EXPERIMENTAL STUDY ON VIBRATION OF A ROTATING BLADE

Y.C. Fan, M.S. Ju and Y.G. Tsuei
Structural Dynamics & Control Laboratory
Department of Mechanical Engineering
National Cheng Kung University
Tainan, Taiwan 701, R. O. C.

ABSTRACT
The vibration of a rotating blade is investigated in this work. A rotor system is built and natural frequencies of the rotating blade are measured and compared with the numerical results from a finite element analysis. The experimental setup has a strain gage-based telemetry system and a piezoelectric shaker that rotates with the rotor. The finite element model of the beam is derived based on the Timoshenko beam theory. The effects of varying rotating speeds and stagger angles on the blade natural frequencies are studied. The results indicate that the natural frequencies calculated from the finite element model and the experimental values are in good agreement. It is found that the blade natural frequencies increase with the rotating speed in a nonlinear way. The effects of the stagger angle on the measured natural frequencies are not clear.

NOMENCLATURE
A cross sectional area of blade
b width of the blade
E Young's modulus
G shear modulus
I_y, I_z area moment of inertia of blade cross section about y and z axes
J polar moment of inertia of the blade
K(Ω) stiffness matrix of the assembled FEM model of the blade
M mass matrix of the FEM model
T(Ω) centrifugal force field
κ shear correction factor

u, v, w elastic displacement components of a point of the blade at x, y, and z directions
θ, α, β rotating angles of a point of the blade neutral axis about x, y and z directions
ρ density of blade material
ϕ stagger angle
Ω rotating speed
ω natural frequency of blade
\dot{\cdot} partial differentiation with respect to x
differentiation with respect to time t

INTRODUCTION
The vibration analysis of rotating blades is an important issue in turbomachinery research. In the past two decades, both analytical methods and experimental measurement techniques have been developed to analyze the vibration behavior of a rotating blade (Leissa 1981). The stability of the blade vibration is affected by the centrifugal force due to the rotation, the stagger angle of the blade, pre-twist and taper of the blades (Mahrenholtz 1984). In this work the effects of the rotating speed and blade stagger angle on the blade natural frequencies are studied. In general, the blade can be modeled as a cantilever beam mounted on the periphery of the rotor. Carnegie (1959) first derives the increment of strain energy of the blade and the hub due to centrifugal force and the Rayleigh's method is employed to solve the vibration problem. The relation between the shifting of fundamental frequency and the staggered angle is also explored. In another work, the finite difference method is employed to solve the blade vibration problem (Carnegie & Thomas 1972a). The shear deformation...
and rotary inertia are considered and the selection of shear coefficients are also discussed (Carneigie et al 1972b). The vibration of pre twisted and tapered blades are also studied. Since the closed form solution for the blade vibration problem is not obtainable so numerical analysis is employed. Thomas (1973) suggests several finite element models based on Timoshenko beam theory. In this work only the geometric boundary conditions are considered. Latter, Thomas and Abbas (1975) develop a model which includes both geometric and a simple natural boundary condition.

Experimental measurement of the blade vibration is important for validating the analytical or numerical results. For non contact measurement of blade vibration, Kulczyk and Davis (1973) develop a laser Doppler instrument. Stargardter (1977) uses mirrors that mounted on the blade to measure the mode shapes from the deflection of a laser light. Frei (1977) uses strain gages and slip rings to measure the vibration of a rotating blade.

In this work, the experimental and finite element analysis of a rotating blade are performed. A strain gage-based telemetry system and a piezo-electric shaker system are employed to measure the natural frequencies of a rotating blade. The results are compared with the finite element analysis. In the finite element model the blade is modeled as a Timoshenko beam and the following factors are considered, namely, the stiffness effect of the centrifugal field, the staggered angle, support elasticity and the coupled vibrations of the disk and its blades.

METHODS

Experimental setup

The block diagram of the experimental setup is shown in Figure 1. There are six subsystems, namely, (a) piezo-electric shaker and a slip ring to excite the blade, (b) telemetry system to transmit and receive strain gage signal, (c) servo motor system to drive the rotor to a constant speed, (d) velocity detector system to measure the rotating speed, (e) a spectrum analyzer to collect and calculate the frequency response function in real time. The steel shaft measures a diameter of 47 mm and the aluminum alloy rotor measures 229 mm in diameter and 101 mm in thickness. The blade is a flat aluminum beam measured 253 mm in length, 25.4 mm in width and 1.55 mm in thickness. The Young’s modulus of the blade material is 70 GPa, 27 GPa for shear modulus and 2710 Kg/m³ for the density. The blade is bolted to a stand that can be indexed to different stagger angles (Figure 2a). The stand is fastened to the rotor by four bolt screws. Two dynamic strain gages are attached on one side of the blade surface at 10 cm and 15 cm from the root surface respectively. Since the telemetry system has only one channel the strain signals are measured one after one. Attention should be paid to the connection between the shaker and the blade. Due to the lack of a slip ring channel for the load cell of the shaker, the shaking force can not be measured directly. The exciting signal from the spectrum analyzer instead of the shaking force is used as the input for the blade system and the strain signal as the output. The shaking force is transmitted by the connector (or stinger) to the blade. The dynamic coupling between the shaker and the blade due to the stiffness of the connector may introduce errors in the measured frequency response functions. A straight and an U-shaped connectors (Fig.2b) are used in the tests when the rotor is still or the so-called static case. Random tests using the two connectors and an impulsive test with an accelerometer attached at the free end of the blade for the static case are performed to explore the dynamic coupling due to the connector. With the U-shaped connector and the strain gages, the first three modes of the blade in the static case are measured by using the swept sine tests with the exciting frequency around each resonant frequency found from the random test. The measured natural frequencies are compared with the analytical solutions based on Euler beam theory and the finite element results. These data are used as references for the dynamic (or rotating) cases.

In the dynamic cases, the continuous random signal is employed and two sets of experiments are performed, namely, the speed-up and the slow-down tests. In the speed-up, the rotating speed is made to increase from zero rpm to 800 rpm with an increment of 200 rpm. In the slow-down, the rotating speed is made to decrease from 800 rpm to zero rpm with a decrement of 200 rpm. The servo motor is controlled by a PID controller to rotate at each desired speed. When the rotor speed reaches the steady state, the analyzer sends a uniformly distributed signal to the shaker driver. After the transient response dies out, the analyzer sends a uniformly distributed signal to the shaker driver. After the transient response dies out, a total number of 100 frequency response function (FRF) measurements are taken and averaged to yield an accurate estimate. The frequency band is from zero Hz to 800 Hz. The blade is tested with stagger angles ranged from zero degree to 70 degrees. When the stagger angle is higher than 50 degrees the highest rotor speed is limited to 600 rpm due to the aerodynamic resisting force.

Modeling of the rotating blade

From the Timoshenko beam theory and the Hamiltonian principle the governing equations for the rotating blade can be written as (Figure 3)

\[ E A u'' + \rho \Omega^2 A u = \rho A \ddot{u} \]

\[ \kappa G A (v' - \beta')' + \rho \Omega^2 A v + [T(x) v']' = \rho A \ddot{v} \]

\[ (w' + \alpha')' + [T(x) w']' = \rho A \ddot{w} \]

\[ EI_y \alpha'' - \kappa G A (\alpha + w') + \rho \Omega^2 l_y \alpha = \rho l_y \ddot{\alpha} \]
EI \ddot{I}_2 \beta - \kappa GA \beta + \rho \Omega^2 I_2 \beta = \rho I_2 \beta \tag{1}

\kappa G J \theta + \rho \Omega^2 I_y \theta = \rho J \theta

with the following boundary conditions:

at the fixed end \( x = 0 \)

\( u(x, t) = v(x, t) = w(x, t) = \alpha(x, t) = \beta(x, t) = \theta(x, t) = 0, \)

at the free end \( x = L \)

\( u'(L, t) - \beta(L, t) = v'(L, t) - \alpha(L, t) = \alpha'(L, t) = \beta'(L, t) = \theta'(L, t) = 0 \)

where \( u, v, \) \( w \) are the displacements and \( \alpha, \beta, \theta \) are the rotation angles of a point on the neutral axis in the \( x, y \) and \( z \) direction respectively. The " \( ' \) " denotes the partial differentiation with respect to \( x \) and the over dot denotes time derivative. \( \Omega \) is the rotating speed, \( \rho \) is the density, \( E \) is the Young's modulus, \( A \) is the cross section area, \( \kappa \) is the shear correction coefficient, \( G \) is the shear modulus and \( T(x) \) is the centrifugal force factor. Detailed derivation of the model can be found in (Fan 1992, Chen & Chen 1993). Since closed form solution for Equation (1) is not exist so the Finite Element Method (FEM) is employed to obtain an approximate solution. The Galerkin method is employed to develop the discrete finite element model of the system. Beam elements are used and the discrete model can be written as

\[ M \ddot{q}(t) + K(\Omega)q(t) = 0 \tag{2} \]

where \( q \) is the vector that consists of all the nodal variables of the system. At a constant rotating speed \( \Omega \)

let

\[ q = A e^{i \omega t} \tag{3} \]

and substitute Equation (3) into Equation (2) yields

\[ -\omega^2 M^\top A + K(\Omega)A = 0 \tag{4} \]

where \( \omega \) is the natural frequency of the blade rotating at speed \( \Omega \).

RESULTS

Static case

The natural frequencies of the blade obtained from the impulsive testing are 28 Hz, 180 Hz and 508 Hz (Figure 4a). The FRFs measured with the two connectors show significant difference in both the number of modes and the natural frequencies (Figure 4b). By using the U-shaped connector, a set of natural frequencies which is close to that of the impulsive testing are obtained (30.5 Hz, 190.5 Hz and 527 Hz). A swept-sine testing reveals that the resonant peak at 60 Hz is a noise from the electronic devices. The dynamic coupling between the shaker and the blade is strong when the straight stinger is employed. Based on this finding, the U-shaped connector is adopted in all the dynamic cases. The FRFs measured with the two strain gages show that the natural frequencies are: 30.5 Hz for the first mode, 190.5 Hz for the second mode and 527 Hz for the third mode (Figure 5). Swept-sine tests around these frequencies yield resonant frequencies that agree well with the analytical and FEM results (Table 1). The relative errors are all below 1.5%.

Dynamic cases

For zero stagger angle, the natural frequencies of the rotating blade increase with the rotating speed. Detailed shifts of the first three natural frequencies are shown in Figures 6, 7 and 8. The first mode frequency is shifted by 1 Hz for the rotating speed of 200 rpm, by 2.5 Hz for 400 rpm, by 4 Hz for 600 rpm and by 6 Hz for 800 rpm. The second mode frequency is shifted by 2 Hz for the rotating speed of 200 rpm, by 2.5 Hz for 400 rpm, by 4.5 Hz for 600 rpm and by 7 Hz for 800 rpm. Similar trend can be observed for the third mode. Effects of stagger angle

The measured and calculated natural frequencies of the first two modes for different stagger angles are compared. For zero stagger angle, the experimental results for the first mode follow the FEM results well (Figure 9). For a stagger angle of 30 degrees most of the first mode natural frequencies except at 700 rpm are higher than the FEM results (Figure 10). For a higher stagger angle (70 degrees) the deviations between the experimental and FEM results get significant (Figure 11). The absolute errors are less than 0.6 Hz. The increase of natural frequency is a nonlinear function of the rotating speed. Similar trend can be observed for the second mode (Figures 12, 13 & 14). The absolute errors between the measured and the calculated second mode frequencies are larger than that of the first mode. The largest absolute errors are 1.5 Hz, 1.5 Hz and 1.8 Hz for stagger angles of 0, 30 and 70 degrees respectively. The relative errors are less than 1%. For the stagger angle of 70 degrees most of the FEM results are under estimated.

DISCUSSION AND CONCLUSION

In the static test, the blade natural frequencies obtained from the impulsive testing are smaller than those obtained from the random excitations by means of the strain gage. This is due to the mass of the miniature accelerometer is comparable to that of the blade. The increase of mass reduces the natural frequencies. However, it indicates there are only three modes within the bandwidth and the dynamic coupling effect of the straight stinger is higher than the U-shaped connector. The good agreements of the measured natural frequencies and the analytical results by Euler beam theory and the FEM reveal that very accurate natural frequencies of the blade are obtained by current setup. The U-shaped connector design can reduce the dynamic coupling between the shaker and the blade. Although the piezo-electric shaker has a load cell at the moving head but due to the lack of a slip ring channel only the exciting signal instead of the exciting force is available. By employing the U-shaped connector the coupling
effect is reduced and reliable FRFs of the rotating blade can be obtained. The gage numbered 2 is located near the nodal point of the third mode so no peak appears around 527 Hz.

In the dynamic tests, the increase of natural frequency for all the modes is a nonlinear function of the rotating speed. The increment of natural frequency is due to the fact that the centrifugal force field is proportional to the square of the rotating speed. Thus, the higher the rotating speed is the larger the frequency increment. Although the rotor has a balance weight, due to the facility limitation in this project it is not fully balanced. The forced vibration of the rotor does introduce forced vibration of the blade. In the band of 0-30 Hz peaks corresponding to the harmonics of the rotating speed can be observed. This reveals the importance of dynamically balancing the rotor bearing system especially for turbomachines with higher capacity. An unbalanced rotor may cause an excessive blade vibration. The attainable speed is limited by the stagger angle. For high stagger angle the highest rotating speed is 600 rpm. It is the reactive force from the blade air gives a limit on the rotating speed. The causes of the error between the experimental and the FEM results may be that in the FEM modeling the aerodynamic force is neglected.

However, in the real experiments the rotating blade is acted by the air resistance. For more accurate measurements, a vacuum chamber must be used to get rid of the aerodynamic effect especially when much higher speed such as four thousands rpm is required. For rotating speed below 600 rpm the experiment results are in good agreement with the FEM predictions.

In summary, the natural frequencies of a rotating blade have been measured and the results are in good agreement with the FEM results. The U-shaped connector between the shaker and the blade has less dynamic coupling than the straight stinger. The increments of the natural frequencies vary nonlinear to the rotating speeds. The effects of the stagger angle on the measured natural frequencies are not clear in our rotating speed range.

ACKNOWLEDGEMENT

This research was partially supported by the Taiwan Power Company. The authors are grateful to Mr. H.K. Chen for helping us in the finite element analysis.

REFERENCES


Mahrenholtz, O., 1984, Dynamic Aspect of Rotors: Stability and System Identification, CISM, Udine.


FIGURE 3 GEOMETRIC MODEL OF A ROTATING BLADE

FIGURE 2 (A) LOCATIONS OF STRAIN GAGES AND ROOT SUPPORT

FIGURE 2 (B) DETAILS OF TWO CONNECTOR DESIGNS

FIGURE 4 (A) BLADE FRF MEASURED BY USING AN IMPULSIVE TEST WITH AN ACCELEROMETER

FIGURE 4 (B) COMPARISON OF DYNAMIC COUPLING EFFECTS OF THE TWO CONNECTORS

FIGURE 5 FRF OF THE BLADE UNDER STATIC CASE
FIGURE 6 SHIFTING OF FIRST MODE NATURAL FREQUENCY AT DIFFERENT ROTATING SPEEDS

FIGURE 7 SHIFTING OF SECOND MODE NATURAL FREQUENCY AT DIFFERENT ROTATING SPEEDS

FIGURE 8 SHIFTING OF THIRD MODE NATURAL FREQUENCY AT DIFFERENT ROTATING SPEEDS

FIGURE 9 COMPARISON OF FEM AND MEASURED FIRST MODE NATURAL FREQUENCIES (φ = 0°)

FIGURE 10 COMPARISON OF FEM AND MEASURED FIRST MODE NATURAL FREQUENCIES (φ = 30°)
FIGURE 11 COMPARISON OF FEM AND MEASURED FIRST MODE NATURAL FREQUENCIES ($\phi$ = 70°)

FIGURE 12 COMPARISON OF FEM AND MEASURED SECOND MODE NATURAL FREQUENCIES ($\phi$ = 0°)

FIGURE 13 COMPARISON OF FEM AND MEASURED SECOND MODE NATURAL FREQUENCIES ($\phi$ = 30°)

FIGURE 14 COMPARISON OF FEM AND MEASURED SECOND MODE NATURAL FREQUENCIES ($\phi$ = 70°)

Table 1: Comparison of natural frequencies (Hz) for the static case

<table>
<thead>
<tr>
<th></th>
<th>first mode (error)</th>
<th>second mode (error)</th>
<th>third mode (error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>30.71</td>
<td>190.10</td>
<td>527.16</td>
</tr>
<tr>
<td>Euler beam theory*</td>
<td>30.41 (0.98%)</td>
<td>190.6 (0.26%)</td>
<td>533.7 (1.24%)</td>
</tr>
<tr>
<td>Euler beam theory+</td>
<td>30.48 (0.75%)</td>
<td>191.02 (0.48%)</td>
<td>534.92 (1.47%)</td>
</tr>
<tr>
<td>Finite element method (Timoshenko beam)</td>
<td>30.40 (1.01%)</td>
<td>190.50 (0.21%)</td>
<td>533.40 (1.19%)</td>
</tr>
</tbody>
</table>

* 2014 - T6 aluminum alloy + 1100 - T14 aluminum alloy