A REDUCTION METHOD FOR STABILITY AND DYNAMIC RESPONSE ANALYSIS OF NONLINEAR MECHANICAL SYSTEMS

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ABSTRACT

The stability, transient response, and steady state response of nonlinear mechanical systems is studied using reduction method. The steady state periodic response is investigated using the harmonic balance method. An implicit integration method for predicting transient response is proposed. The stability of the steady state periodic response is studied using Floquet theory. A reduction is introduced to analyze the system dynamic behaviors in modal coordinates to reduce the working space. The method reduces the system degrees of freedom to only those coordinates related to the system nonlinear components. The merit of the method is demonstrated by an example of flexible rotor system with nonlinear bearing supports.

NOMENCLATURE

- $\{n\}$, $\{\delta\}$: Fourier coefficient vectors of the $i$th harmonic
- $[b(t)]$, $[F(t)]$: coefficient vectors defined by Eq. (24)
- $[C_T]$: dissipation matrix
- $[D_\epsilon]$: matrix defined by Eq. (7)
- $[F]$: nonlinear force vector
- $[F_\epsilon]$: force vector defined by Eq. (7)
- $[\tilde{F}_\epsilon]$: Fourier coefficient vectors of $[F_\epsilon]$ and $[Q_\epsilon]$; $i$th harmonic
- $[G]$: gyroscopic matrix
- $[J]$: gravity force vector
- $[I]$: identity matrix
- $[K]$: stiffness matrix
- $[K_c]$: modal stiffness matrix
- $[M]$: mass matrix
- $N$: number of system DOFs
- $N_S$: number of harmonics
- $N_P$: polynomial order
- $n_1$, $n_2$: number of "linear, nonlinear" coordinates
- $M$: number of retained modes
- $[P_{0}(t_i)]$: vector defined by Eq. (26b)
- $[Q]$: specified time dependent force vector
- $[\tilde{Q}_\epsilon]$: unbalance force vector
- $[Q_\epsilon]$: modal vector defined by Eq. (7)
- $[q]$: generalized coordinate vector
- $[r_{l}],[q_{l}]$: "linear, nonlinear" coordinate vector
- $[s]$: Fourier coefficient vector, $i$th harmonic
- $[S]$: connectivity matrix
- $[S_e]$: matrix defined by Eq. (26a)
- $S$: factor defined by Eq. (16)

INTRODUCTION

It is very important to investigate the dynamic behavior of nonlinear systems of various natural phenomena such as self-excited vibrations, jump phenomena, and multiple solutions to improve the design. Effective methods, which are available in the literature for studying the response of large order nonlinear dynamic systems, mostly concentrated on numerical integration procedures. There is a large number of work addressing small order systems using a variety of quantitative and qualitative methods. Many of these methods are very successful and also provide considerable insight into the behavior of nonlinear systems. Most of the approaches, however, become mathematically intractable for the application to large order problems.

A few analysts have addressed the dynamic analysis of large order nonlinear system with particular emphasis on the steady state response analysis, which is of considerable interest for real systems. The method of harmonic balance has been successfully utilized by Yamauchi (1983), Choi and Noah (1987), Poplawski (1988), and Shiau and Jean (1991a). Choi and Noah (1987) utilized discrete Fourier transform procedures in conjunction with harmonic balance and also included subharmonic response components. The works by Poplawski (1988) and by Shiau and Jean (1991a) are similar to that of Choi and Noah; however, they added a condensation procedure which reduces the iteration problem so that it only involves those coordinates associated with the nonlinear components. This algorithm is a generalization of the same strategy developed by McLean and Hahn (1983) for analyzing the centered circular orbit response of large rotor systems with squeeze film dampers. Their works were performed in physical coordinates directly.

The weighted residual method of Galerkin has been successfully employed by many authors [Urabe (1965); Urabe and Reiter (1966); Brommundt (1975)]. Brommundt presented a detailed discussion of the Urabe-Reiter version of Galerkin's method for studying the periodic response of rotor-dynamic systems including the autonomous and non-autonomous cases. They also investigated bifurcations as a
means of locating hidden periodic solutions and discussed the stability analysis of the located periodic solutions. Brommundt also mentioned the possibility of condensation but did not present any details for its implementation.

The collocation method as formalized by Samoilenko and Romo (1979) for periodic response studies was utilized by Nataraj (1987) and Nataraj and Nelson (1989) along with a component mode synthesis strategy to render the procedure tractable for large order systems. Jean and Nelson (1991) presented a collocation method that could be utilized for large order nonlinear systems directly in physical coordinates. This was accomplished by utilizing a condensation procedure similar to that used by Shiau and Jean (1991a) and McLean and Hahn (1983). Harmonic, subharmonic, and superharmonic response components can be included in the work of Jean and Nelson (1991). Both autonomous and non-autonomous cases can be studied by using these approaches. Shiau and Hwang (1991) studied nonlinear rotor-bearing systems by using the combined methodology of the harmonic balance and the collocation methods.

The methods used for the transient analysis of dynamic systems are usually based on numerical integration. An approach named the discrete time-transfer matrix method was developed by Kumar and Sankar (1986) for the transient analysis of large order systems. The method discretizes a system into many subsystems and the equations of motion for each subsystem are formulated. The time response of the state at either ends of any subsystem is related by a transfer matrix, which is a function of initial and boundary conditions, from numerical integration procedures. Subbiah and Reiger (1988) and Subbiah et al. (1988) have successfully utilized this approach for large order rotor dynamic systems. Shiau and Jean (1991b) also did an approach for the transient analysis of large order nonlinear rotor systems; their work was directly performed in physical coordinates and a reduction process was included too.

In this study, techniques for the dynamic behavior analysis of large order mechanical systems with nonlinear characteristics are proposed, with special emphasis on the rotordynamic systems. For the steady state response analysis, a method based on the harmonic balance method for evaluating the periodic response of the system is developed. A method based on an implicit numerical integration is employed for predicting the transient response of the system. A reduction process based on the localized physical nature of the nonlinear effects of the system is included in the analysis. In this case, the problem will relate to only those coordinates of system nonlinear components. The solution procedures are similar to the works by Shiau and Jean (1991a, 1991b), which were carried out in physical coordinates. The present paper deals with the problem in modal coordinates in order to reduce the working space. Moreover the stability of the localized periodic response is studied by using Floquet theory. The merit of the methods is demonstrated by a 52 DOFs system with only four of the coordinates related to nonlinear components.

EQUATIONS FORMULATION

The nonlinearities mostly considered in rotordynamic systems are due to bearings, dampers, couplings, seals, etc. They are localized at few stations of the entire rotor system. The nonlinear forces due to these mechanisms (named nonlinear components) are functions of displacement, velocity, and possibly acceleration of the system at related stations. For the convenience, the physical coordinates related to the nonlinear forces are called nonlinear coordinates and denoted as a \((n_1 \times 1)\) vector \(\{q_1\}\). The physical coordinates describing the system motion at the pre-selected stations excluding the nonlinear coordinates are referred to linear coordinates and denoted as a \((n_1 \times 1)\) vector \(\{q\}\).

It is possible to consider the original nonlinear system as a linear structure with nonlinear forces applied at few associated locations, i.e., the forces are treated as pseudo applied forces. The linear structure can be modelled by some modelling techniques, such as the finite element method (Nelson and McVaugh, 1976), the generalized polynomial expansion method (Shiau and Hwang, 1991), etc. The system equation of motion subjected to the nonlinear forces is generally of the second order form

\[
[M]\{\ddot{q}\} + [C]\dot{\{q\}} + [K]\{q\} = \{F\} + \{\Phi\}
\]

(1)

where \([M]\) is known as the \(N \times N\) mass/inertia matrix which is a positive definite real symmetric matrix; \([C]\) is the dissipation matrix which is generally a non-symmetric; \([G]\) is the gyroscopic matrix which is skew symmetric; \([K]\) is the stiffness matrix which is generally non-symmetric and is probably dependent on the spin speed of the rotor system. The vector \([Q]\) is a specified time dependent force vector such as the rotating unbalance and the gravity load, etc. The vector \([F]\) is associated with the forces related to the nonlinear components of the system and is considered as a function of \(\{q_1\}\) and \(\{q\}\). To this in equation (1). \(\{q\}\) is a \((N \times 1)\) generalized coordinate vector. For the finite element method (FEM), \(\{q\}\) consists of translational and rotational displacements at each pre-defined station.

For the generalized polynomial expansion method (GPEM), since the translational displacements of the system are described by polynomial functions in terms of the rotor axial coordinate, \(\{q\}\) is the vector including the time dependent coefficients of the polynomial functions.

The advantage of using a modal representation is that the associated problem size can be reduced truncating the high modes which will lead to a subsequent saving in computer time. The modal representation of equation (1) requires the establishment of modal matrix which can be obtained by solving an eigenvalue problem. The eigenvalue problem based on the non-rotating symmetric undamped homogenous system can be expressed as

\[
\omega^2 [M]\{\Phi\} = [K]\{\Phi\},
\]

(2)

which has spin speed independent real modes. In equation (2), \(\omega^2\) is the real-valued eigenvalue and \(\{\Phi\}\) is the corresponding eigenvector. The eigenvectors satisfy the orthogonality conditions

\[
[\Phi]^T [M]\{\Phi\} = [I]
\]

(3)

\[
[\Phi]^T [K]\{\Phi\} = [\omega^2]
\]

(4)

where \([\Phi]\) is the modal matrix and has been normalized in such a way that equation (3) is an identity matrix \([I]\) and equation (4) is an undamped diagonal eigenvalue matrix \([\omega^2]\). Since the modal vectors satisfy the orthogonality conditions, they form a linearly independent set. The generalized coordinates can be expressed in terms of the modal coordinates \(\{\hat{q}\}\) using the transformation of

\[
\{q\} = [\Phi]\{\hat{q}\}\]

(5)

with truncation of modes from \(N\) to \(n_1\). Substituting equation (5) into equation (1), premultiplying by \([\Phi]^T\), and making use of equations (3) and (4), it yields

\[
[\Phi]^T [M]\{\ddot{\hat{q}}\} + [\Phi]^T [C]\{\dot{\hat{q}}\} + [\Phi]^T [K]\{\hat{q}\} = [\Phi]^T \{F\} + \{\Phi\}^T [\Phi]\{\hat{q}\}
\]

(6)

where

\[
\]

(7)

\[
[\Phi]_{i,1} = [\Phi]^T [K][\Phi]_{i,1}
\]

(8)

\[
[\Phi]_{i,1} = [\Phi]^T [Q]_{i,1}
\]

(9)

\[
[\Phi]_{i,1} = [\Phi]^T [F]_{i,1}
\]

(10)

It should be noted that the modal data based on other eigenvalue problem rather than equation (2) is possible when implementing the modal transformation.

STEADY STATE PERIODIC RESPONSE

For the periodic response analysis of the rotor system, a solution is assumed to be approximated by a finite Fourier series as

\[
\{\eta\} = \{q_0\} + \sum_{n=1}^{N_f} \left\{a_n \cos(n\omega t) + b_n \sin(n\omega t)\right\}
\]

(11)

where \(\omega\) is a set of preselected frequency contents which can be the sub, ultra-sub, super, and ultra-super harmonics of the fundamental component (Jean and Nelson, 1991). The fundamental period is denoted as \(T\). For the simplicity of presentation, if one choose

\[
\omega_0 = 0
\]

(12)

\[
\{b_n\} = \{0\}
\]

(13)

Equation (8) yields

\[
\{\eta\} = \sum_{n=1}^{N_f} \left\{a_n \cos(n\omega t) + b_n \sin(n\omega t)\right\}
\]

(14)

Substitution of equation (10) into equation (6) yields a set of nonlinear algebraic equations in terms of unknown Fourier coefficients as (Shiau and Jean, 1991a)

\[
\{a_n\} = \{F_n\}^{-1} \left\{\{Q_n\}, \{F_1\}\right\}
\]

(15)
The vectors \( \{q_{2j}\} \) and \( \{F_n\} \) are the Fourier coefficients of the linear and nonlinear vector \( \{q_2\} \). The linear vector \( \{q_2\} \) will be of the form

\[
\{q_{2j}\} = \{q_{2j}\} + j\omega_0 [D_d] + [K_n]
\]

where \( [q_{2j}] = \{q_{2j}\} + j\omega_0 [D_d] + [K_n] \) and \( \{q_{2j}\} \) is a small subset of physical coordinates. There exists a connectivity matrix \([S]\) in such a way that

\[
[S]{q_2} = [S]{q_2} + j\omega_0 [D_d] + [K_n]
\]

Premultiplying equation (11) by \([<172]\) and utilizing equation (12) implies that

\[
[Q^r]_n = [S]_n [Q^r]_n = [4.2177.7.7]_{1,0}
\]

where \( \{Q^r\}_n \) is the Fourier coefficient vector of the unbalance force, the index \( n \) denotes the sequence number of the \( n \) related to the unbalance excitation frequency, \( \{q_2\} \) denotes the complex vector of nonlinear and gravity load, the Fourier coefficient vector \( \{Q^r\}_n \) will be of the form

\[
\{Q^r\}_n = [\Phi^T]_n \{q^r\}_n + [\gamma]_n : i = 0, 1, 2, \ldots, N_f
\]

It should be noted that the generalized coordinate vector \( \{q\} \) may base on the FEM, the GPEM, or other modelling methods. Substituting equation (5) into equation (11), it yields

\[
[S]{q_2} = [S]{q_2} + j\omega_0 [D_d] + [K_n]
\]

Premultiplying equation (11) by \([<172]\) and utilizing equation (20), it yields

\[
[S]{q_2} = [S]{q_2} + j\omega_0 [D_d] + [K_n]
\]

This equation implies that

\[
[S]{q_2} = [S]{q_2} + j\omega_0 [D_d] + [K_n]
\]

where \( \{r_{2j}\} \) is the complex Fourier coefficient associated with the \( l \)-th harmonic component of the response \( \{q_2\} \). Therefore, equation (11) yields

\[
[S]{q_2} = [S]{q_2} + j\omega_0 [D_d] + [K_n]
\]

Premultiplying equation (11) by \([<172]\) and utilizing equation (20), it yields

\[
[S]{q_2} = [S]{q_2} + j\omega_0 [D_d] + [K_n]
\]

Since the nonlinear forces are generally localized at few stations, the nonlinear coordinate vector \( \{q_2\} \) is a small subset of physical coordinates. There exists a connectivity matrix \([S]\) in such a way that

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\]

Since the nonlinear forces are generally localized at several stations, the nonlinear coordinate vector \( \{q_2\} \) is a small subset of physical coordinates. There exists a connectivity matrix \([S]\) in such a way that

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\[
\{Q^r\}_n = [\Phi^T]_n \{q^r\}_n + [\gamma]_n : i = 0, 1, 2, \ldots, N_f
\]
of lubricant = 0.00266 N-s/m². The short bearing theory developed by Mohan and Hahn (1974) is utilized to evaluate the hydrodynamic forces of the dampers. The system is subjected to the gravity load and an unbalance excitation due to the eg eccentricity of 10.16 mm in the disc located at station 12.

The finite element modelling technique results in a 52 degrees of freedom system with four of those being nonlinear coordinates (n2 = 4), which are associated with the translation displacements of the journal in the squeeze film planes individually, are listed in Table 4. Because the system is axially symmetric as shown in Figure 1, the eigen properties are the same in both XY and XZ planes.

The GPEM (Shiau and Hwang, 1991) is also used to study the system characteristic. The method approximates the motion of the system in the XY and XZ-plane individually by a polynomial of order (Np - 1), which is written as

\[ \omega^2[M] \phi = [K] \phi \]

which is an eigenvalue problem associated with the undamped non-rotating system. The first 18 modes of the system, which consist of nine modes in XY and XZ planes individually, are listed in Table 4. Because the system is axially symmetric as shown in Figure 1, the eigen properties are the same in both XY and XZ planes.

The GPEM (Shiau and Hwang, 1991) is utilized to evaluate the hydrodynamic forces of the dampers. The polyharmonic modal model (Hahn, 1974) is employed to evaluate the hydrodynamic forces of the dampers. The method approximates the motion of the system in the XY and XZ-plane individually by a polynomial of order (Np - 1), which is written as

\[ [\mathbf{G}] \mathbf{a} = [\mathbf{C}] \mathbf{a} \]

which is an eigenvalue problem associated with the undamped non-rotating system. The first 18 modes of the system, which consist of nine modes in XY and XZ planes individually, are listed in Table 4. Because the system is axially symmetric as shown in Figure 1, the eigen properties are the same in both XY and XZ planes.

The GPEM leads to a set of 2N, nonlinear differential equations which has symmetric mass and stiffness matrices. The generalized coordinate vector is

\[ \{q\}_{\text{GPEM}} = \begin{bmatrix} \{C_{ij}\} \\ \{C_{ij}\} \end{bmatrix} \]

The system mass and stiffness matrices are used to generate the modal data. Table 4 also shows the first 18 eigenvalues associated with the GPEM.

The periodic response analysis, the first four harmonic components are included in the assumed solution. The periodic response over the range of 0 ≤ \( \Omega \) ≤ 3000 rad/s is computed with a speed increment of 20 rad/s. The results are obtained using a PC-380 computer.

Figure 2 indicates the static component of the system periodic response in the vertical (gravity) direction for the journals at stations 3 and 13. It should be noted that the amplitude of the component in the horizontal direction is very small compared to that in the vertical direction, it is not shown in the paper. Figures 3 and 4 show the amplitude of the semi-major axis of the first and second elliptic vibration components of the forced response of the journal at station 3, as a function of spin speed. Similarly figures 5 and 6 show those for the journal at station 13. It can be seen that the first three critical speeds of the first harmonic response components are approximately at 400, 720, and 2400 rad/s. It is found that the journals of the dampers will be lifted up when the spin speed closes to the first harmonic resonances. It is also seen from figures 4 and 6 that the higher modes are required for accurately predicting the response of the component at high spin speeds.

The stability of the located periodic solutions is evaluated from the eigenvalues of the so called Floquet transition matrix (Friedmann et al., 1977). Consider the eigenvalue of the Floquet transition matrix (FTM) as \((\lambda_1 + \lambda_2)\) and \((\lambda_1 A + \lambda_2 A)\) if for all eigenvalues of the FTM, the system at the equilibrium state is said to be stable. For the present analysis, since the maximum value of \((\lambda_1 A + \lambda_2 A)\) of the FTM associated with the periodic solutions shown in figures 2–6 has been computed to be less than 1. This implies that all the modes are stable.

Figures 7 and 8 show the transient response of the journals at stations 3 and 13 respectively for spin speed of 700 rad/s. The steady state response of this case is shown in figure 9. The precession is in the counterclockwise direction. It is seen that the amplitude of the response based on the PEM is slightly different from that based on the GPEM. However, the amplitude of the steady state orbits shown in figure 9, which has about mean movement of 4.869 × 10⁻³ m with amplitude of 3.249 × 10⁻³ m and mean movement of 3.884 × 10⁻³ m for the journal at station 3, is approximately in correspondence with the result obtained by the periodic solution strategy.

### DISCUSSION AND CONCLUSIONS

Many large order mechanical vibration systems are of nonlinearity. Especially for rotor bearing systems, the nonlinearities are usually associated with a few discrete components. In this case, the system model can be considered as primarily linear, with the nonlinear effects from a small subset of the system coordinates. Methods in applying this physical nature of localized nonlinear effects have been presented for studying the dynamic response as well as the system stability. A new method based on the harmonic balance method has been proposed for evaluating the periodic response of the system. An implicit numerical integration is employed for predicting the transient response of the system. The analyses described in this paper have been performed in modal coordinates in order to reduce the working space. The solution procedures result in a set of nonlinear algebraic equations in terms of Fourier coefficients of the modal coordinates for the HBM and in terms of modal coordinates at a time instant for the integration scheme. A reduction process have been introduced to reduce the number of the nonlinear algebraic equations to those coordinates only related to the nonlinear components of the system.

A flexible rotor system with nonlinear squeeze film dampers has been studied to illustrate the merit of the procedures. The system is modeled as a 52 degrees of freedom system with only four of the coordinates related to the nonlinear squeeze film dampers. For the periodic solution strategy if \( N_p = 4 \) is used and the HBM is directly applied in the physical domain, it will generate 162 nonlinear algebraic equations. Moreover, if the algorithm by Shiau and Jean (1991a) is employed, it will lead to 36 equations with the working space of the matrix operations is 48. The present method will result in 36 nonlinear algebraic equations and the working space is 18. Similarly, the analyses described as above can be applied in the transient response strategy. It is shown that the reduction method proposed in this study is of better efficiency and it is highly recommended for the dynamic analysis of large order mechanical vibration systems with nonlinear characteristics.

### ACKNOWLEDGEMENT

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### REFERENCES


TABLE 4: EIGENVALUES IN BOTH XY AND XZ PLANES (rad/s)

<table>
<thead>
<tr>
<th>No.</th>
<th>FEM</th>
<th>GPEM</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>311.1</td>
<td>312.9</td>
</tr>
<tr>
<td>2</td>
<td>667.9</td>
<td>673.2</td>
</tr>
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<td>3080.5</td>
</tr>
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</tr>
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<td>6035.9</td>
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<td>8</td>
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<td>7839.5</td>
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<tr>
<td>9</td>
<td>12980.2</td>
<td>12883.4</td>
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</table>

FIGURE 1: ROTOR SCHEMATIC PLOT.

TABLE 1: ROTOR CONFIGURATION DATA.

<table>
<thead>
<tr>
<th>Element Number</th>
<th>Element Length (cm)</th>
<th>Inner Radius (cm)</th>
<th>Outer Radius (cm)</th>
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</thead>
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<td>2.95</td>
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<td>7.92</td>
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</table>

E=20.68 x 10^5 N/m^2, ρ=8193 kg/m^3

FIGURE 2: CONSTANT OFFSET OF ELLIPTIC VIBRATION VS. SPIN SPEED.

TABLE 2: CONCENTRATED DISC DATA.

<table>
<thead>
<tr>
<th>Station Number</th>
<th>Mass (kg)</th>
<th>Diametral Inertia (kg m^2)</th>
<th>Polar Inertia (kg m^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.38</td>
<td>0.0982</td>
<td>0.1957</td>
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<td>2</td>
<td>7.88</td>
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<td>7.70</td>
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<td>21.71</td>
<td>0.2224</td>
<td>0.4448</td>
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</table>

TABLE 3: ISOTROPIC BEARING PROPERTIES.

<table>
<thead>
<tr>
<th>Station Number</th>
<th>Stiffness (N/cm)</th>
<th>Damping (N/s/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>17510</td>
<td>0</td>
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<tr>
<td>6</td>
<td>769950</td>
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</tbody>
</table>

FIGURE 3: FIRST HARMONIC SEMI-MAJOR AXIS OF JOURNAL RESPONSE AT STATION 3 VS. SPIN SPEED.


**FIGURE 4:** SECOND HARMONIC SEMI-MAJOR AXIS OF JOURNAL RESPONSE AT STATION 3 VS. SPIN SPEED.

**FIGURE 5:** FIRST HARMONIC SEMI-MAJOR AXIS OF JOURNAL RESPONSE AT STATION 13 VS. SPIN SPEED.

**FIGURE 6:** SECOND HARMONIC SEMI-MAJOR AXIS OF JOURNAL RESPONSE AT STATION 13 VS. SPIN SPEED.

**FIGURE 7:** TRANSIENT RESPONSE OF THE JOURNAL AT STATION 3. PRECESSION IN COUNTERCLOCKWISE DIRECTION. $\omega = 700 \text{rad/s}$, $n_0 = 18$, $N_P = 16$; ---: FEM, --: GPEM.

**FIGURE 8:** TRANSIENT RESPONSE OF THE JOURNAL AT STATION 13. PRECESSION IN COUNTERCLOCKWISE DIRECTION. $\omega = 700 \text{rad/s}$, $n_0 = 18$, $N_P = 16$; ---: FEM, --: GPEM.

**FIGURE 9:** STEADY STATE RESPONSE OF THE JOURNALS. $\omega = 700 \text{rad/s}$, $n_0 = 18$, $N_P = 16$; ---: FEM, --: GPEM.

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