A STUDY ON NEW IDENTIFICATION TECHNIQUE WITH APPLICATION TO SQUEEZE FILM BEARING SYSTEM

Ting Nung Shiau  
Department of Mechanical Engineering  
National Chung Cheng University  
Chia Yi, Taiwan, Republic of China

Chun Pao Kuo  
Chung Shan Institute of Science and Technology  
Taiwan, Republic of China

G. J. Sheu and P. L. Kuo  
Institute of Aeronautics and Astronautics  
National Cheng Kung University  
Tainan, Taiwan, Republic of China

ABSTRACT

Two identification techniques, which are the Method of Feasible Directions (MFD) based on optimization concept and the Hybrid Method (HM) combining the merits of the State Variable Filter method (SVF) and MFD, are proposed for the parameters identification of rotor system with squeeze-film damper (SFD). The parameters of SFD, including hydrodynamic inertia, damping and stiffness, are identified and the results obtained by using MFD and HM are compared to those using SVF. The accuracy and efficiency of using these techniques are demonstrated by the experimental simulation with various noise levels and different initial values. The results indicate that the choice of initial values is of no significant effect on SVF method and the accuracy of SVF depends on the level of noise. However, for MFD method, it is almost independent of noise effect but significantly affected by the choice of initial values. The Hybrid Method (HM) is proposed to overcome these handicaps and found of better accuracy and efficiency than SVF and MFD. Then, it is highly recommended for the parameters identification of system with noise effect.

NOMENCLATURE

- $c_{hx}, c_{hy}, c_{hx}, c_{hy}$: direct and cross damping coefficients of squeeze film damper, respectively
- $c_{ax}, c_{ay}$: direct damping coefficients without squeeze film effect
- $D$: design variables vector
- $D(i)$: design variables
- $F(t), F_{m}(t)$: excitation and measured excitation force vector, respectively
- $F(D)$: objective function
- $F_{x}, F_{y}$: forces of squeeze film in $x$ and $y$ directions
- $g_{f}, g_{x}, g_{y}$: identified parameter matrices
- $f_{x}, f_{y}$: excitation forces in $x$ and $y$ directions
- $g_{j}(D)$: inequality constraint functions
- $h$: initial condition vector
- $h$: clearance between the journal and the housing
- $J$: least squares cost function
- $K_{xx}, K_{yy}, K_{xy}, K_{yx}$: nondimensional direct and cross stiffness coefficients of system, respectively
- $k_{hx}, k_{hy}, k_{hx}, k_{hy}$: direct and cross stiffness coefficients of squeeze film damper, respectively
- $k_{ax}, k_{ay}$: direct stiffness coefficients without squeeze film effect
- $M_{xx}, M_{yy}, M_{xy}, M_{yx}$: nondimensional direct and cross inertia coefficients of system, respectively
- $m_{x}$: the inertia mass of the system without squeeze film effect
- $m_{hx}, m_{hy}, m_{hx}, m_{hy}$: direct and cross inertia coefficients of squeeze film damper, respectively
- $N$: number of samples on each measured signal

Presented at the International Gas Turbine and Aeroengine Congress and Exposition  
Cincinnati, Ohio — May 24–27, 1993
INTRODUCTION

Since the rotor bearing system in modern high speed rotating machinery often experiences large vibration problem which may be caused by rotor shaft unbalance and/or dynamic instability of the system, the fundamental work is to reduce the high vibration amplitude to increase the fatigue life and performance of system. Many investigators [Mohan and Hahn, 1974; Cunningham et al., 1974; Bansal and Hibner, 1978] have shown that squeeze-film damper (SFD) can be used to significantly attenuate the vibration problem and stabilize the rotordynamics of turbomachinery. However, it has also been shown that an improperly designed squeeze-film damper can induce large dynamic loads. Therefore, it is very important to accurately estimate the properties of the squeeze-film damper. Some researchers [Modest and Tichy, 1978; Tichy, 1983; Tichy and Chen, 1985; San Andres, 1985; San Andres and Vance, 1986] have shown that, except the primary damping and stiffness, the fluid inertia effect of squeeze film damper should also be taken into account in the design analysis.

This leads to the necessity of estimating the dynamic coefficients of SFD including inertia, damping and stiffness. A number of techniques dealing with identification of bearing coefficients have been developed for the linear or linearized system. The identification methods for squeeze-film damper can be categorized in time domain as well as in frequency domain techniques. A frequency domain approach for estimating direct- and cross-damping coefficients of SFD was given by Burrows and Sahinkaya (1982). A refined frequency-domain parameter estimation algorithm was also developed by Burrows et al. (1987) to determine twelve squeeze-film linear dynamic coefficients.

Ramli et al. (1987) presented a time-domain technique for estimating the direct damping and inertia coefficients of a squeeze-film bearing based on transient response experiments. It involves the application of invariant imbedding method (IIM), which is of recursive way using a nonlinear estimation algorithm [Detchmendy and Sridhar, 1966]. It also has been applied to estimate the damping coefficients from the force response data by Stanway (1983) using synchronous excitation. Ellis et al. (1988) presented the comparison of IIM and state variable filter (SVF) method in identifying the dynamic coefficients including inertia, damping and stiffness of squeeze-film damper with model of single degree of freedom. Both methods yield a recursive scheme for sequentially estimating all coefficients. The comparison indicates that the SVF method is much superior, not only for computational efficiency but also for accuracy. The state variable filter approach was further applied to estimate the complete set of twelve linear hydrodynamic coefficients by Ellis et al. (1990).

This study is proposing two new identification techniques, one is the technique of optimization method—Method of Feasible Directions (MFD), the other is Hybrid Method (HM). The MFD starts from a global viewpoint by choosing a period of experimental data in time domain, and using curve fitting concepts together with optimization method. While SVF method is to update the parameters step by step locally with the condition of minimum difference between each measured data and estimated value in time-series. The concepts of optimization method is distinctly different from those of SVF method. The Hybrid Method, which combines the merits of SVF and MFD, is expected to be of better accuracy and efficiency.

EQUATIONS OF MOTION

Consider a simple squeeze-film damper as shown in Fig.1. The mass, damping and stiffness of the system with empty-oil condition are denoted by \( m_s \), \( c_s \) and \( k_s \). The equations of motion of rotor bearing system with squeeze-film damper can be written as

\[
\frac{dx}{dt} + \frac{c_s x}{m_s} + \frac{k_s x}{m_s} = f_x
\]

\[
\frac{dy}{dt} + \frac{c_s y}{m_s} + \frac{k_s y}{m_s} = f_y
\]

where \( F_x \) and \( F_y \) are squeeze-film forces and functions of displacements, velocities, and accelerations of the journal motion. And \( f_x \) and \( f_y \) are the excitation forces applied on the journal. It should be noted that the system cross coefficients including \( c_{sy}, c_{ys}, k_{sys} \) and \( k_{syy} \) are neglected compared to the hydrodynamic effects. Let \( F_x \) and \( F_y \) to be of linear form as follows:

\[
F_x = m_h z_y \frac{dz}{dt}^2 + m_h x_y \frac{d^2 y}{dt^2} + c_s z_x \frac{dz}{dt} + c_h s_x \frac{d^2 x}{dt^2} + k_{syy} \frac{dy}{dt} + k_{sxy} z + k_{esy} y
\]

\[
F_y = m_h y_x \frac{dx}{dt}^2 + m_h y_y \frac{d^2 z}{dt^2} + c_s y_x \frac{dy}{dt} + c_h y_x \frac{d^2 y}{dt^2} + k_{sxy} \frac{dx}{dt} + k_{esy} z + k_{syy} y
\]

where the subscripts "s" and "h" denote the structural system parameters and hydrodynamic components respectively. Define the nondimensional variables as follows:

\[
X \equiv x/h, \quad Y \equiv y/h, \quad \tau \equiv \omega t, \quad (') \equiv d/dr
\]

where \( h \) is the clearance between the journal and the housing, \( \omega \) is the natural frequency of the system. Substituting Eqs.(3)–(5) into Eqs.(1) and (2), one can obtain the equations of motion in nondimensional form as follows:
\[ \begin{bmatrix} M_{zz} & M_{zy} \\ M_{yz} & M_{yy} \end{bmatrix} \begin{bmatrix} \ddot{X} \\ \ddot{Y} \end{bmatrix} + \begin{bmatrix} C_{xz} & C_{xy} \\ C_{yz} & C_{yy} \end{bmatrix} \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} + \begin{bmatrix} K_{xz} & K_{xy} \\ K_{yz} & K_{yy} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \end{bmatrix} \]  

(6)

where

\[ \begin{align*}
M_{zz} &= \omega^2 (m_z + m_{hzz}), \\
M_{zy} &= \omega^2 m_{hzy}, \\
M_{yz} &= \omega^2 (m_z + m_{hzy}), \\
M_{yy} &= \omega^2 (m_z + m_{hyy}), \\
C_{xz} &= \omega (c_{xzz} + c_{hxx}), \\
C_{xy} &= \omega c_{hxy}, \\
C_{yz} &= \omega c_{hzy}, \\
C_{yy} &= \omega (c_{yy} + c_{hyy}), \\
K_{xz} &= \eta (k_{xzz} + k_{hxx}), \\
K_{xy} &= \eta k_{hxy}, \\
K_{yz} &= \eta k_{hzy}, \\
K_{yy} &= \eta (k_{yy} + k_{hyy})
\end{align*} \]

Eqn.(6) can be symbolically written as

\[ M \ddot{Z} + C \dot{Z} + KZ = F \]  

(8)

where

\[ M = \begin{bmatrix} M_{zz} & M_{zy} \\ M_{yz} & M_{yy} \end{bmatrix}, \quad C = \begin{bmatrix} C_{xz} & C_{xy} \\ C_{yz} & C_{yy} \end{bmatrix}, \quad K = \begin{bmatrix} K_{xz} & K_{xy} \\ K_{yz} & K_{yy} \end{bmatrix}, \]

\[ Z = \begin{bmatrix} X \\ Y \end{bmatrix}, \quad F = \begin{bmatrix} f_x \\ f_y \end{bmatrix} \]

It should be noted that the coefficient matrices of Eqn.(8) can be expressed as:

\[ \begin{align*}
M &= M_s + M_h, \\
C &= C_s + C_h, \\
K &= K_s + K_h
\end{align*} \]

Premultiplying Eqn.(8) by \( M^{-1} \), it yields

\[ \ddot{Z} + a \ddot{Z} + b \dot{Z} + BZ = BF \]  

(9)

where

\[ a = M^{-1}C, \quad b = M^{-1}K, \quad B = M^{-1} \]

Eqn.(9) is a more convenient form in the identification process. Obviously, if \( a, b, \) and \( B \) are known, the real system coefficients \( M, C, \) and \( K \) can be identified. The concept for the identification of squeeze-film parameters may be stated first to identify the parameters \( M_s, C_s, \) and \( K_s \) with empty-oil condition, and secondly to identify the parameter matrices \( M, C, \) and \( K \) with full-oil condition. Then, the parameters \( m_{hxx}, m_{hxy}, m_{hxy}, m_{hyy}, c_{hxx}, c_{hxy}, c_{hxy}, k_{hxx}, k_{hxy}, k_{hxy}, k_{hyy} \) can be determined through Eqn.(7).

**IDENTIFICATION TECHNIQUES**

1. **State Variable Filter Method (SVF)**

The concept of SVF method is to estimate the parameter values \( a, b, \) and \( B \) shown in Eqn.(9) by minimizing the difference between the measured response \( Z_m(t_i) \) and estimated response \( Z(t_i) \) at the sample times \( t_i \) [Gawthrop et al. (1988)]. Consider the system with input \( F(t) \) and output \( Z(t) \). And the relationship between the measured and the estimated input and output can be expressed as:

\[ F_m(t_i) = F(t_i) + \epsilon_F(t_i) \]

\[ Z_m(t_i) = Z(t_i) + \epsilon_Z(t_i) \]

where \( \epsilon_F(t_i) \) and \( \epsilon_Z(t_i) \) are the observation noise of input and output at the sampling time \( t_i \). The subscript “m” denotes measured response. Since the identification problem is of “linear in-the-parameter” type, it is possible to derive an alternative “linear least-square” algorithm, which is computationally more efficient. The first step is to cast Eqn.(9) into the following form:

\[ \ddot{Z} + c \ddot{Z} + d \dot{Z} = BF + fZ + gZ \]  

(10)

where \( f = c - a \) and \( g = d - b \).

The system coefficients of Eqn.(10), \( c \) and \( d \), are known constant matrices.

It is noticed that the response of the system is now linearly related to the values of the parameters in \( f, g \) and \( B \). The identification of all the individual elements of \( f, g \) and \( B \) requires the filter response to a total of ten inputs. The filter is specified by the values of the elements in \( c \) and \( d \). Consider the following “auxiliary differential equations”:

\[ \ddot{\lambda}_{ij} + c \dot{\lambda}_{ij} + d \lambda_{ij} = \phi_{ij} \quad (i, j = 1, 2) \]  

(11)

\[ \ddot{\mu}_{ij} + c \dot{\mu}_{ij} + d \mu_{ij} = \psi_{ij} \quad (i, j = 1, 2) \]  

(12)

\[ \ddot{\rho}_i + c \dot{\rho}_i + d \rho_i = 0 \quad (i = 1, 2) \]  

(13)

where \( \lambda_{ij}, \mu_{ij}, \rho_i, \phi_{ij} \) and \( \psi_{ij} \) are all two-vectors. The forcing functions for the auxiliary equations are of the following forms:

\[ \phi_{11} = \begin{bmatrix} X \\ 0 \end{bmatrix}, \quad \phi_{12} = \begin{bmatrix} Y \\ 0 \end{bmatrix}, \quad \varphi_{21} = \begin{bmatrix} 0 \\ X \end{bmatrix}, \quad \phi_{22} = \begin{bmatrix} 0 \\ Y \end{bmatrix} \]

\[ \psi_{21} = \begin{bmatrix} 0 \\ f_x \end{bmatrix}, \quad \psi_{22} = \begin{bmatrix} 0 \\ f_y \end{bmatrix} \]

(14)

The comparison of Eqns.(11)-(13) to Eqn.(10) with appropriate initial conditions [Roberts et al. (1990)], gives

\[ Z = \sum_{i=1}^{2} \sum_{j=1}^{2} (f_{ij} \dot{\lambda}_{ij} + g_{ij} \lambda_{ij} + B_{ij} \mu_{ij}) + \sum_{i=1}^{2} (p_i \dot{\rho}_i + q_i \rho_i) \]  

(15)

where \( f_{ij}, g_{ij} \) and \( B_{ij} \) are the corresponding elements of matrices \( f, g \) and \( B \) respectively. The parameters \( p_i, q_i \) and the filter responses \( \rho_i \) (i=1,2) in Eqn.(15) have been introduced to allow for the effect of the nonzero initial conditions. A convenient choice of initial conditions for the auxiliary equations is given as follows:

\[ \lambda_{ij}(0) = \dot{\lambda}_{ij}(0) = \mu_{ij}(0) = \dot{\mu}_{ij}(0) = 0 \quad (i, j = 1, 2) \]

\[ \rho_i(0) = 0, \quad \dot{\rho}_i = h \quad (i = 1, 2) \]  

(16)

Here the vector \( h \) is chosen such that its \( i-th \) element is of a
value of unity—i.e.,
\[ h = [h_k] \]
where
\[ h_k = \begin{cases} 0, & \text{for } k \neq i \\ 1, & \text{for } k = i \end{cases} \]

An incremental solution of Eqns. (11) to (13) by time marching through the measured displacement and force data with given initial conditions results in sequence of values for \( \lambda_{ij}, \mu_{ij}, \rho_i \) and their time derivatives. Define an augmented matrix \( \Gamma(t) \) as follow:

\[ \Gamma(t) = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \ldots & \lambda_{1N} \\ \lambda_{21} & \lambda_{22} & \ldots & \lambda_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{N1} & \lambda_{N2} & \ldots & \lambda_{NN} \end{bmatrix} \]

which is constructed at each time interval. By comparison with Eqn. (18), one can show that

\[ Z(t) = \Gamma(t)\Theta \]

where \( \Theta \) is the parameter vector

\[ \Theta = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{bmatrix} \]

The unknown parameter vector \( \Theta \) can be estimated by minimizing the difference between \( Z(t_i) \) from Eqn. (18), and the measured data, \( Z_m(t_i) \) at the sample times \( t_i \). It is usually convenient to minimize in least square sense, i.e., to minimize the least square cost function.

\[ J = \sum_{i=1}^{N-1} [Z_m(t_i) - \Gamma(t_i)\Theta]^2 \]

The sequential least-square estimation (SLSE) is employed to estimate \( \Theta(t_i) \) of \( \Theta \), sequentially, for \( i=0,1,2,...,(N-1) \) by marching in time steps, through the data. As described by Hsia (1979), one can establish the auxiliary matrix \( P \) as follow:

\[ P = \left( \sum_{i=1}^{N-1} \Gamma_i^T \Gamma_i \right)^{-1} \]

And repeat the calculation for each time steps of the following Eqn.

\[ \Theta(i+1) = \Theta(i) + \Gamma(i+1)P(i+1) \]

where \( \theta_i \) is the parameter vector, \( P(i) \) is the Jacobian matrix, \( \Gamma(i) \) is the Hessian matrix, \( \Theta(i) \) is the parameter vector at time \( t_i \), and \( \Gamma(i) \) is the Hessian matrix at time \( t_i \).

The optimization technique employed in this study requires two procedures: (1) determining the search directions; and (2) deciding the search-step size. Based on the features of MFD, the searching direction can be determined. It is noted that both objective and constraint functions are implicit nonlinear in design variables \( D \).

The design variable vector \( D \) is defined as:

\[ D = \{ D(1), ..., D(n) \}^T, \quad n=12 \]

The objective function \( F(D) \) is defined as:

\[ F(D) = \sum_{n=1}^{N-1} [ (\hat{X}_n - X_n)^2 + (\hat{Y}_n - Y_n)^2 ] \]

The \( \hat{X} \) and \( \hat{Y} \) are the estimated displacement response which can be solved by direct integrating Eqn. (6) of the form:

\[ \begin{bmatrix} D(1) \\ D(2) \\ D(3) \\ D(4) \\ D(5) \\ D(6) \\ D(7) \\ D(8) \end{bmatrix} = \begin{bmatrix} [X] \\ [Y] \end{bmatrix} + \begin{bmatrix} [D(3)] \\ [D(4)] \\ [D(5)] \\ [D(6)] \end{bmatrix} \]

The constraint functions are chosen as follows:

\[ g_j(D) = | \hat{X}_j/A_x - X_j/A_x | - e < 0 \]

where \( A_x \) and \( A_y \) are the reference amplitude of measured shaft displacements in x and y directions, \( j \) and \( k \) denote the number of the peak, valley and zero points of measured displacements in \( x \) and \( y \) coordinates respectively, and \( e \) is a scalar factor which can be adjusted. It is noted that both objective and constraint functions are implicit nonlinear in design variables \( D \).

The optimization technique employed in this study requires two procedures: (1) determining the search directions; and (2) deciding the search-step size. Based on the features of MFD, the searching direction can be determined. Then the step size is obtained using one-dimensional search technique. The procedures are summarized as follows:

Step 1: Guess initial values and set larger values of factors \( e \) in Eqn. (26), i.e. to form a larger scope of feasible region.

Step 2: Find a set of optimum values (local minimum) by means of MFD.

Step 3: Reduce the value of \( e \) by multiplying a scalar, say 0.5 or 0.2, i.e. to reduce the scope of feasible region.

Step 4: Use the previous optimum values as initial values and find another set of optimum values by means of MFD.
3. Hybrid Method

The Hybrid Method combines the merits of both State Variable Filter (SVF) method and Method of Feasible Directions (MFD). It will be described more details in the numerical results and discussion. In general, it uses the identified results of SVF as the initial values for MFD.

NUMERICAL RESULTS AND DISCUSSION

The objective of this study is to develop the identification techniques with good accuracy and efficiency for identifying the parameters values of fluid inertia, damping, and stiffness of SFD in rotor system. Fig. 2 shows the equipment of system identification which includes the measurements of inputs and outputs. As shown by Ellis et al. (1990), random signals are subsequently applied as the forcing inputs which will lead to good parameter estimates under typical experimental conditions.

Since electronic noise and digital quantization will inevitably exist in experimental data, the identification techniques with the cure of measurement noise is the major concern. To assess the influence of measurement noise, Gaussian independent random numbers from computer’s pseudo-random number generator are added to the simulated data, Zn(t), which can easily be obtained by numerically solving the equations of motion. A conventional measurement of the relative noise level is defined as follow:

\[
\text{Noise level} = \left[ \frac{1}{N} \sum_{i=1}^{N} e_Z^2(t_i) / \sum_{i=1}^{N} Z_n^2(t_i) \right]^{1/2}
\]  

(27)

where \(e_Z(t_i)\) and \(Z_n(t_i)\) are the components of the noise and the noiseless response vectors in the \(i\)-th instant of time.

The forced response are measured for cases with and without oil in SFD to separate the fluid inertia, hydrodynamic damping and stiffness from the entire rotor bearing system. An example is employed to demonstrate the accuracy and the efficiency of the techniques. The identified results of a squeeze-film damper using SVF, MFD and HM are obtained for various noise levels. The initial conditions for all the parameters to be identified are set to be zero for the SVF and HM, which is denoted by IC0. For MFD method, the initial values are usually experience dependent.

The example of rotor system with following positive cross parameter values is considered:

\[
M = \begin{bmatrix}
M_{xx} & M_{xy} \\
M_{yx} & M_{yy}
\end{bmatrix} = \begin{bmatrix}
1.39 & 0.08 \\
0.07 & 1.38
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
C_{xx} & C_{xy} \\
C_{yx} & C_{yy}
\end{bmatrix} = \begin{bmatrix}
3.5 & 0.6 \\
0.56 & 3.4
\end{bmatrix}
\]

\[
K = \begin{bmatrix}
K_{xx} & K_{xy} \\
K_{yx} & K_{yy}
\end{bmatrix} = \begin{bmatrix}
1.25 & 0.11 \\
0.12 & 1.37
\end{bmatrix}
\]

The results of using SVF, MFD and HM are shown in Tables 1(a)–1(g). Figs. 3(a)–3(d) show the results of four fluid inertia coefficients using SVF. It can be seen that for high level of noise, the accuracy of SVF is decreased. The damping and stiffness terms are not shown since the noise effects are not significant. It should be noted that the initial condition effects on the results of SVF are almost negligible. However, as shown in Figs. 4(a)–4(d), the results of MFD is almost independent of noise effects. On the contrast, as shown in Figs. 5(a)–5(d), the different initial conditions will significantly affect the identified results. The following three different sets of initial conditions are studied.

(1) Initial condition 1 (IC1):

\[
M_{xx} = 1.3, \quad M_{xy} = 0.0, \quad C_{xx} = 5.0 \\
M_{yx} = 0.0, \quad M_{yy} = 1.3, \quad C_{yy} = 0.0 \\
C_{xy} = 0.0, \quad K_{xx} = 1.0, \quad K_{xy} = 0.0 \\
C_{yy} = 5.0, \quad K_{yx} = 0.0, \quad K_{yy} = 1.0
\]

(2) Initial condition 2 (IC2):

\[
M_{xx} = 1.0, \quad M_{xy} = 0.0, \quad C_{xx} = 2.0 \\
M_{yx} = 0.0, \quad M_{yy} = 1.0, \quad C_{yy} = 0.0 \\
C_{xy} = 0.0, \quad K_{xx} = 1.5, \quad K_{xy} = 0.0 \\
C_{yy} = 2.0, \quad K_{yx} = 0.0, \quad K_{yy} = 1.5
\]

(3) Initial condition 3 (IC3):

\[
M_{xx} = 0.5, \quad M_{xy} = 0.0, \quad C_{xx} = 1.0 \\
M_{yx} = 0.0, \quad M_{yy} = 0.5, \quad C_{yy} = 0.0 \\
C_{xy} = 0.0, \quad K_{xx} = 1.0, \quad K_{xy} = 0.0 \\
C_{yy} = 1.0, \quad K_{yx} = 0.0, \quad K_{yy} = 1.0
\]

In order to overcome these handicaps, the Hybrid Method (HM) is proposed to combine the merits of both SVF and MFD, i.e., it employs the identified results of SVF as the initial condition for the use of MFD. Figs. 6(a)–6(h) show that the results of Hybrid method is of better accuracy than those of SVF and MFD.

CONCLUSION

Two identification techniques, which are the Method of Feasible Directions (MFD) and Hybrid Method (HM), have been proposed for the parameters identification of rotor bearing system with squeeze-film damper. The results of using these techniques are compared to those of State Variable Filter (SVF) method. The conclusions can be summarized as follows:

(1) The SVF method is insensitive to the choice of initial values as well as the low level noise as many authors presented. However, for high noise level, the accuracy of using SVF method will be decreased.

(2) The accuracy of using MFD method depends on the choice of initial values. However, it is insensitive to the noise level. If the initial values are chosen properly, which is usually experience dependent, the MFD method will lead to good accuracy and efficiency. In addition, it is required to well define the optimization problem to yield good results.

(3) The Hybrid Method, which combines the merits of SVF and MFD methods, is proposed for the parameters identification. The primary advantage of using Hybrid Method is that it is not only independent to the choice of initial values but also insensitive to noise level. Since most of sys-
tem measurements are of certain level of noise, the Hybrid Method is highly recommended.

REFERENCES


Hisia, T. C., 1979, “System Identification: Least-Square Method”.

Table 1(a) The estimated results of SVF method, MFD and Hybrid Method in identification of $M$, $C$, and $K$ with noise level = 0.0

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$M_{xx}$</th>
<th>$M_{yy}$</th>
<th>$M_{yz}$</th>
<th>$M_{yx}$</th>
<th>$C_{xx}$</th>
<th>$C_{yy}$</th>
<th>$C_{xy}$</th>
<th>$C_{yx}$</th>
<th>$K_{xx}$</th>
<th>$K_{yy}$</th>
<th>$K_{yz}$</th>
<th>$K_{xy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True values</td>
<td>1.39</td>
<td>0.08</td>
<td>0.07</td>
<td>1.38</td>
<td>3.5</td>
<td>0.6</td>
<td>0.56</td>
<td>3.4</td>
<td>1.25</td>
<td>0.11</td>
<td>0.12</td>
<td>1.37</td>
</tr>
<tr>
<td>SVF method</td>
<td>1.2809</td>
<td>0.0536</td>
<td>0.0442</td>
<td>1.2752</td>
<td>3.5047</td>
<td>0.5978</td>
<td>0.5564</td>
<td>3.3984</td>
<td>1.2500</td>
<td>0.1101</td>
<td>0.1200</td>
<td>1.3699</td>
</tr>
<tr>
<td>% error</td>
<td>7.85%</td>
<td>32.96%</td>
<td>36.88%</td>
<td>7.59%</td>
<td>0.14%</td>
<td>0.36%</td>
<td>0.64%</td>
<td>0.05%</td>
<td>0.00%</td>
<td>0.12%</td>
<td>0.02%</td>
<td>0.01%</td>
</tr>
<tr>
<td>MFD method</td>
<td>1.3528</td>
<td>0.0513</td>
<td>0.0487</td>
<td>1.3519</td>
<td>3.4885</td>
<td>0.5901</td>
<td>0.5554</td>
<td>3.3795</td>
<td>1.2417</td>
<td>0.1011</td>
<td>0.1174</td>
<td>1.3602</td>
</tr>
<tr>
<td>% error</td>
<td>2.67%</td>
<td>35.93%</td>
<td>30.43%</td>
<td>2.03%</td>
<td>0.33%</td>
<td>1.66%</td>
<td>0.82%</td>
<td>0.60%</td>
<td>0.66%</td>
<td>8.10%</td>
<td>2.13%</td>
<td>0.72%</td>
</tr>
<tr>
<td>Hybrid Method</td>
<td>1.3750</td>
<td>0.0521</td>
<td>0.0564</td>
<td>1.3399</td>
<td>3.4904</td>
<td>0.5854</td>
<td>0.5565</td>
<td>3.3902</td>
<td>1.2468</td>
<td>0.1079</td>
<td>0.1169</td>
<td>1.3643</td>
</tr>
<tr>
<td>% error</td>
<td>1.08%</td>
<td>34.82%</td>
<td>19.41%</td>
<td>2.91%</td>
<td>0.27%</td>
<td>2.43%</td>
<td>0.63%</td>
<td>0.29%</td>
<td>0.25%</td>
<td>1.92%</td>
<td>2.59%</td>
<td>0.42%</td>
</tr>
</tbody>
</table>

Table 1(b) The estimated results of SVF method, MFD and Hybrid Method in identification of $M$, $C$, and $K$ with noise level = 0.005

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$M_{xx}$</th>
<th>$M_{yy}$</th>
<th>$M_{yz}$</th>
<th>$M_{yx}$</th>
<th>$C_{xx}$</th>
<th>$C_{yy}$</th>
<th>$C_{xy}$</th>
<th>$C_{yx}$</th>
<th>$K_{xx}$</th>
<th>$K_{yy}$</th>
<th>$K_{yz}$</th>
<th>$K_{xy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True values</td>
<td>1.39</td>
<td>0.08</td>
<td>0.07</td>
<td>1.38</td>
<td>3.5</td>
<td>0.6</td>
<td>0.56</td>
<td>3.4</td>
<td>1.25</td>
<td>0.11</td>
<td>0.12</td>
<td>1.37</td>
</tr>
<tr>
<td>SVF method</td>
<td>1.2859</td>
<td>0.0562</td>
<td>0.0441</td>
<td>1.2737</td>
<td>3.5064</td>
<td>0.6001</td>
<td>0.5579</td>
<td>3.3969</td>
<td>1.2505</td>
<td>0.1114</td>
<td>0.1198</td>
<td>1.3696</td>
</tr>
<tr>
<td>% error</td>
<td>7.49%</td>
<td>29.78%</td>
<td>36.95%</td>
<td>7.70%</td>
<td>0.18%</td>
<td>0.02%</td>
<td>0.38%</td>
<td>0.09%</td>
<td>0.04%</td>
<td>1.29%</td>
<td>0.13%</td>
<td>0.03%</td>
</tr>
<tr>
<td>MFD method</td>
<td>1.3572</td>
<td>0.0546</td>
<td>0.0679</td>
<td>1.3663</td>
<td>3.4781</td>
<td>0.5879</td>
<td>0.5370</td>
<td>3.3754</td>
<td>1.2421</td>
<td>0.1032</td>
<td>0.1212</td>
<td>1.3677</td>
</tr>
<tr>
<td>% error</td>
<td>2.36%</td>
<td>31.74%</td>
<td>2.94%</td>
<td>0.99%</td>
<td>0.63%</td>
<td>2.02%</td>
<td>4.11%</td>
<td>0.72%</td>
<td>0.63%</td>
<td>6.15%</td>
<td>0.97%</td>
<td>0.17%</td>
</tr>
<tr>
<td>Hybrid Method</td>
<td>1.3428</td>
<td>0.0611</td>
<td>0.0400</td>
<td>1.3044</td>
<td>3.4837</td>
<td>0.5647</td>
<td>0.5514</td>
<td>3.3845</td>
<td>1.2247</td>
<td>0.1045</td>
<td>0.1196</td>
<td>1.3576</td>
</tr>
<tr>
<td>% error</td>
<td>3.40%</td>
<td>23.59%</td>
<td>42.83%</td>
<td>5.48%</td>
<td>0.46%</td>
<td>5.88%</td>
<td>1.53%</td>
<td>0.46%</td>
<td>2.02%</td>
<td>5.04%</td>
<td>0.35%</td>
<td>0.90%</td>
</tr>
</tbody>
</table>
Table 1(c) The estimated results of SVF method, MFD and Hybrid Method in identification of $M$, $C$, and $K$ with noise level = 0.01

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$M_{xx}$</th>
<th>$M_{yy}$</th>
<th>$M_{zz}$</th>
<th>$M_{xy}$</th>
<th>$C_{xx}$</th>
<th>$C_{yy}$</th>
<th>$C_{yz}$</th>
<th>$K_{xx}$</th>
<th>$K_{yy}$</th>
<th>$K_{yz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True values</td>
<td>1.39</td>
<td>0.08</td>
<td>0.07</td>
<td>1.38</td>
<td>3.5</td>
<td>0.6</td>
<td>0.56</td>
<td>3.4</td>
<td>1.25</td>
<td>0.11</td>
</tr>
<tr>
<td>SVF method</td>
<td>1.2917</td>
<td>0.0586</td>
<td>0.0434</td>
<td>1.2719</td>
<td>3.5045</td>
<td>0.6031</td>
<td>0.5573</td>
<td>3.3951</td>
<td>1.2511</td>
<td>0.1118</td>
</tr>
<tr>
<td>% error</td>
<td>7.07%</td>
<td>26.76%</td>
<td>38.08%</td>
<td>7.84%</td>
<td>0.13%</td>
<td>0.51%</td>
<td>0.48%</td>
<td>0.14%</td>
<td>0.109%</td>
<td>0.1197</td>
</tr>
<tr>
<td>MFD method</td>
<td>1.3630</td>
<td>0.0549</td>
<td>0.0758</td>
<td>1.3634</td>
<td>3.4776</td>
<td>0.5952</td>
<td>0.5347</td>
<td>3.3790</td>
<td>1.2431</td>
<td>0.1023</td>
</tr>
<tr>
<td>% error</td>
<td>1.94%</td>
<td>31.36%</td>
<td>8.35%</td>
<td>1.99%</td>
<td>0.64%</td>
<td>1.13%</td>
<td>4.52%</td>
<td>0.62%</td>
<td>0.55%</td>
<td>7.02%</td>
</tr>
<tr>
<td>Hybrid Method</td>
<td>1.3480</td>
<td>0.0495</td>
<td>0.0359</td>
<td>1.3384</td>
<td>3.4947</td>
<td>0.5993</td>
<td>0.5563</td>
<td>3.3842</td>
<td>1.2421</td>
<td>0.1036</td>
</tr>
<tr>
<td>% error</td>
<td>3.02%</td>
<td>38.10%</td>
<td>48.78%</td>
<td>3.01%</td>
<td>0.15%</td>
<td>0.12%</td>
<td>0.67%</td>
<td>0.63%</td>
<td>5.85%</td>
<td>3.56%</td>
</tr>
</tbody>
</table>
Table 1(d) The estimated results of SVF method, MFD and Hybrid Method in identification of $M$, $C$, and $K$ with noise level = 0.03

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$M_{xx}$</th>
<th>$M_{yy}$</th>
<th>$M_{xy}$</th>
<th>$C_{xx}$</th>
<th>$C_{yy}$</th>
<th>$C_{xy}$</th>
<th>$K_{xx}$</th>
<th>$K_{yy}$</th>
<th>$K_{xy}$</th>
<th>$K_{yx}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True values</td>
<td>1.39</td>
<td>0.07</td>
<td>0.07</td>
<td>1.38</td>
<td>3.5</td>
<td>0.6</td>
<td>0.56</td>
<td>3.4</td>
<td>1.25</td>
<td>0.11</td>
</tr>
<tr>
<td>SVF method</td>
<td>1.3154</td>
<td>0.0714</td>
<td>0.0452</td>
<td>1.2690</td>
<td>3.5020</td>
<td>0.6131</td>
<td>0.5572</td>
<td>3.3880</td>
<td>1.2537</td>
<td>0.1156</td>
</tr>
<tr>
<td>% error</td>
<td>5.37%</td>
<td>10.69%</td>
<td>35.42%</td>
<td>8.04%</td>
<td>0.66%</td>
<td>2.19%</td>
<td>0.50%</td>
<td>0.35%</td>
<td>0.29%</td>
<td>0.06%</td>
</tr>
<tr>
<td>MFD method</td>
<td>1.3623</td>
<td>0.0496</td>
<td>0.0701</td>
<td>1.3651</td>
<td>3.4810</td>
<td>0.5918</td>
<td>0.5471</td>
<td>3.3683</td>
<td>1.2432</td>
<td>0.1031</td>
</tr>
<tr>
<td>% error</td>
<td>2.00%</td>
<td>37.98%</td>
<td>0.15%</td>
<td>1.08%</td>
<td>0.54%</td>
<td>1.36%</td>
<td>2.31%</td>
<td>0.93%</td>
<td>0.54%</td>
<td>6.23%</td>
</tr>
<tr>
<td>Hybrid Method</td>
<td>1.3599</td>
<td>0.0334</td>
<td>0.0321</td>
<td>1.3207</td>
<td>3.4830</td>
<td>0.5960</td>
<td>0.5553</td>
<td>3.3790</td>
<td>1.2434</td>
<td>0.0992</td>
</tr>
<tr>
<td>% error</td>
<td>2.17%</td>
<td>58.26%</td>
<td>54.17%</td>
<td>4.30%</td>
<td>0.49%</td>
<td>0.66%</td>
<td>0.84%</td>
<td>0.62%</td>
<td>0.53%</td>
<td>9.78%</td>
</tr>
</tbody>
</table>
Fig. 5(a) Results of estimated parameter $M_{xx}$ using MFD method for various initial values at noise level = 0.01.

Fig. 5(b) Results of estimated parameter $M_{xy}$ using MFD method for various initial values at noise level = 0.01.

Fig. 5(c) Results of estimated parameter $M_{yx}$ using MFD method for various initial values at noise level = 0.01.

Fig. 5(d) Results of estimated parameter $M_{yy}$ using MFD method for various initial values at noise level = 0.01.

Table 1(e) The estimated results of SVF method, MFD and Hybrid Method in identification of $M$, $C$, and $K$ with noise level = 0.05.

<table>
<thead>
<tr>
<th>Case 1. Noise level = 0.05, $dr = 0.1$, number of data points = 500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>True values</td>
</tr>
<tr>
<td>SVF method</td>
</tr>
<tr>
<td>% error</td>
</tr>
<tr>
<td>MFD method</td>
</tr>
<tr>
<td>% error</td>
</tr>
<tr>
<td>Hybrid Method</td>
</tr>
<tr>
<td>% error</td>
</tr>
</tbody>
</table>
Table 1(f) The estimated results of SVF method, MFD and Hybrid Method in identification of $M$, $C$, and $K$ with noise level = 0.1

<table>
<thead>
<tr>
<th>Case 1. Noise level = 0.1, $dr = 0.1$, number of data points = 500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>True values</td>
</tr>
<tr>
<td>SVF method</td>
</tr>
<tr>
<td>% error</td>
</tr>
<tr>
<td>MFD method</td>
</tr>
<tr>
<td>% error</td>
</tr>
<tr>
<td>Hybrid Method</td>
</tr>
<tr>
<td>% error</td>
</tr>
</tbody>
</table>
Fig. 6(e) Results of estimated parameter $C_{xx}$ using Hybrid Method for various noise levels at ICO

Fig. 6(f) Results of estimated parameter $C_{yy}$ using Hybrid Method for various noise levels at ICO

Fig. 6(g) Results of estimated parameter $K_{xx}$ using Hybrid Method for various noise levels at ICO

Fig. 6(h) Results of estimated parameter $K_{yy}$ using Hybrid Method for various noise levels at ICO

Table 1(g) The estimated results of SVF method, MFD and Hybrid Method in identification of $M$, $C$, and $K$ with noise level = 0.2

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$M_{xx}$</th>
<th>$M_{yy}$</th>
<th>$M_{xz}$</th>
<th>$M_{xy}$</th>
<th>$C_{xx}$</th>
<th>$C_{yy}$</th>
<th>$C_{xz}$</th>
<th>$C_{xy}$</th>
<th>$K_{xx}$</th>
<th>$K_{yy}$</th>
<th>$K_{xz}$</th>
<th>$K_{xy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True values</td>
<td>1.39</td>
<td>0.08</td>
<td>0.07</td>
<td>1.38</td>
<td>3.5</td>
<td>0.6</td>
<td>0.56</td>
<td>3.4</td>
<td>1.25</td>
<td>0.11</td>
<td>0.12</td>
<td>1.37</td>
</tr>
<tr>
<td>SVF method</td>
<td>1.6327</td>
<td>0.1346</td>
<td>0.1935</td>
<td>1.4026</td>
<td>3.5593</td>
<td>0.7137</td>
<td>0.5460</td>
<td>3.3121</td>
<td>1.2897</td>
<td>0.1005</td>
<td>0.1316</td>
<td>1.4027</td>
</tr>
<tr>
<td>% error</td>
<td>17.46%</td>
<td>68.27%</td>
<td>176.48%</td>
<td>1.63%</td>
<td>1.70%</td>
<td>18.95%</td>
<td>2.51%</td>
<td>2.59%</td>
<td>3.18%</td>
<td>8.61%</td>
<td>9.69%</td>
<td>2.38%</td>
</tr>
<tr>
<td>MFD method</td>
<td>1.3977</td>
<td>0.0527</td>
<td>0.0995</td>
<td>1.3605</td>
<td>3.4699</td>
<td>0.6139</td>
<td>0.5541</td>
<td>3.3034</td>
<td>1.2521</td>
<td>0.1153</td>
<td>0.1143</td>
<td>1.3590</td>
</tr>
<tr>
<td>% error</td>
<td>0.56%</td>
<td>34.10%</td>
<td>42.2%</td>
<td>1.41%</td>
<td>0.86%</td>
<td>2.32%</td>
<td>1.06%</td>
<td>2.84%</td>
<td>0.17</td>
<td>4.85%</td>
<td>4.72%</td>
<td>0.80%</td>
</tr>
<tr>
<td>Hybrid Method</td>
<td>1.4152</td>
<td>0.0526</td>
<td>0.1271</td>
<td>1.3821</td>
<td>3.4746</td>
<td>0.6262</td>
<td>0.5514</td>
<td>3.3048</td>
<td>1.2517</td>
<td>0.1078</td>
<td>0.1146</td>
<td>1.3533</td>
</tr>
<tr>
<td>% error</td>
<td>1.81%</td>
<td>34.21%</td>
<td>81.53%</td>
<td>0.16%</td>
<td>0.73%</td>
<td>4.37%</td>
<td>1.54%</td>
<td>2.80%</td>
<td>0.013%</td>
<td>2.03%</td>
<td>4.48%</td>
<td>1.22%</td>
</tr>
</tbody>
</table>