OPTIMAL DESIGN OF SPLIT RING DAMPERS FOR GAS TURBINE ENGINES

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ABSTRACT

Vibration analysis techniques used for labyrinth air seals are presented, along with a derivation of the methods used to calculate the dissipation capability of split ring dampers. After defining the problem in classical optimization format, the methods used to automate the process are presented, along with the results for a particular test case. For the case considered, the optimal designs were found to lie at or near the design space boundaries.

INTRODUCTION

The annular air seal is commonly used in gas turbine engines to prevent gas flow from one section of the engine to another. The rotating part of the seal is designed to maintain clearances on the order of 0.125mm - 0.25mm with the mating static structure, thus producing an effective seal. (Figure 1)

The seal is a full hoop structure which can vibrate with both axial and circumferential nodal patterns. If the natural frequency of any of these modes is near a driving frequency present in the system, vibrations of excessive magnitude may develop, leading to premature failure. For this reason, it is common to provide additional damping to

Figure 1 - Annular Air Seal

Figure 2 - Split Ring Damper
the system in the form of a split ring damper. The damper is a wire formed into a ring containing a gap which rides in a groove provided for it on the seal. (Figure 2) It may have a circular, rectangular, or other cross-section. Centripetal forces generated by engine rotation cause the ring to be forced against the seal surface. Vibrational modes of the system result in relative motion between the ring and the seal surface which causes energy to be dissipated, thus lowering the amplitude of vibration to an acceptable level.

Current design methods for split-ring dampers are based largely on past experience. In this paper we will present a method for automatically determining the optimal damper design for circular or rectangular cross sections.

**SPLIT DAMPER ANALYSIS**

The vibrational modes of an annular air seal have two identifying indices. The first is \( m \), which is the number of axial nodes in the vibration pattern. The second is \( N \), which is the number of full wavelengths that are present around the circumference of the seal. Each combination of \( m \) and \( N \) defines a unique vibrating shape or mode (Figure 3).

![Figure 3 - Vibrational Patterns for Shell Structures](image)

(Figure 3). Since the axial length of the seal is normally much less than the circumferential length, it is customary to restrict consideration to axial modes of \( m = 1 \), while circumferential modes up to \( N = 12 \) are normally considered. Assuming negligible damping for now, the natural frequency associated with each of these modes may be obtained by solving the generalized eigenvalue problem:

\[
[M] \dot{x} + [K] x = 0 \quad (1)
\]

These frequencies are then compared with the driving frequencies from the rotor and also with the natural frequencies of the mating static structure. The damper is designed to maximize dissipation at the natural frequency which comes closest to one of the potential drivers in the structure surrounding it.

Current design methods for split-ring dampers are based largely on past experience. In this paper we will present a method for automatically determining the optimal damper design for circular or rectangular cross sections.

**ASSUMPTIONS**

(Figure 3) shows a typical damper ring configuration with the nomenclature to be used in this paper. The underlying assumptions for the system are:

1. Seal thickness and sectional properties are constant.
2. Coefficient of friction, \( \mu \), is constant.
3. The seal radial thickness is small compared to its radius.
4. Hooke's Law applies: only elastic deformation is considered.
5. The assumed radial displacement of the seal nodal pattern is \( w = B \cdot \cos(N\phi) \).
6. The seal body will undergo only inextensible flexural vibration. (Neutral axis length remains constant)
7. Due to the surface friction force the damper ring may experience a significant extension of its neutral axis.
STRAINS IN THE SEAL AND DAMPER

The following analysis is based on work presented by Alford[1-4]. For in plane bending of a circular ring:

\[
\frac{M}{EI} = \frac{1}{R^2} \left[ w + \frac{d^2 w}{d\theta^2} \right] \tag{2}
\]

Using (2), the bending strain on the surface is:

\[
\varepsilon_{\text{bend}} = \frac{C}{R^2} \left[ w + \frac{d^2 w}{d\theta^2} \right] \tag{3}
\]

For the seal, the bending strain is compressive at the interface, while it is tensile for the damper ring. Therefore,

\[
\varepsilon_{s,\text{bend}} = \frac{C_s}{R_s^2} \left[ N^2 - 1 \right] \cos (N\theta) \tag{4}
\]

\[
\varepsilon_{d,\text{bend}} = \frac{C_d}{R_d^2} \left[ N^2 - 1 \right] \cos (N\theta)
\]

The opposing strains at the interface cause friction forces in the circumferential direction, which will cause the ring to undergo elongation or contraction. Over some portion of the region, the friction forces may be sufficient to prevent slipping. For now it will be assumed that no slipping occurs at the interface over the region, \(0 < \theta < \theta_0\), where \(\theta_0\) is to be determined. For a \(\theta\) that is greater than \(\theta_0\), slipping will occur, and the friction force over the region will be maximum and will cause the damper ring to contract in compression. Therefore, the total strain in the damper ring at the interface is the sum of the bending strain and the frictional strain.

To determine \(\theta_0\), the point where the slipping motion ceases, consider the seal with a deflected shape of amplitude \(R\), where slippage is impending at \(\theta = \pi/2N\), but friction forces are just high enough to prevent it. The strain in the seal is equal to the strain on the damper ring for a no slip condition. \(\varepsilon_{s,\text{bend}}\) is compressive because the seal elongation strain is zero (inextensible) and its bending strain is compressive at the interface, while the damper ring surface bending strain is tensile.
We may alternately calculate the strain caused by friction at any point in the ring by dividing the frictional force at that point by the product of the damper area and its modulus of elasticity. The frictional force at any point is obtained by integrating the force per unit length over the length of application. Let \( F \) be the frictional force per unit length at the interface prior to the onset of slipping, where \( F = \mu P \), and \( \theta = 0 \).

Therefore,

\[
F = \frac{PR_d}{A_dE} \int F \cos(\theta) \, d\theta
\]

Substituting the expression for \( \epsilon_{d, friction} \) from (5) yields:

\[
F = \frac{NA_dEC_dB}{R_d^3} \left[ \frac{C_s(R_d^2)}{C_d(R_s^2)} + 1 \right] (N^2 - 1) \cos(N\theta)
\]

Thus, when there is no slipping \( F_{max} \), will occur at the nodes, when \( \theta = n/2N \), \( \sin(N\theta) = 1 \). When slipping is impending at the nodes, \( F_{max} = P \), \( \sin(N\theta) = 1 \) and \( B = B_o \) which is the modal amplitude corresponding to no slippage for a given radial load per unit length, \( P \). Equation 8 may then be solved for \( B \).

\[
B = \frac{1}{\frac{NA_dEC_d}{R_d^3} \left[ \frac{C_s(R_d^2)}{C_d(R_s^2)} + 1 \right] (N^2 - 1)}
\]

The "lock-up" load \( P_i \) required to stop all slipping for a given amplitude \( B \) may also be obtained.

\[
P_i = \frac{NA_dEC_dB}{\mu R_d^3} \left[ \frac{C_s(R_d^2)}{C_d(R_s^2)} + 1 \right] (N^2 + 1)
\]

If \( B > B_i \) for some \( P \), slipping will proceed over the region \( \theta_o \leq \theta \leq n/2N \). At the point, \( \theta = \theta_o \), \( F \) is equal to \( \mu P \). Combining equations (9) and (10) we obtain:

\[
F = \mu P \sin(N\theta) = \mu P \frac{B}{B_i} \sin(N\theta)
\]

At \( \theta = \theta_o \),

\[
\sin(N\theta_o) = \frac{P}{P_i} = \frac{B_i}{B}
\]

CALCULATION OF SLIP BETWEEN DAMPER AND SEAL

To find the slip, the tangential displacement of the damper ring surface, \( \nu_r \), must be calculated over the region where slip is occurring, and compared to the tangential displacement of the seal. In this region, the friction force per unit length is at a maximum and therefore,

\[
\epsilon_{d, friction} = -\frac{\mu PR_d}{A_dE} \frac{(\pi - \theta)}{2N^2} \sin(N\theta)
\]

since the ring is assumed to be extensible,

\[
\epsilon_{d, friction} = -\frac{w}{R_d} \frac{1}{\frac{R_d}{d\theta}}
\]

and,

\[
\frac{dV_d}{d\theta} = -\frac{\mu P R_d^2}{A_dE} \left( \frac{\pi}{2N^2} - \theta \right) - B \cos(N\theta)
\]

Equation (15) can be integrated to yield:

\[
\nu_d = \frac{B}{N} \sin(N\theta) - \frac{\mu P R_d^2}{2A_dE} \left( \frac{\pi}{2N^2} - \theta \right)^2 + C
\]

The tangential displacement of the damper ring, \( \nu_r \), at the interface radius, \( R_i \), can be obtained by adding the bending component and the tensile component of the ring's displacement.

\[
\nu_r = -\frac{B}{N} \sin(N\theta) - \frac{\mu P R_d^2}{2A_dE} \left( \frac{\pi}{2N^2} - \theta \right)^2 + \frac{C R_i}{N R_d^2} (N^2 - 1) \sin(N\theta) + C
\]

For the seal body at the interface, the technique is similar. However, the bending strain displacement has the opposite sign and the "stretching" term is equal to zero since inextensibility is assumed. Let \( \nu_s = \) the displacement of the seal at the interface and \( s = \) slip.
Then $s = v_0 - v$, relative displacement. Using the above equations and combining the integration constants, and noting that at $\theta = \theta_0$, $s = 0$, the expression for slip is:

$$s = \frac{\mu PR_d^2}{EA_d} \left[ -\frac{1}{N^2} \frac{R_f}{R_d} \left( \cot \theta_0 \cdot N \theta_0 - \frac{\pi}{2} \right) \right]$$

for $\theta_0 \leq \theta \leq \pi/2N$. For all other locations, $s = 0$.

**WORK CALCULATION**

The work, $W$, done by the friction force in a full cycle may be obtained by multiplying the friction force by the relative displacement and integrating over the slip region.

$$W = 16 \frac{(\mu P)^2 R_d^2}{N^2 EA_d} \left[ -\frac{R_f}{R_d} \left( \cot \theta_0 \cdot N \theta_0 - \frac{\pi}{2} \right) \right]$$

The power, $D$, dissipated by the damper ring is obtained by multiplying the work per cycle by the frequency of vibration, $f$. The absolute value is necessary since slip is defined as a negative in equation (19).

$$D = 16f \frac{(\mu P)^2 R_d^2}{N^2 EA_d} \left[ -\frac{R_f}{R_d} \left( \cot \theta_0 \cdot N \theta_0 - \frac{\pi}{2} \right) \right]$$

Note that the energy dissipated is a function of $\theta_0$, the angle where slippage starts. As shown in (12), this angle is a non-linear function of the modal amplitude, $B$. Therefore, the damping is a non-linear function of the modal amplitude, $B$. In particular, a finite modal amplitude, $B$, exists where no slipping occurs, and no energy is dissipated by the damper.

**DAMPER PERFORMANCE METRIC**

The non-linear dependence of the dissipation on the amplitude of vibration, which is unknown at the design stage, makes it difficult to compare the performance of various damper designs. Therefore, it was decided to base the comparison on an assumed vibrational amplitude which results in a vibratory hoop stress in the seal equal to the allowable vibratory stress obtained from a Goodman Diagram analysis. A relative dissipation ratio, $Q$, can then be formed, which is the ratio between the total system energy and the energy dissipated by the damper.

$Q = \frac{\text{Seal Vibrational Energy + Energy dissipated by Damper}}{\text{Energy dissipated by Damper}}$  \hspace{1cm} (21)

Note that the vibrational energy of the seal is a function of the modal amplitude, $B$, which is measured at the free end of the seal, while the energy dissipated by the damper is calculated based on the radial amplitude at the particular axial location of the seal.

For design purposes $Q$ may be used to measure the merits of one damper design versus another. As $Q$ becomes smaller, the percentage of total system energy that is dissipated by the damper increases. For the assumed $B_{\text{damp}}$, the damper with the smaller $Q$ value would provide the largest dissipation, and thus the greatest reduction in actual response amplitude. The optimal damper design is then that design which minimizes the value of $Q$.

**DESIGN VARIABLE DEFINITION**

$Q$ is a function of four or five design variables, depending on the damper cross section desired. They are: 1. ($x_1$): damper diameter (or width for rectangular cross section), 2. ($x_2$): interface radius, $R_f$, 3. ($x_3$): axial location along the seal, 4. ($x_4$): the coefficient of friction between the seal and damper, and 5. ($x_5$): damper aspect ratio = height/width if a rectangular cross section is desired (Figure 6). The interface radius and axial location design variables were normalized by dividing by the maximum interface radius, and the seal length respectively.

**CONSTRAINTS**

The problem will be constrained by two strength and one stability criteria. First, the additional damper load must not increase the total hoop stress at the knife edge tip, $\sigma_{\text{total}} = \sigma_{\text{ext}} + \sigma_{\text{damp}}$, to greater than 80% of $\sigma_{\text{PSY5}}$.
Second, the total bending stress at the seal base must not exceed $\sigma_{2\text{max}}$. Finally, damper separation is not allowed. If the damper’s inertia is greater than the force per unit length applied to the damper, i.e. the radial load is insufficient to cause the ring to maintain contact with the seal, separation will occur. Since the damper radial load is caused strictly by centrifugal forces, the equation for separation may be written as:

$$\rho A_d \sigma_2 B < \rho A_d R_d \sigma_2$$

$$\rho A_d (2 \pi f)^2 B < \rho A_d R_d \left(\frac{RPM + 2\pi}{60}\right)^2$$

or

$$3600 \frac{B}{R_d} \left(\frac{f}{RPM}\right)^2 - 1.0 < 0$$

In addition, a series of side constraints are used. The damper diameter will be limited such that $1.0mm \leq x_1 \leq 7.5mm$. This is based on size limitations of the seal, and the need for attachment material. The normalized interface radius will be limited to $0.95 \leq x_3 \leq 1.0$. For values smaller than 0.95, the material added to the seal to house the damper increases in radial height to a point where additional stiffness is added to the seal, and the neutral axis of the seal may also shift. The normalized axial location will be limited such that $0 \leq x_3 \leq 1.0$. The coefficient of friction will be limited such that $0.4 \leq x_4 \leq 0.7$. Finally, if a rectangular cross section is used, the aspect ratio will be limited to $0.8 \leq x_5 \leq 1.25$. These limits are used so that the damper will not have a tendency to "roll" over, or be too long axially to fit on the seal. With the objective function, design variables, and constraints defined, the problem can now be summarized in classical optimization format.

$$\text{minimize} \quad Q(x) ; \quad x=(x_1, x_2, x_3, x_4, x_5)$$

subject to:

$$g_1(x) = 0.8 \sigma_{2\text{max}} - \sigma_{\text{hoop}} \geq 0 \quad \text{- Hoop Stress}$$

$$g_2(x) = 0.2\sigma_{2\text{max}} - \sigma_{\text{bending}} \geq 0 \quad \text{- Bending Stress}$$

$$g_3(x) = 1 - 3600 \frac{B}{R_d} \left(\frac{f}{RPM}\right)^2 \geq 0 \quad \text{- Separation}$$

$$g_4(x) \quad \text{thru} \quad g_{13} \quad \text{or} \quad g_{15}(x) \geq 0 \quad \text{- Side Constraints}$$

The minimization of $Q$ was accomplished using the Keefer and Gottfried exterior penalty function method as described in Vanderplaats[5], based on it's robustness and ability to handle starting points outside the feasible region. A pseudo-objective function, $Q'$, is defined as:

$$Q'(x) = Q(x) - r_p \sum_{i=1}^{n} \left(\min[0, g_i(x)]\right)^2$$

The penalty multiplier, $r_p$, is a positive variable which serves to increase(penalize) the pseudo-objective function for any violated constraints. Low values of $r_p$ will typically result in solutions in the infeasible region. Therefore the penalty multiplier is increased as the search progresses to force convergence on the constrained optimum. Normalization of the design variables and constraints so that all values fall in the region $0.0 \leq x_i \leq 1.0$ ensures that all constraint violations are given equal weight.

OPTLIB [6], an optimization code library produced by the University of Missouri which contains a menu of general optimization algorithms, was used to perform the search for the optimum damper design. For this study, Powell’s method was selected to generate successive search directions through the design space. The one-dimensional searches were performed using the Golden Section algorithm.

The program starts with an initial damper design, as defined by assigned values of $x_1, x_2, x_3, x_4, x_5$. Each starting vector lies in the basin of attraction of some local minimum. Ten different starting vectors were used for both the circular and rectangular damper to attempt to find the best local minimum.

With the initial damper geometry known, the additional load on the seal caused by the damper mass can be calculated. The change in steady state stress is found by using the method of superposition. Since the problem is purely linear, for a given damper load at a known axial station, the new stress in the seal can be calculated using equation (25), where $(\sigma\delta)_{\text{basic}}$ is known from a static F.E. analysis.

$$\frac{(\sigma\delta)_{\text{new}}}{(\delta)} = \frac{(\sigma\delta)_{\text{old}}}{(\delta)} + \frac{\text{damper load}}{(\delta)_{\text{basic}}}$$

The OPTLIB code, which performs the search, calls a FORTRAN subroutine to evaluate the objective function and constraints. The axial location is sent to the interpolation subroutine to update the ratios and find the amplitude of vibration at the current axial location from the mode shape data. Finally, Q and the constraints are calculated and returned to the optimization program. The program generates new values of the design variables according to the strategy outlined above. The process is repeated until the lowest value of Q is found. The final values of the design variables are output, along with the values of the constraints.

**RESULTS**

The optimization code was run on the test case shown in Figure 1. In order to test for local minima of the objective function, various starting points for the design variables were chosen. The results from the program are summarized in Table 1 for circular dampers and Table 2 for rectangular cross sections, with the optimum designs highlighted.

As can be seen, for the test case studied, both the optimum circular and rectangular dampers were found
### Table 1 - Results of Circular Cross Section Optimization

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### Table 2 - Results of Rectangular Cross Section Optimization

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7
to have the minimum interface radius, and the maximum coefficient of friction.

The circular damper had the largest diameter allowed, located at the far end of the seal and showed no local minima within the design space. Even at the boundaries, where the optimum was found, the constraints of stress magnitude and damper separation tended to be well within the allowable bounds.

For the different starting points, the rectangular cross section exhibits local minima with damper heights of approximately 6.75mm and 7.5mm.

**DISCUSSION**

Figure 7 - Q and Dissipation as Functions of Amplitude

Figure 7 is a plot of total energy dissipation and Q, as functions of B, the amplitude of radial motion of the damper axial station along the seal. For any given mode shape and the assumed response amplitude, the seal's total energy is fixed. Therefore, the numerator of Q becomes a constant plus the dissipation produced by the damper. The damper dissipation is a function of the amplitude at the axial location chosen. Since seal energy is constant for the mode shape, moving to the far end of the seal has the effect of increasing B, which will decrease Q.

A larger diameter wire would also produce similar results by providing additional pressure load which also increases dissipation, but is limited by the stress constraints. Finally, a larger coefficient of friction will provide a larger amount of work for a given displacement.

From equation (17), it can be seen that with all other variables being held constant, slip increases linearly with respect to $c_u$, the distance between the seal's neutral axis and the interface surface. In addition, for a given damper, reducing $R_f$ and $R_d$ by the same amount causes $R_f/R_d$ to become larger which increases dissipation.

The test case tended to validate these observations for the circular cross section. For the rectangular cross section, however, the optimum damper was located only 95% of the distance from the base to the end, and the hoop stress constraint was active. This move in location indicates that for optimum dissipation, the pressure produced by the damper is a stronger contributor than the axial location. For this case it is more advantageous to move the seal towards the base, even though the local amplitude is lower, in order to satisfy the stress constraints, rather than sacrifice pressure load by making the damper cross section smaller.

For the rectangular cross section tested, an aspect ratio greater than 1 would add additional distance between the neutral axes, but the optimum damper was found to have an aspect ratio less than one. For this particular test case then, we may conclude that damper load is a stronger influence on overall effectiveness than the distance between neutral axes.

**EVALUATION OF Q AS A DESIGN METRIC**

Figure 8 - Q and Dissipation as Functions of Modal Amplitude

The relative dissipation ratio, Q, is computed using an assumed modal amplitude which may not actually occur. For this amplitude, the lowest value of Q corresponds to the largest amount of dissipation by the damper. For a
given seal, larger values of dissipation (smaller Q) will result in larger reductions of vibratory amplitude.

When the seal system is in the design stage, it is difficult to determine the actual vibratory amplitude. Modal analysis can be used to approximate the response only if the internal damping and forcing function are assumed. Figure 8 is a plot of dissipation and Q as functions of the modal amplitude. Note that a different definition of B is used than in Figure 9. Here, B is the assumed amplitude which is used to calculate seal energy for a given axial location of the damper. As can be seen, as B decreases and approaches B", the lock up amplitude, Q tends to increase in a quadratic fashion. As B increases above B", Q tends to be linearly related. Therefore, if B_{damped,actual} is less than the assumed B, Q will be higher, but infinite life will be achieved. If B_{damped,actual} is greater than the assumed B, the design does not meet design requirements, and must be redesigned.

Figure 9 - Q as a Function of Modal Amplitude

For the given circular optimum damper design, x_1 = 7.5mm, x_2 = 200.3mm, x_3 = 1.0, x_4 = 0.700, various damped amplitudes were assumed, and the corresponding Q values calculated. The results are presented in Figure 9. Also shown are 4 randomly generated damper designs. The design variables chosen are summarized in Table 3. The results are similar for the rectangular damper. As can be seen, over the range of amplitudes shown, the optimum damper design still provides the lowest Q value. Only random damper #4 comes close to the optimum, but this is due to the fact that the optimum damper would be near lock-up for lower B values in this regime. If in fact, the locked-up amplitude for the optimum damper is converted back to a vibratory stress, it is seen that infinite life would result.

Table 3 Random Dampers

<table>
<thead>
<tr>
<th>Trial</th>
<th>x_1 (mm)</th>
<th>x_2 (mm)</th>
<th>x_3</th>
<th>x_4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5</td>
<td>222.5</td>
<td>0.92</td>
<td>0.550</td>
</tr>
<tr>
<td>2</td>
<td>2.0</td>
<td>215.9</td>
<td>0.84</td>
<td>0.580</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>211.4</td>
<td>0.60</td>
<td>0.700</td>
</tr>
<tr>
<td>4</td>
<td>2.8</td>
<td>200.3</td>
<td>0.90</td>
<td>0.300</td>
</tr>
</tbody>
</table>

The important thing to note from the graph is that although the Q value for the optimal damper design is not expected to be attained when the hardware is manufactured, the optimum design will provide the maximum amount of dissipation possible for the given constraints. When hardware is produced, vibration tests can be performed to evaluate the actual reduction in vibratory amplitude.

CONCLUSIONS

It has been demonstrated that by using standard optimization techniques, it is possible to design an optimum split ring damper. For the test case presented, the optimum damper existed on the bounds of the design space for the circular damper, and near the boundary for the rectangular cross section. It is shown that the relative dissipation ratio, Q, is a valid metric for comparison of candidate designs, since the damper selected based on Q evaluated for some assumed amplitude would also be the optimum for any other reasonable amplitude of response. Using the optimization technique increases the likelihood that the vibrational amplitude will be reduced to an acceptable level, since no other damper design, subject to these constraints, can provide greater dissipation than the optimum design.

REFERENCES

