ABSTRACT
The stability, transient response, and steady state response of nonlinear mechanical systems is studied using reduction method. The steady state periodic response is investigated using the harmonic balance method. An implicit integration method for predicting transient response is proposed. The stability of the steady state periodic response is studied using Floquet theory. A reduction is introduced to analyze the system dynamic behaviors in modal coordinates to reduce the working space. The method reduces the system degrees of freedom to only those coordinates associated with the nonlinear components. The merit of the method is demonstrated by an example of flexible rotor system with nonlinear bearing supports.

INTRODUCTION
It is very important to investigate the dynamic behavior of nonlinear systems of various natural phenomena such as self-excited vibrations, jump phenomena, and multiple solutions to improve the design. Effective methods, which are available in the literature for studying the response of large order nonlinear dynamic systems, mostly concentrated on numerical integration procedures. There is a large number of work addressing small order systems using a variety of quantitative and qualitative methods. Many of these methods are very successful and also provide considerable insight into the behavior of nonlinear systems. Most of the approaches, however, become mathematically intractable for the application to large order problems.

A few analysts have addressed the dynamic analysis of large order nonlinear system with particular emphasis on the steady state response analysis, which is of considerable interest for real systems. The method of harmonic balance has been successfully utilized by Yamauchi (1983), Choi and Noah (1987), Poplawski (1988), and Shiau and Jean (1991a). Choi and Noah (1987) utilized discrete Fourier transform procedures in conjunction with harmonic balance and also included subharmonic response components. The works by Poplawski (1988) and by Shiau and Jean (1991a) are similar to that of Choi and Noah; however, they added a condensation procedure which reduces the iteration problem so that it only involves those coordinates associated with the nonlinear components. This algorithm is a generalization of the same strategy developed by McLean and Hahn (1983) for analyzing the centered circular orbit response of large rotor systems with squeeze film dampers. Their works were performed in physical coordinates directly.

The weighted residual method of Galerkin has been successfully employed by many authors [Urabe (1965); Urabe and Reiter (1966); Brommundt (1975)]. Brommundt presented a detailed discussion of the Urabe-Reiter version of Galerkin's method for studying the periodic response of rotodynamic systems including the autonomous and non-autonomous cases. They also investigated bifurcations as a
means of locating hidden periodic solutions and discussed the stability analysis of the located periodic solutions. Bromundt also mentioned the possibility of condensation but did not present any details for its implementation.

The collocation method as formalized by Samoilenko and Rontm (1979) for periodic response studies was utilized by Nataraj (1987) and Nataraj and Nelson (1989) along with a component mode synthesis strategy to render the procedure tractable for large order systems. Jean and Nelson (1991) presented a collocation method that could be utilized for large order nonlinear systems directly in physical coordinates. This was accomplished by utilizing a condensation procedure similar to that used by Shiau and Jean (1991a) and McLean and Hahn (1983). Harmonic, subharmonic, and superharmonic response components can be included in the work of Jean and Nelson (1991). Both autonomous and non-autonomous cases can be studied by using these approaches. Shiau and Hwang (1991) studied nonlinear rotor-bearing systems by using the combined methodology of the harmonic balance and the collocation methods. The methods used for the transient analysis of dynamic systems are usually based on numerical integration. An approach named the discrete time-transfer matrix method was developed by Kumar and Sankar (1986) for the transient analysis of large order systems. The method discretizes a system into many subsystems and the equations of motion for each subsystem are formulated. The time response of the state at either ends of any subsystem is related by a transfer matrix, which is a function of initial and boundary conditions, from numerical integration procedures. Subbiah and Reiger (1988) and Subbiah et al. (1988) have successfully utilized this approach for large order rotor dynamic systems. Shiau and Jean (1991b) also did an approach for the transient analysis of large order nonlinear rotor systems; their work was directly performed in physical coordinates and a reduction process was included too.

In this study, techniques for the transient behavior analysis of large order mechanical systems with nonlinear characteristics are proposed, with special emphasis on the rotordynamic systems. For the steady state response analysis, a method based on the harmonic balance method for evaluating the periodic response of the system is developed. A method based on an implicit numerical integration is employed for predicting the transient response of the system. A reduction process based on the localized physical nature of the nonlinear effects of the system is included in the analysis. In this case, the problem will relate to only those coordinates of system nonlinear components. The solution procedures are similar to the works by Shiau and Jean (1991a, 1991b), which were carried out in physical coordinates. The present paper deals with the problem in modal coordinates in order to reduce the working space. Moreover the stability of the localized periodic response is studied by using Floquet theory. The merit of the methods is demonstrated by a 52 DOFs system with only four of the coordinates related to nonlinear components.

EQUATIONS FORMULATION

The nonlinearities mostly considered in rotordynamic systems are due to bearings, dampers, couplings, seals, etc. They are localized at few stations of the entire rotor system. The nonlinear forces due to these mechanisms (named nonlinear components) are functions of displacement, velocity, and possibly acceleration of the system at related stations. For the convenience, the physical coordinates related to the nonlinear forces are called nonlinear coordinates and denoted as a (\(n_1 \times 1\)) vector \(\{q_2\}\). The physical coordinates describing the system motion at the pre-selected stations excluding the nonlinear coordinates are referred to linear coordinates and denoted as a \((n_1 \times 1\)) vector \(\{q_1\}\).

It is possible to consider the original nonlinear system as a linear structure with nonlinear forces applied at few associated locations, i.e., the forces are treated as pseudo applied forces. The linear structure can be modelled by some modelling techniques, such as the finite element method (Nelson and McVaugh, 1976), the generalized polynomial expansion method (Shiau and Hwang, 1991), etc. The system equation of motion subjected to the nonlinear forces is generally of the second order form

\[
[M]\{q\} + [C] + [\Omega(C)]\{q\} + [K]\{q\} = \{Q\} + \{F\}
\]

(1)

where \([M]\) is known as the \(N \times N\) mass/inertia matrix which is a positive definite real symmetric matrix; \([C]\) is the dissipation matrix which is generally a non-symmetric; \([\Omega(C)]\) is the gyroscopic matrix which is skew symmetric; \([K]\) is the stiffness matrix which is generally non-symmetric and is probably dependent on the spin speed of the rotor system. The vector \(\{Q\}\) is a specified time dependent force vector such as the rotating unbalance and the gravity load, etc. The vector \(\{F\}\) is associated with the forces related to the nonlinear components of the system and is considered as a function of \(\{q_2\}\) in this study. \(\{q\}\) is a \((N \times 1)\) generalized coordinate vector. For the finite element method (FEM), \(\{q\}\) consists of translational and rotational displacements at each pre-defined station. For the generalized polynomial expansion method (GPEM), since the translational displacements of the system are described by polynomial functions in terms of the rotor axial coordinate, \(\{q\}\) is the vector including the time dependent coefficients of the polynomial functions.

The advantage of using a modal representation is that the associated problem size can be reduced truncating the high modes which will lead to a subsequent saving in computer time. The modal representation of equation (1) requires the establishment of modal matrix which can be obtained by solving an eigenvalue problem. The eigenvalue problem based on the non-rotating symmetric undamped homogenous system can be expressed as

\[
\omega^2[M]\{\phi\} = [K]\{\phi\},
\]

(2)

which has spin speed independent real modes. In equation (2), \(\omega^2\) is the real-valued eigenvalue and \(\{\phi\}\) is the corresponding eigenvector. The eigenvectors satisfy the orthogonality conditions

\[
[\Phi]^T[M][\Phi] = [I]
\]

(3)

\[
[\Phi]^T[K][\Phi] = [\omega^2]
\]

(4)

where \([\Phi]\) is the modal matrix and has been normalized in such a way that equation (3) is an identity matrix \([I]\) and equation (4) is an undamped diagonal eigenvalue matrix \([\omega^2]\). Since the modal vectors satisfy the orthogonality conditions, they form a linearly independent set. The generalized coordinates can be expressed in terms of the modal coordinates \(\{\eta\}\) using the transformation of

\[
\{q\} = \Phi[\eta] = \Phi(\{\eta\} + \{Q\} + \{F\})
\]

(5)

with truncation of modes from \(N_0\) to \(N\). Substituting equation (5) into equation (1), premultiplying by \([\Phi]^T\), and making use of equations (3) and (4), it yields

\[
[\Phi]\{\eta\} + [\Phi][D_\eta] + [\Phi][K]\{\eta\} + [\Phi][\Omega(C)][\Phi]\{\eta\} + [\Phi][\Phi]^T\{Q\} + [\Phi]^T\{F\} = \{Q\} + \{F\}
\]

(6)

where

\[
[D_\eta] = [\Phi]^T([C] + [\Omega(C)][\Phi])
\]

(7)

\[
[K_\eta] = [\Phi]^T[K][\Phi]
\]

(8)

\[
[Q_\eta] = [\Phi]^T\{Q\}
\]

(9)

\[
[F_\eta] = [\Phi]^T\{F\}
\]

(10)

It should be noted that the modal data based on other eigenvalue problem rather than equation (2) is possible when implementing the modal transformation.

STEADY STATE PERIODIC RESPONSE

For the periodic response analysis of the rotor system, a solution is assumed to be approximated by a finite Fourier series as

\[
\{\eta\} = \{\eta_0\} + \sum_{i=1}^{N_0} \{a_i\} \cos(\omega_i t) + \{b_i\} \sin(\omega_i t)
\]

(11)

where \(\omega_i\) is a set of preseleced frequency contents which can be the sub, ultra-sub, super, and ultra-super harmonics of the fundamental component (Jean and Nelson, 1991). The fundamental period is denoted as \(T\). For the simplicity of presentation, if one choose

\[
\omega_0 = 0
\]

(12)

\[
\{b_0\} = \{0\}
\]

(13)

equation (8) yields

\[
\{\eta\} = \sum_{i=1}^{N_0} \{a_i\} \cos(\omega_i t) + \{b_i\} \sin(\omega_i t)
\]

(14)

Substitution of equation (10) into equation (6) yields a set of nonlinear algebraic equations in terms of unknown Fourier coefficients as (Shiau and Jean, 1991a)

\[
\{a_i\} = \frac{1}{T [\Phi]^T \{\Phi\} + [\Phi]^T \{\Phi\}} \{[\Phi]^T \{Q_\eta\} + \{F_\eta\}\}
\]

(15)

where

\[
\{a_i\} = [\Phi]^T \{\Phi\}^{-1} \{[\Phi]^T \{Q_\eta\} + \{F_\eta\}\}
\]

(16)
\[ \{r_n\} = \{q_{n0}\} + j\{h_n\}, \quad j = \sqrt{-1} \]  

\[ \{T_n\} = \{h_n\} + \omega_n^2\{f\} - j\omega_n\{D_n\} \]  

\[ \{F_n\} = S \int_0^T \{F_n\} e^{j\omega t} dt \]  

\[ \{Q_n\} = S \int_0^T \{Q_n\} e^{j\omega t} dt \]

and

\[ S = \begin{cases} 1/T, & i = 0; \\ 2/T, & i = 1, 2, \ldots, N_f \end{cases} \]

The vectors \( \{Q_n\} \) and \( \{F_n\} \) are the Fourier coefficients of the linear and nonlinear force vectors respectively. If the specified time dependent force vector \( \{f(t)\} \) only consists of the rotating unbalance and the gravity load, the Fourier coefficient vector \( \{Q_n\} \) will be of the form

\[ \{Q_n\} = \theta^T \left( \begin{array}{c} \{Q_n\} \Delta_t + \{0\} \end{array} \right) \quad i = 0, 1, 2, \ldots, N_f \]

where \( \theta \) is the Fourier coefficient vector of the unbalance force, the index \( j \) denotes the sequence number of the \( w_i \) related to the unbalance excitation frequency, \( \{0\} \) is the complex gravity force vector, and \( \Delta_t \) is the Kroncker Delta. Equation 11 includes \( n_0(N_f + 1) \) linear algebraic equations in terms of \( n_0(N_f + 1) \) complex variables, \( \{r_n\}_i \), \( i = 0, 1, 2, \ldots, N_f \). It should be noted that the HBMM will result in \( N(N_f + 1) \) complex unknowns in physical domain, i.e., without modal truncation. It is easy to see that when \( n_0 \) is much less than \( N \), the computational effort of solving the iteration problem can be substantially saved in the modal domain with mode truncation. However, the computational effort for the iteration problem can be further reduced by introducing a second reduction based on the localized physical nature of the system nonlinearity.

Since the nonlinear forces are generally localized at few stations, the nonlinear coordinate vector \( \{q_{2n}\} \) is a small subset of physical coordinates. There exists a connectivity matrix \([S]\) in such a way that

\[ \{q_2\} = [S]\{q\} \]  

It should be noted that the generalized coordinate vector \( \{q\} \) may base on the FEM, the GPEM, or other modelling methods. Substituting equation (5) into equation (18), the nonlinear coordinate vector \( \{q_{2n}\} \) can be expressed in terms of modal coordinates as

\[ \{q_{2n}\} = [S]\{q\} \]  

This equation implies that

\[ \{r_{2n}\} = [\Phi_2]\{r_n\} \]  

where \( \{r_{2n}\} \) is the complex Fourier component of the response \( \{q_{2n}\} \). Premultiplying equation (11) by \([\Phi_2]\) and utilizing equation (20), it yields

\[ \{r_{2n}\} = [\Phi_2]^{-1}\{Q_n\} + \{F_n\} \]

Since the nonlinear force vector \( \{F_n\} \) is a function of \( \{q_{2n}\} \) and \( \{F_n\} \), \( \{F_n\} \) is a function of \( \{r_{2n}\} \) only. Equation 21 is a set of \( n_2(N_f + 1) \) algebraic equations in terms of \( n_2(N_f + 1) \) unknown nonlinear coefficients which are only associated with the nonlinear coordinates. If the number of nonlinear coordinates \( n_2 \) is small compared to \( n_0 \), the procedure results in a system of equations where the order is substantially reduced again from equation (11). Thereby it provides reduction in computational effort for solving the iteration problem and most likely increasing the probability of locating solutions. When \( \{r_{2n}\} \) are successfully solved, the solution of \( \{q_{2n}\} \) and its derivatives with respect to time can be computed from the assumed time solution. After knowing \( \{r_{2n}\} \) and \( \{q_{2n}\} \), \( \{q_{2n}\} \) can be obtained from the fourth expression of equation (7). Therefore the Fourier coefficient vector of the modal coordinate \( \{q_n\} \) can be calculated from equation (11). Finally, the periodic response of the generalized coordinates can be obtained from the transformation of equation (5). It should be noted that the solution procedures except for solving \( \{r_{2n}\} \), which requires iterations, only involves direct substitutions.

STABILITY ANALYSIS

The investigation of the stability of periodic responses for the nonlinear dynamic system requires a perturbation. It will result in a linearized periodic system. An efficient numerical treatment of stability for the linear periodic system has been established by Friedmann et al. (1977). It is used in this work for the stability analysis.

TRANSIENT RESPONSE

The transient response of a rotor system is also dealt with from the modal system, equation (6). For the transient analysis, the first and the second order derivatives of the system displacement with response to time at any time instant \( t_i \) can be expressed as a linear function of the displacement at the time (Kumar and Sankar, 1986). They are of the following forms

\[ \{q(i)\} = C_T^{2}\{q(t_i)\} + \{B(t_i)\} \]

\[ \{q(i)\} = C_T^{1}\{q(t_i)\} + \{C(t_i)\} + \{E(t_i)\} \]

where

\[ C_T = 2/\Delta T \]

\[ \{B(t_i)\} = -C_T^{2}\{q(t_i)\} + \{q(t_i-1)\} \]

\[ \{E(t_i)\} = -C_T^{1}\{q(t_i-1)\} - \{q(t_i-1)\} \]

\[ \Delta T \] is the time interval. The substitution of equations (22) and (23) into equation (6) results in a set of \( n_0 \) nonlinear algebraic equations as follows:

\[ \{S_n\}_i^2\{q(t_i)\} + \{P_n(t_i)\} - \{P_n(t_i)\} = \{0\} \]

or

\[ \{q(t_i)\} + \{S_n\}_i^{-1}\{P_n(t_i)\} - \{P_n(t_i)\} = \{0\} \]

where

\[ \{S_n\} = C_T^{2}\{I\} + C_T^{1}\{K\} + \{K\} \]

\[ \{P_n(t_i)\} = \{B(t_i)\} + \{D\}_i\{E(t_i)\} - \{Q_n(t_i)\} \]

Since the \( C_T \), \( \{D\}_i \), and \( \{K\} \) are known matrices at this point. Thus with knowing the initial conditions \( \{q(t_i-1)\} \) and \( \{q(t_i-1)\} \), the value of the matrix \( \{S_n\}_i \) and the vector \( \{P_n(t_i)\} \) can be obtained at time instant \( t_i \). The response of the modal coordinates at time instant \( t_i \) can be calculated from equation (25). However, similar to the periodic response analysis, equation (25) can be further reduced to only relate to the nonlinear coordinates because of the localized physical nature of the nonlinear effects. Recall that

\[ \{q_2(t_i)\} = \{\Phi_2\}\{q(t_i)\} \]

and the nonlinear force vector \( \{F_n(t_i)\} \) is a function of the nonlinear coordinates \( \{q_2(t_i)\} \). Premultiplying equation (25a) by \([\Phi_2]\) yields

\[ \{q_2(t_i)\} + \{\Phi_2\}_k^{-1}\{F_n(t_i)\} - \{F_n(t_i)\} = \{0\} \]

Equation (28) is a set of \( n_2 \) nonlinear algebraic equations in terms of nonlinear coordinates \( \{q_2(t_i)\} \) only. It is evident that if \( n_2 \ll n_0 \ll N \), the computational effort for the iteration problem can be substantially saved by solving equation (28). If a solution for \( \{q_2(t_i)\} \) is calculated using equation (28), then the associated value of \( \{F_n(t_i)\} \) can be obtained with direct substitutions. Since \( \{S_n\}_i^{-1} \) and \( \{P_n(t_i)\} \) are known at this time \( t_i \), \( \{q\} = [\Phi_2]\{q_2(t_i)\} \), the value of generalized coordinates \( \{q(t_i)\} \) can be obtained directly from the following multiplication

\[ \{q(t_i)\} = -[\Phi_2]^{-1}_k\{F_n(t_i)\} + \{F_n(t_i)\} \]

For the response of the system at next time instant \( t_{i+1} \), one can repeat the above procedures with considering the response at \( t_i \) as initial conditions.

NUMERICAL EXAMPLE

A flexible rotor system with squeeze film dampers (Chen et al., 1988) is employed to illustrate the use of the present algorithm. The rotor system is schematically shown in Figure 1. It is simulated as 12 elements (13 stations) and each station is of four degrees of freedom including two translations and two rotations in the finite element analysis (Nelson and McVaugh, 1976). The detail rotor configurations and material properties are listed in Table 1. The rotor includes four rigid discs located at stations 1, 2, 4, and 12 with mass properties shown in Table 2. The system is supported on a rigid foundation by isotropic unbonded bearings with properties listed in Table 3. Two identical squeeze film dampers are located at
stations 3 and 13. Each acts in parallel with the support stiffness. The dampers have properties: radius=30.8mm; length=234mm; radial clearance=152μm; viscosity of lubricant=0.00266 Ns/m². The short bearing theory developed by Mohan and Hahn (1974) is utilized to evaluate the hydrodynamic forces of the dampers. The system is subjected to the gravity load and an unbalance excitation due to the cg eccentricity of 10.16mm in the disc located at station 12.

The finite element modelling technique results in a 52 degrees of freedom system with four of those being nonlinear coordinates (n₂ = 4), which are associated with the translation displacements of the journal in the squeeze film system is subjected to the gravity load and an unbalance excitation due to the cg eccentricity of 10.16mm in the disc located at station 12.

The GPEM (Shiau and Hwang, 1991) is also used to study the system characteristics. The method approximates the motion of the system in the XY and XZ plane individually by a polynomial of order (N₁ - 1), which is written as

\[ w^2[M]v = [K]v \]  \hspace{1cm} (30)

which is an eigenvalue problem associated with the undamped non-rotating system. The first 18 modes of the system, which consist of nine modes in XY and XZ planes individually, are listed in Table 4. Because the system is axially symmetric as shown in Figure 1, the eigen properties are the same in both XY and XZ planes.

The GPEM is symmetric and independent of the rotor speed. The modal data is directly obtained by using

\[ \sum_{n=1}^{N_2} C_{nm}(1) \eta^{n-1} \]  \hspace{1cm} (31a)

\[ \sum_{n=1}^{N_2} C_{nm}(1) \eta^{n-1} \]  \hspace{1cm} (31b)

The GPEM leads to a set of 2N₂ nonlinear differential equations which has symmetric mass and stiffness matrices. The generalizied coordinate vector is

\[ \{q\}_GPM = \begin{bmatrix} \{G_m\} \\ \{C_m\} \end{bmatrix} \]  \hspace{1cm} (32)

The system mass and stiffness matrices are used to generate the modal data. Table 4 also shows the first 18 eigenvalues associated with the GPEM.

For the periodic response analysis, the first four harmonic components are included in the assumed solution. The periodic response over the range of 0 ≤ Ω ≤ 3000 rad/s is computed with a speed increment of 20 rad/s. The results are obtained using a PC-386 computer.

Figure 2 indicates the static component of the system periodic response in the vertical (gravity) direction for the journals at stations 3 and 13. It should be noted that the amplitude of the component in the horizontal direction is very small compared to that in the vertical direction, it is not shown in the paper. Figures 3 and 4 show the amplitude of the semi-major axis of the first and second elliptic vibration components of the forced response of the journal at station 3, as a function of spin speed. Similarly figures 5 and 6 show those for the journal at station 13. It can be seen that the first three critical speeds of the first harmonic response component are approximately at 400, 720, and 2400 rad/s. It is found that the journals of the dampers will be lifted up when the spin speed closes to the first harmonic resonance. It is also seen from figures 4 and 6 that the higher modes are required for accurately predicting the response of the component at high spin speeds.

The stability of the located periodic solutions is evaluated from the eigenvalues of the so called Floquet transition matrix (Friedmann et al., 1977). Consider the eigenvalue of the Floquet transition matrix (FTM) as \((\lambda + j\omega)\). If \((\lambda + j\omega) < 1\) for all eigenvalues of the FTM, the system at the equilibrium state is said to be stable. For the present analysis, since the maximum value of \((\lambda + j\omega)\) of the FTM associated with the periodic solutions shown in figures 2–6 has been computed to be less than 1, this implies that all the motions are stable.

Figures 7 and 8 show the transient response of the journals at stations 3 and 13 respectively for spin speed of 700 rad/s. The steady state response of this case is shown in Figure 9. The precession is in the counterclockwise direction. It is seen that the amplitude of the response based on the FEM is slightly different from that based on the GPEM. However the amplitude of the steady state orbits in Figure 10, which has about mean movement of 4.488 × 10⁻⁶ meter with amplitude of 1.219 × 10⁻⁵ meter for the journal at station 3 and 1.329 × 10⁻⁵ meter mean movement of 7.869 × 10⁻⁵ meter for the journal at station 13, is approximately in correspondence with the result obtained by the periodic solution strategy.

DISCUSSION AND CONCLUSIONS

Many large-order mechanical vibration systems are of nonlinearity. Especially for rotor bearing systems, the nonlinearities are usually associated with a few discrete components. In this case, the system model can be considered as primarily linear, with the nonlinear effects from a small subset of the system coordinates. Methods in applying this physical nature of localized nonlinear effects have been presented for studying the dynamic response as well as the system stability. A method based on the harmonic balance method has been proposed for evaluating the periodic response of the system. An implicit numerical integration is employed for predicting the transient response of the system. The analyses performed in modal coordinates in order to reduce the working space. The solution procedures result in a set of nonlinear algebraic equations in terms of Fourier coefficients of the modal coordinates for the HBM and in terms of modal coordinates at a time instant for the integration scheme. A reduction process have been introduced to reduce the number of the nonlinear algebraic equations to those coordinates only related to the nonlinear components of the system.

A flexible rotor system with nonlinear squeeze film dampers has been studied to illustrate the merit of the procedures. The system is modeled as a 52 degrees of freedom system with only four of the coordinates related to the nonlinear squeeze film dampers. For the periodic solution strategy if \(N₂ = 4\) is used and the HBM is directly applied in the physical domain, it will generate 468 nonlinear algebraic equations. However, if the method is applied in the modal domain (\(n₂ = 18\)), it will generate 162 nonlinear algebraic equations. Moreover, if the algorithm by Shaiu and Jean (1991a) is employed, it will lead to 36 equations with the working space of the matrix operations is 48. The present method will result in 36 nonlinear algebraic equations and the working space is 18. Similarly, the analyses described as above can be applied in the transient response strategy. It is shown that the reduction method proposed in this study is of better efficiency and it is highly recommended for the dynamic analysis of large order mechanical vibration systems with nonlinear characteristics.

ACKNOWLEDGEMENT

The authors would like to thank Prof. H.D. Nelson at Texas Christian University for his valuable suggestions.

REFERENCES


TABLE 4: EIGENVALUES IN BOTH XY AND XZ PLANES (rad/s)

<table>
<thead>
<tr>
<th>No.</th>
<th>FEM</th>
<th>GPEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>311.1</td>
<td>312.9</td>
</tr>
<tr>
<td>2</td>
<td>667.9</td>
<td>673.2</td>
</tr>
<tr>
<td>3</td>
<td>1167.1</td>
<td>1178.5</td>
</tr>
<tr>
<td>4</td>
<td>2156.3</td>
<td>2162.8</td>
</tr>
<tr>
<td>5</td>
<td>3039.4</td>
<td>3080.5</td>
</tr>
<tr>
<td>6</td>
<td>4250.4</td>
<td>4362.8</td>
</tr>
<tr>
<td>7</td>
<td>6035.9</td>
<td>6449.2</td>
</tr>
<tr>
<td>8</td>
<td>7268.1</td>
<td>7839.5</td>
</tr>
<tr>
<td>9</td>
<td>12980.2</td>
<td>12883.4</td>
</tr>
</tbody>
</table>
FIGURE 4: SECOND HARMONIC SEMI-MAJOR AXIS OF JOURNAL RESPONSE AT STATION 3 VS. SPIN SPEED.

FIGURE 5: FIRST HARMONIC SEMI-MAJOR AXIS OF JOURNAL RESPONSE AT STATION 13 VS. SPIN SPEED.

FIGURE 6: SECOND HARMONIC SEMI-MAJOR AXIS OF JOURNAL RESPONSE AT STATION 13 VS. SPIN SPEED.

FIGURE 7: TRANSIENT RESPONSE OF THE JOURNAL AT STATION 3. PRECESSION IN COUNTERCLOCKWISE DIRECTION. \( \omega = 700 \text{rad/s}, n_p = 18, N_p = 16; \) : FEM, ---: GPEM.

FIGURE 8: TRANSIENT RESPONSE OF THE JOURNAL AT STATION 13. PRECESSION IN COUNTERCLOCKWISE DIRECTION. \( \omega = 700 \text{rad/s}, n_p = 18, N_p = 16; \) : FEM, ---: GPEM.

FIGURE 9: STEADY STATE RESPONSE OF THE JOURNALS. \( \omega = 700 \text{rad/s}, n_p = 18, N_p = 16; \) : FEM, ---: GPEM.