velocity-sensors were operated in a constant temperature mode using a TSI model IFA 100 Intelligent Flow Analyzer. The IFA 100 allows simultaneous operation of up to four channels. A DISA M20 temperature bridge was used to operate the temperature-sensor in the constant current mode. For future turbulent power and thermal power spectral analysis, TSI Model 157 signal conditioners were used to low pass filter signals from all three sensors. The 'X'-wires of the three-wire sensor were operated at overheat ratios of 1.43 and 1.66. The 1.2 μm temperature sensor was operated with a very low overheat ratio. The probe current was set at 0.1 mA and an amplifier gain of 3500 was used. For convenience, the velocity wires are called hot wires and the temperature wire is called cold wire in this study. The TSI IFA 100 is also equipped with a square wave generator with a frequency range of 0.3-30 kHz and amplitude range of 0-4.5 V. The square wave generator was used to optimize the frequency response of each velocity-wire prior to calibration to ensure minimum under or over damping of the wire response. The optimum frequency response found for each velocity wire was approximately 200 kHz. The frequency response of the temperature sensor was experimentally determined ranging from 4800-6400 Hz depending on the velocity using the DISA M20 constant current bridge (see Wang, Keller, and Zhou, 1992 for details). The data from all three sensors were subsequently sampled at 2 kHz for 20 seconds with the low pass filter set at 1 kHz.

The wind tunnel, the test wall power supply, and the cooling water supply were started at least 12 hours prior to the experimentation. A global measurement for wall temperature distribution was performed by scanning the temperature approximately every two hours. Each time an average of three different scans, with each scan made at a sample rate of 1 channel/second, was obtained. During the measurement of each boundary layer temperature profile, a check of the steadiness of the local wall temperature was performed before, midway, and at the end of each measurement. Both the global and local check served to monitor the steadiness of the wall and the free-stream temperature. For all thermocouple measurements a Metrabyte IEEE-488 general interface I/O expansion board was used. A Fluke 8842A 5 1/2 digit digital multimeter and a Fluke 2205A 100-channel switch controller were interfaced with the IEEE-488 board. Special low voltage scanner modules (Model 2205A-600), each with silver-coated shields, were installed in the switch controller to provide a voltage resolution of 1 μV for thermocouple emf measurements. For each case a uniform heat flux of 335 W/m² was applied to the test wall and the free-stream temperature was maintained at approximately 15° C. The resulting wall temperatures ranged from 24° C to 41° C.

Conditional Sampling Technique

Conditional sampling consists of three primary stages: the choice of a criterion function, the determination of a threshold value, and the generation of an intermittency function. The determination of the threshold value and intermittency function are discussed below.

To determine the appropriate threshold value for each criteria function, a “dual-slope method” was used. This method is based on the cumulative distribution functions used by Corrsin and Kistler (1955) and was extended by Kuan and Wang (1990). This method uses a graphical approach to find the threshold value at each location. A program was used to generate the cumulative distribution of intermittency as a function of threshold value. For each data reading, the criterion function was compared to the threshold value. If the value was greater than the threshold, the reading was considered turbulent. If the value was less than the threshold and the next two readings also less than the threshold, the reading was considered non-turbulent. Once all the readings for a given location were categorized, the final intermittency for that threshold was determined. The threshold value was then increased and the process repeated. The resulting intermittency distribution function was then plotted as shown in Figure 4. When presented in a semi-log coordinate, two straight lines of different slopes were apparent (therefore, named dual-slope method) and the threshold value at the intersection of these two lines was taken as the initial estimate. Further refinement was required to find an optimum value. The reasoning behind this method is as follows. The background noise and fluctuations in the non-turbulent portion are close to a Gaussian probability density, f(q). The fluctuations in the turbulent portions also have a Gaussian probability density but with a larger standard deviation (Figure 5a). By choosing the appropriate criterion function and desensitizing it, the intersecting region of these two curves is minimized (Figure 5b). The area of intersecting regions depends on the actual flow behaviors. An inevitable overlap region will represent the probability of indeterminate discrimination of turbulence from non-turbulence. For each threshold value 32768 (2¹⁵) data readings were processed.

After the sampled data was reduced, the intermittency function was obtained. The value of this function is 1 if the flow is turbulent and is 0 otherwise. Due to inherent spurious dropouts amidst turbulent signals, a holding time was introduced to smooth out these spurious dropouts. Hadley and Keffer (1974) suggested an optimum holding time based on the Kolmogorov length scale, \( \eta = (\nu^3/\epsilon)^{1/4} \). The recommended holding time will be \( \eta \) divided by the convective velocity of the smallest eddies. However, the probe resolution and the digital sample rate must also be considered. The actual holding...
time is therefore suggested by Hedley and Keffer (1974) to be approximately 15-35 times this Kolmogorov scale. Hedley and Keffer used a value of 4 times their sampling time interval which was 0.0004 seconds. Since the eddy size in transitional flow is larger than the eddy size in a fully turbulent flow, the holding time was assigned a larger interval for the transitional flow. For this investigation, with a sample rate of 2 kHz the holding time was set equal to three sampling time intervals which corresponds to approximately 200 times the Kolmogorov scale for the fully turbulent boundary layer (baseline case).

RESULTS AND DISCUSSION

Criterion Functions

All criterion functions were generated from the output signals of the three-wire sensor. The streamwise and cross-stream velocities (U and V), the temperature (T), and the corresponding correlations (ut, vt, and uv) were used. Sixteen locations were investigated for the zero-pressure gradient boundary layer (baseline case). Each cross-stream location was selected based on the distribution of streamwise Reynolds normal stress (u'). Station 5 (ReX = 6.13 x 10^5), was the first measuring location to indicate signs of transition in the form of turbulent bursts. A location of y/δ* = 1.2 corresponding to the maximum peak in u' was investigated for this station. For each remaining station in the transition region, stations 6 through 8 (ReX = 7.43 x 10^5 through ReX = 9.87 x 10^5), three cross-stream locations were selected corresponding to the maximum peak in u', the plateau region following this maximum peak, and a point near the edge of the boundary layer. For stations 9 through 13 (ReX = 11.2 x 10^5 through ReX = 16.2 x 10^5), a single location near the edge of the boundary layer was investigated. A near-wall point was also investigated for the fully turbulent boundary layer of station 13. Similar points were chosen for each pressure gradient case. Three representative u' distributions for stations 5, 8, and 13 of the baseline case and the corresponding locations of investigation are shown in Figure 6. An example of the signals and correlations from the baseline case for station 6 with \( r = 0.5 \) are shown in Figure 7. It is apparent that turbulent/non-turbulent discrimination from the direct use of T, U, or ut would be difficult. For V, vt, and uv the turbulent portions are more clearly defined (labeled 'A' in Figure 7) but several questionable regions still exist (labeled 'B'). The raw signals shown in Figure 7 are inappropriate for use as criterion functions especially with the presence of unsteady oscillations in the non-turbulent portion. A means of desensitizing the signal to the non-turbulent fluctuations

Figure 6: Distribution of streamwise Reynolds normal stress for zero-pressure gradient (shaded symbols represent points investigated).

Figure 7: Signals and correlations for \( r = 0.5, y/δ* = 1.1 \) (baseline case). A: clearly defined turbulent region. B: questionable region.
must be implemented. The method most commonly used is to high-pass filter the signal or to differentiate the signal with respect to time and square it, thus emphasizing the high frequency components.

A comparison of the effects of using an ideal digital high-pass filter and taking the derivative of an example signal is shown in Figure 8a through 8d. For all differentiation throughout the analysis, a second order central-difference technique was utilized. A 0.5 second sample of the Reynolds shear stress sampled at 2 kHz with a 1 kHz analog filter is shown in Figure 8a. The frequency response of the first time derivative and the ideal digital high-pass filter with a 200 Hz cutoff frequency are shown in Figure 8b. The first time derivative behaves as a high-pass filter with a linear phase and a frequency response with a slope of 35 dB/decade. Applying both the time derivative and the ideal digital high-pass filter to the signal shown in Figure 8a and squaring, results in the criterion functions shown in Figure 8c. The resulting probability distributions for each case are indicated in Figure 8d. For the transitional flow signal, no significant differences are observed between using the first time derivative filter and the ideal digital high-pass filter. These results are significant for several reasons. First, using a digital filter in post-acquisition allows more flexibility than using a high-pass analog filter during acquisition. This allows for post-acquisition filter adjustment for different signals and flow conditions. Second, using a low order derivative is easier to implement and requires less computational time than a higher order digital filter. A higher order filter requires more terms to implement in the time domain than a low order derivative thus increasing computational time. Implementation of an ideal digital high-pass filter must be done in the frequency domain which requires performing an FFT and IFFFT resulting in more than an order of magnitude increase in computational time.

Nine separate criterion functions were investigated for the baseline case and the three pressure gradient cases. An example of the criterion functions investigated corresponding to the signals in Figure 7 are shown in Figure 9. Six criterion functions were based on the second derivative of the signals while three criterion functions were based on the square of the first derivative. In both these figures, 0.5 seconds of data (1000 data points) are shown from the 16.38 second (32768 point) record. The intermittency determined from the "dual-slope method" for each criterion function was compared to that obtained by direct observation (the eyeball method). In all cases the discrepancy between the two methods was within 5%. From Figure 9 it is observed that differences in turbulence discrimination exist between the various criterion functions. A larger uncertainty is observed in using the second derivative of the streamwise Reynolds heat flux, $d^2u/dt^2$ (CF7).
demarcation between the turbulent and non-turbulent portions for this criterion function is not as pronounced as the others. This correlation could not be desensitized to low frequency unsteadiness resulting in several false turbulence readings. This low frequency unsteadiness was more apparent in several of the other signals (not shown in this paper). The intermittency values obtained using criterion functions based on temperature (CF1) or a single velocity signal (CF2 through CF5) were comparable within 5% throughout the transition region. No advantage was gained by combining velocity signals (CF5), as recommended by Hedley et al. (1974). Using criterion functions based on uv (CF6 and CF9) or vt (CF8) resulted in intermittency values 0.14 to 0.38 lower in the outer boundary layer region ($y/8^* > 4.0$) than the values found from the single-signal-based criterion functions. These discrepancies occurred in the late transitional and early turbulent regions (stations 8 through 13). The range of intermittency values determined for several locations of the baseline case are presented in Figure 10. The large variation in the outer boundary layer is apparent. This same procedure was repeated for each pressure gradient case with similar results (Figures 11 and 12). A complete listing of all the intermittency and threshold values for each criterion function was documented by Keller (1993). The intermittency determined for each criterion function from the "dual-slope method" was compared to the eyeball method for verification and was always within 5%. It is apparent from these results that near-wall intermittency values were similar regardless of the criterion function. Only in the outer boundary layer were significant differences observed. The results from using the temperature based scheme, CF1, were consistent with the results from the other single signal schemes (CF2 through CF5) for all cases investigated. No differences were found using the temperature based criterion function to support the use of separate thermal intermittency factor.

From the above results the criterion functions were divided into two groups, the single signal schemes (CF1 through CF5) and the correlation schemes (CF6 through CF9). One criterion function from each group was selected for further investigation. CF2 was chosen from the first group and CF6 from the second. Each of these criterion functions showed the greatest demarcation between the turbulent and non-turbulent portion of the flow for their respective groups. In addition, these two criterion functions are the ones most commonly used by researchers.

**Best Criterion Function**

Several factors were considered for determining which criterion function is the best choice for use in the transitional boundary layer. These factors include: 1) a sharpness in demarcation between turbulent and non-turbulent portions of the flow, 2) a small variation of threshold value throughout the transition region, 3) a low uncertainty in determining the threshold value, and 4) a low sensitivity of the resulted intermittency to the uncertainty in choosing the threshold value. A single location in the mid-transition region was selected for detailed comparison of the two chosen criterion functions. A location from station 6 for $y/8^* = 1.1$ was selected. For this location, both criterion functions indicated an intermittency value of approximately 0.5.

A detailed view of the two criterion functions and the resultant intermittency function is shown in Figure 13. The raw signals up to 0.5 seconds of Figure 13 are previously shown in Figure 7. Both criterion functions are of the same order of magnitude within the regions labeled 'A' through 'E' in Figure 13. However, each criterion function weights different areas within each region differently. For example, for region 'E', CF6 indicates intense turbulent activity towards the end of the region with less turbulent activity towards the beginning of the region. CF2 indicates the turbulent activity at the beginning and end of region 'E' is of the same order of intensity. The two different criterion functions do not recognize turbulence equivalently; inherent differences exist. It remains to determine which criterion function more accurately represents turbulence. Also shown in Figure 13 is an expanded view of each criterion function in order to
investigate the detailed structure between and within each region. While the regions 'A' through 'E' are of the same order of magnitude, the expanded views show that the areas between these regions are not. CF6 has a much sharper demarcation between the turbulent and non-turbulent portions. This difference results in different variations of threshold value throughout the transition region for each criterion function. For CF2, large variations of the threshold value occur. Typically the threshold value is the smallest very close to the wall and increases nonlinearly, asymptotically approaching a constant value near the edge of the boundary layer. An increase of 500% is typical. No quantitative correlation is found to describe this trend. A similar observation was made by Kuan and Wang (1989 and 1990) using the same criterion function (CF2). For CF6, negligible variation in the threshold value occur in this study. The results from the "dual-slope method" consistently indicate the same threshold value regardless of location. This nature of an almost constant threshold value is especially advantageous in the outer boundary layer in the late transitional and turbulent region because the linear slope representing the Gaussian probability density distribution of the non-turbulent portion in the dual-slope diagram (Figure 4) in these regions becomes very short and vague. The four factors previously mentioned for determining the best threshold value are best satisfied using the Reynolds shear stress. This indicates that the Reynolds shear stress is easier to implement in transitional flow and more accurately indicates the turbulent regions.

While the intermittency factor for the overall record was approximately 0.5 for both criterion functions, the intermittency functions displayed in Figure 13 indicate that regions identified as turbulent were not the same for each criterion function. For example, region 'E' is identified as two turbulent bursts using CF2 but was identified as two large bursts with several smaller bursts using CF6. These smaller bursts are not picked up as the turbulent portion when CF2 is used. Sometimes, using the streamwise velocity may indicate the same overall intermittency factor as the Reynolds shear stress but analysis of the turbulent and non-turbulent portions will most likely not yield the same results.

To further investigate these differences, 30 points through the boundary layer at station 6 were conditionally sampled using both criterion functions. Figure 14 shows the distribution of intermittency through the boundary layer. The discrepancy in outer boundary layer intermittency is apparent for y/δ > 0.4 with Γ from CF6 being consistently lower than Γ from CF2. For y/δ < 0.4 the intermittency values from each criterion function are nearly identical. The conditionally sampled mean velocity profiles for each criterion function are presented in $U^+$ versus $Y^+$ coordinates and are shown in Figure

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**Figure 12:** Intermittency variation for different criterion functions ($K = 0.16 \times 10^{-6}$ and $K = 0.25 \times 10^{-6}$).

**Figure 13:** Comparison of two criterion functions and corresponding intermittency functions, $\Gamma = 0.5$, $y/\delta = 1.1$ (baseline case).

**Figure 14:** Distribution of intermittency through the boundary layer.
15. Both criterion functions result in similar profiles. The non-turbulent portions matched the Blasius profile while the turbulent portions do not exhibit the logarithmic law-of-the-wall region. No differences were discernible between the abilities of each criterion function to separate the mean velocity. The conditionally sampled Reynolds normal stresses are shown in Figure 16 where differences are seen between the results from the two criterion functions. For CF2, the peak intensity in the non-turbulent portion is 11% while the corresponding peak intensity for CF6 is 9%. Both peak intensities occur at approximately \( y/\delta = 0.3 \). The turbulent part from using CF6 indicates higher values in Reynolds normal stress than the results from using CF2. Similar results are observed for the Reynolds shear stress, \( -uv/u^* \) (not shown here). The criterion function using streamwise mean velocity under-evaluated the Reynolds stresses in the turbulent portion and over-evaluated them in the non-turbulent portion.

**Intermittency Distributions**

Using CF6 as the best criterion function, the intermittency distribution through the transition region for each case was determined. The results are shown in Figures 17-20. The results for a fully turbulent boundary layer obtained by Klebanoff (1954) are included for comparison. The dashed lines in each figure represent the uncertainty in determining the boundary layer thickness, \( \delta \). The uncertainty in the mean streamwise velocity for the three-wire sensor is approximately 3% which corresponds to an uncertainty in \( \delta \) of \( \pm 5\% \). This variation in \( \delta \) results in a large variation in \( \Gamma \) for the fully turbulent profile near the edge of the boundary layer. For the baseline case, the intermittency distributions for stations 8 through 13 are seen to match the fully turbulent profile (within the uncertainty band). Station 6 exhibits a peak in intermittency away from the wall similar to that reported by Kuan and Wang (1989 & 1990), Sohn, O’Brien, and Reshotko (1989), and Gostelow and Walker (1990). For the acceleration cases of \( K = 0.07 \times 10^{-6} \) and \( K = 0.16 \times 10^{-6} \) (Figures 18 and 19 respectively), similar observations are made. In the late transitional and early turbulent regions intermittency distributions match the fully turbulent results of Klebanoff. Peak values in intermittency for the early to mid-transitional regions occur away from the wall at approximately \( y/\delta = 0.3 \) for all three accelerating cases. As \( K \) increases, the length of transition increases thus allowing more stations to be measured in the transition region. For \( K = 0.16 \times 10^{-6} \) in Figure 19, three profiles are observed to have intermittency peaks away from the wall. These peaks disappear approximately mid-way through the transition region. Visual inspection of the instantaneous correlation signals verifies that the frequency of breakdown increases to a maximum away from the wall then decreases toward zero in the free-stream. A similar observation was made by Kuan and Wang (1990), who attributed these peaks to the overhang of a typical turbulent spot. Blair (1992) did not observe the peak in his experiment in a transitional boundary layer with a free-stream turbulence of 0.8% and \( K = 0.2 \times 10^{-6} \). However, he reported observing a near-wall minimum and a peak at about \( y/\delta = 0.3 \) in the intermittency distribution in higher turbulence cases (1.9 and 2.5%). Mayle (1991) pointed out the controversy on the peak in intermittency distribution across the boundary layer and...
attributed it to the differences in the turbulent flow discrimination schemes used to determine intermittency. As early as 1958, Dhawan and Narasimha concluded that although the $\Gamma(y)$ variation is probably important to the detailed structure of the turbulent motion associated with the turbulent spots, the value near the wall is the characteristic property for the transition region. These authors believe the peak intermittency at about $y/\delta = 0.3$ in the early to middle transitional boundary layer is real and it reflects the stretching of a turbulent spot away from the wall. This vortex stretching is a very important part of the vortex dynamics during the early transitional process.

The general appearance of the intermittency distribution across the boundary layer of an accelerated flow is very similar to that of a non-accelerated flow; however, a distinctive near-wall minimum exists at about $y/\delta = 0.1$ for most of the stations even as late as $\Gamma = 0.9$ (Figures 18 and 19). The intermittency value increases at a clear trend toward the wall from this minimum, which is not observed in the baseline case.
in Figure 17. For the strongest accelerating case, $K = 0.25 \times 10^{-6}$, the intermittency distributions for stations 8 to 11 are almost identical (see Figure 20). This implies a strong suppression of the growth of turbulent spots. The transition process for this case is not completed at the exit of the test section of the current facility.

To find the intermittency factor, $\Gamma(x)$, through the transition region, the method first developed by Dhawan and Narasimha (1958) was used. Dhawan and Narasimha proposed a "universal" intermittency distribution of the form:

$$\Gamma(x) = 1 - \exp[-0.412(x-x_s)^2/\xi^2]$$

where

$$\xi = x_{\Gamma=0.75} \cdot x_{\Gamma=0.25}.\,$$

The applicability of using this technique in boundary layer flows subjected to pressure gradients was discussed by Narasimha, et al. (1984).

To determine the start of transition, $x_s$, the following procedure is taken following their recommendations. The function $\sqrt{-\ln(1-\Gamma(x))}$ is plotted versus $x$ and a straight line fit to the data between 0.25 < $\Gamma$ < 0.75. The $x$-intercept is $x_s$. An appropriate value of $\Gamma(x)$ to represent the intermittency at each $x$-location must still be determined in light of the non-uniform distribution of intermittency across the boundary layer. Mayle (1991) stated that most researchers which use anemometers to determine the intermittency typically chose the near-wall value to be around $y/\delta = 0.2$. In order to look into this issue, three locations were chosen for the selection of the representative intermittency value, $\Gamma(x)$: the location of the intermittency peak (approximately $y/\delta = 0.3$), the value at $y/\delta = 0.2$ as suggested by Mayle, and the value at the local minimum near the wall ($y/\delta = 0.1$) for each station. Figures 21a and 21b are the results from using the peak value of intermittency (note: since the flow for $K = 0.25 \times 10^{-6}$ never completed transition in the present facility they are excluded from further discussion). For $K = 0.16 \times 10^{-6}$, only the results for $\Gamma(x) > 0.6$ followed a linear relation when plotted in $F(\Gamma)$ versus $x$ coordinate. The representative intermittency distribution obtained from these results are shown in Figure 21b. Too large a deviation from the universal distribution is observed for $K = 0.16 \times 10^{-6}$ to justify using the peak value in $\Gamma$. This procedure was repeated for the intermittency values obtained at $y/\delta = 0.2$ and the results are shown in Figures 21c and 21d. The representative intermittency distribution still shows a large variation from the correlation of Dhawan and Narasimha (1958). The results using the values of intermittency obtained from the minimum near the wall ($y/\delta = 0.1$) are shown in Figures 21e and 21f. For $K = 0.16 \times 10^{-6}$, two linear regions of different slopes are present in the $F(\Gamma)$ versus $x$ coordinate similar to the results of Narasimha et al. (1984) and Blair (1992). Narasimha (1985) termed this sudden change in flow behavior "subtransition" indicating the flow changes from a subcritical to a supercritical state. The near-wall intermittency distribution is seen to match the "universal" distribution of Dhawan and Narasimha with slightly higher values for $K = 0.16 \times 10^{-6}$ in the early transition region. Acharya (1984) and Blair (1992), measured the streamwise distribution of boundary layer intermittency for flows with $K > 0$ and both reported a similar observations. The results of the near-wall intermittency distributions indicate that in order to match the correlation of Dhawan and Narasimha.
(1958), the near-wall value of intermittency at $y^+/\delta = 0.1$ should be used instead of at $y^+/\delta = 0.2$ suggested by Mayle (1991).

CONCLUSIONS

The effects of different criterion functions on the determination of intermittency were investigated for application in heated transitional boundary layers with and without streamwise acceleration. Nine separate criterion functions were investigated for the zero-pressure gradient baseline case and three constant $K$, accelerated cases. The criterion functions were classified into two general categories; single signal schemes, those based in $U$, $V$, and $T$, and correlation schemes, those based on $uv$, $vt$, or $ut$. For the baseline case, criterion functions based on the correlation schemes resulted in intermittency values 0.14 to 0.38 lower in the outer boundary layer region ($y^+/\delta > 4.0$) than the values found from the single signal schemes. Similar differences were found for the accelerated cases. No differences were found using the temperature based criterion function to support the use of a separate thermal intermittency factor in accelerated flows.

Inherent differences exist between each criterion function's turbulence recognition capabilities. Each criterion function weights different areas within a turbulent burst differently. No differences were discernible between the abilities of each criterion function to separate the mean velocity; however the results of using single signal schemes tended to under-evaluate the Reynolds stresses in the turbulent portion and over-evaluate them in the non-turbulent portion. A criterion function based on Reynolds stress, $(\partial u/\partial t)^2$, resulted in the sharpest demarcation between turbulent and non-turbulent portions of the flow. This criterion function also had a negligible variation of threshold value throughout the transition region with the lowest uncertainty in determining the threshold value and the lowest sensitivity of the resultant intermittency to the variation of the threshold value. These results indicate that using the Reynolds shear stress for turbulent/non-turbulent discrimination in a heated transitional boundary layer is superior to a single velocity or temperature criterion function to separate the mean velocity; however the weights different areas within a turbulent burst differently.

In a criterion function has a clear trend of increasing values of intermittency toward the wall from this minimum were observed for the accelerating flow cases. To match the universal intermittency distribution of Dhawan and Narasimha (1958), the values of intermittency at this near-wall minimum $y^+/\delta = 0.1$ should be used as the representative "near-wall" values.

Using a digital time derivative is considered superior for use as a criterion function than an ideal digital high-pass filter since no significant differences are observed between the two methods in determining the probability densities of a transitional flow and the derivative requires less computational time.

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