PASSIVE CONTROL OF FLOW INDUCED VIBRATIONS
BY SPLITTER BLADES

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ABSTRACT

Splitter blades as a passive control technique for flow induced vibrations is investigated by developing an unsteady aerodynamic model to predict the effect of incorporating splitter blades into the design of an axial flow blade row operating in an incompressible flow field. The splitter blades, positioned circumferentially in the flow passage between two principal blades, introduce aerodynamic and/or combined aerodynamic-structural detuning into the rotor. The unsteady aerodynamic gust response and resulting oscillating cascade unsteady aerodynamics, including steady loading effects, are determined by developing a complete first-order unsteady aerodynamic analysis together with an unsteady aerodynamic influence coefficient technique. The torsion mode flow induced vibrational response of both uniformly spaced tuned rotors and detuned rotors are then predicted by incorporating the unsteady aerodynamic influence coefficients into a single-degree-of-freedom aeroelastic model. This model is then utilized to demonstrate that incorporating splitters into axial flow rotor designs is beneficial with regard to flow induced vibrations.

NOMENCLATURE

\( \bar{C}_p \) steady surface static pressure, \( \frac{P_{1}}{2P_{2}}U^2 \)

\( I_{CG} \) gust influence coefficient of airfoil \( n \)

\( I_{CM} \) motion-induced influence coefficient of airfoil \( n \)

\( k \) reduced frequency, \( \omega_b/U_0 \)

\( k_2 \) transverse gust wave number

\( M_{GR} \) gust unsteady aerodynamic moment

\( M_{d} \) motion-induced unsteady aerodynamic moment

\( P \) steady pressure

\( P' \) unsteady pressure

\( Q \) complete flow field

\( \bar{Q} \) steady mean flow

\( U_c \) far field uniform mean flow

\( U_0 \) steady airfoil surface chordwise velocity

\( V_0 \) steady airfoil surface normal velocity

\( u^+ \) streamwise gust component

\( v^+ \) transverse gust component

\( w^+ \) complex gust amplitude

\( \varphi \) interblade phase angle

\( \varphi_d \) detuned interblade phase angle

\( \gamma \) gust direction angle

\( \Phi \) general velocity potential

\( \Phi_0 \) steady velocity potential

\( \Phi' \) unsteady velocity potential

\( \varphi'_{c} \) circulatory unsteady velocity potential

\( \Phi'_{NC} \) noncirculatory unsteady velocity potential

\( \Gamma \) steady circulation constant

\( \Gamma' \) unsteady circulation constant

\( \varepsilon \) level of aerodynamic detuning

\( \Omega_0 \) natural torsional frequency

\( \omega_0 \) reference frequency

INTRODUCTION

Axial flow blade rows which can achieve higher pressure ratios without flow separation are required for the development of advanced gas turbine engines which are more compact, lighter weight, and have increased reliability as compared to current technology engines. Thus these engines will feature high solidity, low aspect ratio blade rows, with the number of airfoils and/or the airfoil chord increased. Unfortunately, these adversely affect aerodynamic efficiency, with the additional airfoils and end wall surface areas resulting in increased losses and the increased blockage reducing the flow capacity. In addition, the effect of flow induced vibrations of these advanced blade row designs must be considered.

One approach to minimizing these performance penalties is the use of partial chord blades, termed splitters, between principal full chord blades. In particular, the splitters help to control trailing edge flow separation without the aerodynamic penalties associated with high solidity full chord blade rows, i.e., the splitters enable the aft rotor blade row to have increased flow turning, thereby increasing...
the work transfer. In addition, the splitters do not affect the blade row flow capacity as it is controlled by the entrance region, with the splitters not affecting the entrance region solidity.

Splitter vanes are routinely incorporated into high pressure ratio centrifugal compressor impellers. However, the incorporation of splitters into the design of axial flow rotors is still a research concept. Wennergren (1974, 1975) utilized splitters in the design of a 3:1 single stage axial flow rotor. The camberline of the splitters was identical to that of the principal blades, with the full span splitters positioned circumferentially at mid passage, although other positions were later considered in cascade experiments, Hoffman, McClure and Sinert (1973) and Seifert and Riffel (1977). Although the lack of an advanced design system for splittered rotors resulted in a rotor that did not meet its efficiency and surge margin goals, the splitters did control the deviation levels, resulting in a significant improvement in flow and pressure rise capacity over a baseline conventional design. To overcome this design system deficiency, Tzou et al. (1990) are developing a design methodology for axial compressor rotors which incorporate splitters. Thus, the incorporation of splitters into advanced axial flow blade row designs may enable the performance goals of advanced gas turbine engines to be achieved.

In addition to performance goals, advanced engines must also have improved reliability. Thus, it is necessary to address the effect of incorporating splitters on flow induced vibrations. In this regard, the incorporation of splitters may not only be beneficial with regard to aerodynamic performance but also serve as a passive control technique for flow induced vibrations, i.e., from an aeromechanical point of view, the splitters introduce both structural and aerodynamic detuning into the rotor. This is particularly important for advanced blade rows which feature very low mechanical damping.

Structural detuning, defined as designed blade-to-blade differences in the natural frequencies of a blade row, has been proposed for passive aerelastic control. Studies of the effect of structural detuning of rotors have shown that it is often detrimental to flow induced vibrations, although beneficial to flutter.

Aerodynamic detuning is a relatively new concept for passive aerelastic control. It is defined as designed blade-to-blade differences in the unsteady aerodynamic flow field of a blade row. Thus, aerodynamic detuning creates blade-to-blade differences in the unsteady aerodynamic forces and moments, thereby affecting the fundamental driving force. This results in the blades not responding in a classical traveling wave mode typical of a conventional uniformly spaced aerodynamically tuned rotor. Studies of rotors operating in both incompressible flow fields, Chiang and Fleeter (1989) and supersonic flow fields with subsonic axial components, Hoyniak and Fleeter (1986) and Fleeter and Hoyniak (1987), have shown that aerodynamic detuning is beneficial to flutter stability and may be either beneficial or detrimental to flow induced vibrations, depending on the particular operating condition and geometric configuration.

Splitter blades introduce combined aerodynamic-structural detuning into a rotor. The aerodynamic detuning results from the differences in the principal blade passage unsteady aerodynamics due to the splitters, with the structural detuning achieved through the higher natural frequencies of the splitter blades as compared to the full chord principal rotor blades. Hence, the incorporation of splitters into a rotor may not only result in improved aerodynamic performance, but also in decreased susceptibility to flutter and flow induced vibration problems. This has been shown for rotors operating in supersonic flow fields with subsonic axial components, with the airfoils modeled as flat plates, Topp and Fleeter (1986), Fleeter, Hoyniak and Topp (1988).

Thus, the splitted rotor concept for increased performance also offers the potential of being beneficial with regard to blade row flow induced vibrations. This may have an impact on performance design systems for splitters. Namely, splitters may be able to be designed for safe operation in regions of the performance map wherein an unsplitted rotor would encounter severe flow induced vibration problems, with blade row aerodynamic performance in these parts of the performance map previously unattainable.

In this paper, the effect on flow induced vibrations of incorporating splitter blades into a subsonic rotor design, including airfoil profile effects, is considered. This involves the investigation of the viability of splitters as a passive torsion mode flow induced vibration control technique of an aerodynamically loaded rotor operating in an incompressible flow field. Thus, this research significantly extends the previous modeling and understanding of the fluid dynamics and aeroelasticity of splittered rotors. This is accomplished by developing a complete first order unsteady aerodynamic model, i.e., the thin airfoil approximation is not utilized, to analyze the unsteady aerodynamic gust response and the oscillating airfoil motion-induced aerodynamics of both conventional uniformly spaced rotors without splitters and rotors incorporating variably spaced splitters between principal blades, including the effects of steady aerodynamic loading.

The analysis of the torsion mode flow induced vibration characteristics of a rotor requires the prediction of the unsteady pressure resulting from an aerodynamic gust and the resulting harmonic torsional motion of the cascade. The steady and unsteady aerodynamics acting on the typical two-dimensional airfoil sections of a blade row are determined by considering: (1) a single principal blade passage with periodic boundary conditions for the conventional tuned uniformly spaced rotor, and (2) two principal blade passages with two passage periodic boundary conditions for the aerodynamically detuned rotor. The flow field is assumed to be linearly comprised of a steady potential mean flow and an harmonic unsteady flow field. The steady and unsteady potential flow fields are individually described by Laplace equations, with both the steady and unsteady potentials further decomposed into circulatory and noncirculatory components. The steady flow field is independent of the unsteady flow. However, the unsteady flow is coupled to the steady flow field through the unsteady boundary conditions on the airfoil surfaces.

A locally analytical solution is developed in which the discrete algebraic equations which represent the flow field equations are obtained from analytical solution in individual grid elements. A body fitted computational grid is utilized. General analytical solution to the transformed Laplace equations are developed by applying these solutions to individual grid elements, with the complete flow field then obtained by assembling these locally analytical solutions.

Mathematical Model - Tuned Cascade

Figure 1 presents a schematic representation of a thick, cambered airfoil cascade at finite mean flow incidence, $\frac{Q}{M}$, to the farfield uniform mean flow, $U_{\infty} = \theta \lambda$, executing torsion mode oscillations with a superimposed convected two-dimensional harmonic gust. The cascade has a stager angle of $\delta$, with $S$ the distance between the airfoils along the stager line and $\theta$ the inlet blade angle. The stagger angle is defined as the angle between the leading edge locus line and the line which is perpendicular to the airfoil chord. The inlet blade angle is defined as the angle between the line tangent to the camberline and the line which is perpendicular to the leading edge locus line. The gust amplitude and harmonic frequency are denoted by $A$ and $\omega$, with the interblade phase angle $\beta$, specified by the ratio of the number of gusts to the number of airfoils in the rotor blade row. The harmonic two-dimensional gust with transverse and streamwise components $u^t$ and $u^s$ propagates in the direction $K = k^t + k^s$ where $k^t = \omega \theta U_{\infty}$ is the reduced frequency and $k^s$ is the transverse gust wave number, i.e., the transverse component of the gust propagation direction vector, with the gust direction angle $\psi$ defined as $\tan \psi = \tan^{-1} \left( \frac{u^s}{u^t} \right)$.

The complete flow field is assumed to be comprised of a steady mean flow and a harmonic unsteady flow field, Equation 1. The unsteady flow field corresponds to either the gust unsteady flow field $\vec{Q}_G$ or the motion-induced unsteady flow field $\vec{Q}_M$. 

\[
\begin{align*}
\vec{Q}_G & = \vec{Q}_0 + \vec{Q}_G \\
\vec{Q}_M & = \vec{Q}_0 + \vec{Q}_M
\end{align*}
\]
\[ \tilde{Q}(x,y,t) = \tilde{Q}_u(x,y) + \tilde{Q}_v(x,y) \exp[ik_1t] \]  

**Steady Flow Field**

A velocity potential function can be defined for the steady flow of an incompressible inviscid fluid. The complete flow field is then described by the Laplace equation.

\[ \nabla^2 \Phi_D(x,y) = 0 \]  
where \( \tilde{Q}_n(x,y) = \nabla \Phi_D(x,y) \)

Since the Laplace equation is linear, the velocity potential can be decomposed into noncirculatory and circulatory components \( \Phi_{NC}(x,y) \) and \( \Phi_{C}(x,y) \).

\[ \Phi_D(x,y) = \Phi_{NC}(x,y) + \Phi_{C}(x,y) \]  
where \( \nabla^2 \Phi_{NC} = 0 \) and \( \nabla^2 \Phi_{C} = 0 \).

To complete the steady flow mathematical model, farfield inlet, farfield exit, airfoil surface, wake dividing streamline and cascade periodic boundary conditions must be specified. The steady farfield inlet flow is uniform, with the mass flow rate specified by the farfield exit boundary conditions. Also, a zero normal velocity is specified on the airfoil surfaces.

The steady velocity potential is discontinuous along the airfoil wake dividing streamline. This discontinuity is satisfied by a continuous noncirculatory velocity potential, with the discontinuity in the circulatory velocity potential equal to the steady circulation, \( \Gamma \). The Kutta condition is also applied, thereby enabling the steady circulation constant to be determined. It is satisfied by requiring the chordwise velocity components on the upper and lower airfoil surfaces to be equal in magnitude at the airfoil trailing edge. In addition, the cascade periodic steady velocity potential boundary conditions are satisfied by requiring both the normal and chordwise velocity components to be continuous between the upper and lower periodic boundaries. Refer to Chiang and Fleeter (1988) for a complete description of the steady flow boundary conditions.

**Unsteady Gust Aerodynamics**

The two-dimensional gust unsteady flow field \( \tilde{Q}_G \) is determined by decomposing the gust generated unsteady flow field into harmonic rotational \( \tilde{Q}_R \) and potential \( \tilde{Q}_P \) components. The rotational gust component is specified consistent with linearized unsteady Euler equations, determined by linearizing the unsteady flow about the steady flow field. The gust is assumed to be convected with the steady mean flow and therefore does not interact with the airfoil cascade. Thus the following solution for the rotational gust is determined by solving the linearized Euler equations in the far upstream where the steady flow field is uniform.

\[ \tilde{Q}_R = u^\tau + v^\tau \]  

where \( u^\tau = -A_{k_2} \exp[ik_1(t-x) - ik_2y] \) and \( v^\tau = A_{k_1} \exp[ik_1(t-x) - ik_2y] \).

It should be noted that in this gust solution, the components \( u^\tau \) and \( v^\tau \) are coupled with the ratio of their amplitudes being \( u^\tau/v^\tau = -k_2/k_1 \). Also, the solution corresponds exactly to the Sears transverse gust, Sears (1941) when \( k_2 = 0 \). However, this gust solution differs from that used in the Horlock (1968) and Naumann and Yeh (1972) models in which the two components are uncoupled, \( u^\tau = \tilde{u}^\tau \exp[ik_1(t-x)] \) and \( v^\tau = \tilde{v}^\tau \exp[ik_1(t-x)], \) where \( \tilde{u}^\tau \) and \( \tilde{v}^\tau \) denote the individual amplitudes of the two independent gust components and the gust and resulting unsteady aerodynamics are independent of the transverse component of the gust propagation direction vector \( K = k_1 + ik_2 \).

The potential gust component \( \Phi_G \) is described by a Laplace equation. The solution is determined by decomposing this potential gust component into circulatory and noncirculatory components \( \Phi_{GC}(x,y) \) and \( \Phi_{GNC}(x,y) \), each of which is individually described by a Laplace equation.

\[ \Phi_G = \frac{\partial\Phi_{GNC}}{\partial x} + i \frac{\partial\Phi_{GC}}{\partial y} \]

\[ \Phi_{GC} = \Phi_{GC\text{farfield inlet}} - \frac{\Delta \Phi^*}{\beta_0} \frac{\partial \Phi_{GNC}}{\partial n} \text{farfield inlet} \]

\[ \Phi_{GNC\text{farfield exit}} = \frac{\Delta \Phi^*}{\beta_0} \frac{\partial \Phi_{GNC}}{\partial n} \text{farfield exit} \]

\[ \Phi_{G^*} = \frac{\partial \Phi_{G^*}}{\partial x} - i k_1 \frac{\partial \Phi_{G^*}}{\partial y} \]

where \( \beta_0 = k_1 \sin \delta \) and \( \Delta \Phi^* \) is the unsteady velocity potential discontinuity at farfield exit.

The airfoil surface boundary conditions specify that the normal velocity of the flow field must be equal to that of the airfoil. The gust generated unsteady rotational and potential flow fields are coupled through the boundary conditions on the noncirculatory gust component. In particular, the airfoil cascade is stationary, with the rotational gust convected with the mean steady flow field. Thus the upwash on the airfoil is determined by requiring the normal component of the unsteady flow field to be zero on the airfoil.
The unsteady aerodynamic moment on a reference airfoil for the gust unsteady pressure, the steady and unsteady gust velocity potentials, the unsteady Bernoulli equation, and the unsteady rotational gust pressure lift and moment are calculated. It is determined from the solution for unsteady surface pressure difference across the airfoil chord.

The unsteady gust velocity potential is discontinuous along the airfoil wake dividing streamline. This discontinuity is satisfied with a continuous noncirculatory velocity potential and a discontinuous circulatory velocity potential. The unsteady circulatory velocity potential discontinuity is specified by requiring the pressure to be continuous across the wake and then utilizing the unsteady Bernoulli equation to relate the unsteady velocity potential and the pressure. Also specified is the continuity of the noncirculatory velocity potential along the wake streamline. The Kutta condition is applied to the unsteady gust flow field thereby enabling the unsteady circulation constant \( \Gamma_G \) to be determined. It is satisfied by requiring no unsteady pressure difference across the airfoil chord at the trailing edge.

\[
\Delta \Phi_G^{\text{wakw}} = \Gamma_G \exp[-i k_1 (x - 1)]
\]  
\[
\Delta \Phi_G^{\text{NG}} = 0
\]

The unsteady dependent variable of primary interest is the unsteady pressure \( P_G \) from which the airfoil unsteady aerodynamic lift and moment are calculated. It is determined from the solution for the steady and unsteady gust velocity potentials, the unsteady Bernoulli equation, and the unsteady rotational gust pressure \( P_R \). The unsteady aerodynamic moment on a reference airfoil for the gust unsteady model \( M_{GR} \) is calculated by integrating the gust unsteady surface pressure difference across the airfoil chord.

\[
M_{GR} = \int [P_G(x,y) - P_G(y)] dy
\]

**Motion-Induced Unsteady Aerodynamics**

The unsteady flow field associated with the harmonic motion of the airfoil cascade \( q \) is assumed to be potential and is therefore described by a Laplace equation. The solution is again determined by decomposing this velocity potential into circulatory and noncirculatory components \( \Phi_M \) and \( \Phi_N \), each of which is individually described by a Laplace equation.

\[
\Phi_M = \Phi_M^{\text{NC}} + \Phi_M^{\text{MC}}
\]

\[
V^2 \Phi_M = 0; \quad V^2 \Phi_N = 0
\]

\[
\frac{\partial^2 \Phi}{\partial t^2} = -\gamma^2 \Phi
\]

The boundary conditions in the farfield inlet, farfield exit, wake dividing streamlines, and cascade periodic boundaries as well as the Kutta condition for the motion-induced circulatory and noncirculatory components are identical to those for the gust potential flow. The only boundary condition that changes is that requiring the normal flow velocity to be equal to the airfoil velocity on the airfoil surface, the upwash condition.

The upwash on the airfoil for the motion-dependent model \( w_M \) is a function of both the position of the airfoil and the steady flow field. Thus, this boundary condition couples the unsteady flow field to the steady aerodynamics. For an airfoil cascade executing harmonic torsion mode oscillations about an elastic axis location at \( x_0 \), the upwash on the airfoil is

\[
w_M(x,y) = \alpha \left[ \begin{array}{c} ik_1 \left( (x - x_0) + y \partial \Phi_0 \right) + U_0 + V_0 \partial \Phi_0 - \frac{1}{2} \left( 1 + \partial \partial \Phi_0 \right) \right] + \left( \begin{array}{c} i k_1 \left( (x - x_0) + y \partial \Phi_0 \right) + U_0 + V_0 \partial \Phi_0 - \frac{1}{2} \left( 1 + \partial \partial \Phi_0 \right) \right] 
\]

where \( U_0 \), \( V_0 \), and \( \Phi_0 \) are the steady airfoil surface velocity components, \( f(x) \) denotes the airfoil profile and \( \alpha \) is the amplitude of the torsional oscillations.

The unsteady pressure for the motion-dependent model \( \Phi_M \) is determined from the solution for the steady flow field, the unsteady pressure potential, and the unsteady Bernoulli equation. The unsteady aerodynamic moment on the reference airfoil is calculated by integrating the unsteady surface pressure difference across the chord.

\[
M_{MR} = \int [P_M(x,y) - P_M(y)] dy
\]

**Locally Analytical Solution**

A boundary fitted computation grid generation technique is utilized for the numerical solution. A Poisson type grid solver is used to fit a C-type grid around a reference airfoil in the cascade. This method permits grid points to be specified along the entire boundary of the computational plane.

Laplace equations describe the complete flow field including the unknown velocity potentials \( \Phi_M, \Phi_C, \Phi_N, \Phi_G, \) and \( \Phi_C \). In the transformed \((\xi, \eta)\) coordinate system, the Laplace equation takes on the following nonhomogeneous form.

\[
\frac{\partial^2 \Phi}{\partial \xi^2} + \frac{\partial^2 \Phi}{\partial \eta^2} + 2 \alpha \beta \frac{\partial \Phi}{\partial \eta} - 2 \gamma \frac{\partial \Phi}{\partial \eta} = F(\xi, \eta)
\]

where \( \Phi \) is a shorthand method of writing these four velocity potentials in the transformed plane, \( F(\xi, \eta) \) contains the cross derivative term \( \partial^2 \Phi \partial \eta \partial \eta \) and the coefficients \( \alpha, \beta, \) and \( \gamma \) are functions of the transformed coordinates \( \xi \) and \( \eta \) which are treated as constants in each individual grid element.

To obtain the analytical solution to the transformed Laplace equation, it is first rewritten as a homogeneous equation by defining a new dependent variable \( \Theta(\xi, \eta) \).

\[
\frac{\partial^2 \Theta}{\partial \xi^2} + \frac{\partial^2 \Theta}{\partial \eta^2} - (\gamma^2 + \alpha \beta) \Theta = 0
\]
The following general solution for \( \hat{\Phi} \) is determined by separation of variables.

\[
\hat{\Phi}(x, \eta) = [A_1 \cos(\lambda_1 x) + A_2 \sin(\lambda_2 x)][B_1 \cos(\eta \mu_1) + B_2 \sin(\eta \mu_2)]
\]

where \( \mu = \left[(\mu_1^2 + \mu_2^2 + \lambda^2)^{1/2}\right] \), and \( \lambda, A_1, A_2, B_1, \) and \( B_2 \) are constants to be determined from the boundary conditions.

Analytical solutions in individual computation grid elements are determined by applying proper boundary conditions to evaluate the unknown constants in this general velocity potential solution. The solution to the global problem is then determined through the applications of the global boundary conditions and the assembly of the locally analytical solutions. The locally analytical method for steady two-dimensional fluid flow and heat transfer problems was initially developed by Chen, Naseri-Neshat and Ho (1981). They have shown that the locally analytical method has several advantages over the finite difference and finite element methods.

**Aerodynamically Detuned Cascade**

For the aerodynamically detuned rotor configuration of interest herein, i.e., variable circumferential spacing and chord length, an analogous cascade unsteady aerodynamic model is developed by considering two passages with two passage periodic boundary conditions.

In this model, the rotor will incorporate splitters (short chord airfoils) between each pair of full chord airfoils. As schematically depicted in Figure 2, the splitters are not required to have the same airfoil shape as the full chord airfoils, nor are they restricted to particular circumferential or axial positions between each pair of the full chord airfoils.

There are two distinct passages: (1) a reduced spacing, or increased solidity, passage; and (2) an increased spacing, or reduced solidity, passage. There are also two distinct sets of airfoils, with the two reference airfoils denoted by \( R_0 \) and \( R_1 \). These individual sets of airfoils can be considered as cascades of uniformly spaced airfoils each with twice the spacing of the associated baseline aerodynamically tuned uniformly spaced cascade. The circumferential spacing between these two sets of airfoils, \( S_1 \) and \( S_2 \), is determined by specifying the level of aerodynamic detuning, \( \varepsilon \).

\[
S_{1,2} = (1 \pm \varepsilon) S
\]

where \( S \) is the spacing of the baseline uniformly spaced cascade, and \( S_1 \) and \( S_2 \) denote the spacings of the detuned cascade.

An interblade phase angle for this aerodynamically detuned cascade configuration can be defined. In particular, each set of airfoils is individually assumed to be executing harmonic torsional oscillations with a constant aerodynamically detuned interblade phase angle, \( \beta_0 \), between adjacent airfoils of each set, Figure 3. Thus, this detuned cascade interblade phase angle is two times that for the corresponding baseline tuned cascade.

\[
\beta_0 = 2\beta_0
\]

where \( \beta_0 \) is the tuned baseline cascade interblade phase angle, defined between adjacent airfoils on the rotor.

For a rotor with \( N \) uniformly spaced blades, Lane (1956) showed that the values of \( \beta_0 \) must satisfy the following condition.

**Steady Aerodynamics**

The steady aerodynamic model developed for the baseline uniformly spaced cascade can be applied directly to the two reference passage model of the detuned cascade with the addition that the steady potential for the detuned cascade \( \Phi_{det}(x,y) \) is decomposed into one noncirculatory \( \Phi_{det}(x,y) \) and two circulatory components, one associated with each of the two reference airfoils, \( \Phi_{CR1}(x,y) \) and \( \Phi_{CR2}(x,y) \).

\[
\Phi_{det}(x,y) = \Phi_{det}(x,y) + \Phi_{CR1}(x,y) + \Phi_{CR2}(x,y)
\]

where \( \nabla^2 \Phi_{det} = 0; \nabla^2 \Phi_{CR1} = 0; \) and \( \nabla^2 \Phi_{CR2} = 0 \).

The circulatory component \( \Phi_{CR1}(x,y) \) is discontinuous along the wake dividing streamline of the reference airfoil \( R_0 \), while \( \Phi_{CR2}(x,y) \) is continuous along the wake of \( R_1 \) and \( R_0 \). The steady potential discontinuity is satisfied by a continuous noncirculatory velocity potential, with the discontinuities in the circulatory components \( \Phi_{CR1} \) and \( \Phi_{CR2} \), equal to the steady circulation, \( \Gamma_{R0} \) and \( \Gamma_{R1} \), respectively. Also, the steady circulation constants \( \Gamma_{R0} \) and \( \Gamma_{R1} \) are determined by simultaneously applying the Kutta condition to the two reference airfoils.

**Unsteady Aerodynamics**

The unsteady potential for the detuned cascade \( \Phi' \) is also decomposed into one noncirculatory \( \Phi_{det}(x,y) \) and two circulatory components, one associated with each of the two reference airfoils \( \Phi_{CR1}(x,y) \) and \( \Phi_{CR2}(x,y) \).

\[
\Phi'(x,y) = \Phi_{det}(x,y) + \Phi_{CR1}(x,y) + \Phi_{CR2}(x,y)
\]

where \( \Phi_{CR1}(x,y) \), \( \Phi_{CR2}(x,y) \), and the unsteady circulation constants \( \Gamma_{R0} \) and \( \Gamma_{R1} \), are defined analogous to the corresponding detuned cascade steady aerodynamic quantities.

**Influence Coefficient Technique**

The unsteady airfoil surface boundary conditions require that the cascaded airfoils oscillate with equal amplitudes. Also, the interblade phase angle between adjacent nonuniformly spaced airfoils must be specified. Neither of these requirements is appropriate for the aerodynamically detuned cascade. To overcome these limitations, an unsteady aerodynamic influence coefficient technique is utilized.

The unsteady aerodynamic moment acting on the two reference airfoils is expressed in terms of influence coefficients.

\[
M_{R0,R1} = W_R [CM]_{R0,R1} + W_R [CM]_{R0,R1} + \alpha_R [CM]_{R0,R1}
\]
where $\alpha_R$ and $\beta_R$ are the unknown complex oscillatory displacements for the reference airfoils $R_0$ and $R_1$, respectively and $W_{R_k}$, $R_k$ are the complex gust amplitudes which are related by the detuned interblade phase angle $\beta_d$ and the level of aerodynamic detuning $\epsilon$.

$$W_{R_k} = W_k \exp[-i(1-\epsilon)\beta_d/2]$$  \hspace{1cm} (19)

The influence coefficients $[CG]_{R_kR_k}$, $[CG]_{R_kR_1}$ and $[CM]_{R_kR_k}$ and $[CM]_{R_kR_1}$ are the unsteady aerodynamic moments acting on the two reference airfoils. They are determined by analyzing the two reference passages with the unsteady cascade model developed herein. $[CG]_{R_kR_k}$ and $[CM]_{R_kR_k}$ are determined by considering a unit amplitude gust acting only on the reference airfoil $R_k$ and unit amplitude motion of only the reference airfoil $R_0$ respectively, and reference airfoil $R_1$ with no gust and stationary. Note that the gust is modeled only through the airfoil surface boundary conditions. Thus, the gust acts either on one or both airfoils. The influence coefficients $[CG]_{R_kR_k}$ and $[CM]_{R_kR_1}$ are obtained in an analogous manner.

**FORCED RESPONSE MODEL**

The equations describing the single-degree-of-freedom torsional motion of the two reference airfoils of the aerodynamically detuned cascade are developed by considering the typical airfoils depicted in Figure 4. The elastic restoring forces are modeled by linear torsional springs at the elastic axis location, with the inertial properties of the airfoils represented by their mass moments of inertia about the elastic axis. The equations of motion, determined by Lagrange's technique, are

$$L_{R_0} \alpha_{R_0} + (1 + 2 i \rho_k) L_{R_1} \alpha_{R_1} \omega_{R_0}^2 \alpha_{R_0} = M_{R_0}$$ \hspace{1cm} (20)

$$L_{R_1} \alpha_{R_1} + (1 + 2 i \rho_k) L_{R_1} \alpha_{R_1} \omega_{R_1}^2 \alpha_{R_1} = M_{R_1}$$

where $L_{R_0}$, $L_{R_1}$ are the stiffness coefficients about the elastic axis, $\omega_{R_0}$ and $\omega_{R_1}$ denote the structural damping coefficients for the reference airfoils, and the undamped natural frequencies are

$$\omega_{R_0}^2 = K_{R_0}/I_{R_0} \alpha_{R_0} \quad \text{and} \quad \omega_{R_1}^2 = K_{R_1}/I_{R_1} \alpha_{R_1}.$$  \hspace{1cm} (21)

Considering harmonic time dependence of the reference airfoils and utilizing the total unsteady aerodynamic moments and Equation 19, the equations of motion are written in matrix form.

$$\begin{bmatrix} \mu_0 & [CM]_{R_0}^0 \\ [CM]_{R_1}^0 & \mu_1 \end{bmatrix} \begin{bmatrix} \alpha_{R_0} \\ \alpha_{R_1} \end{bmatrix} = \begin{bmatrix} \omega_{R_0} \omega_{R_1} \\ \omega_{R_0} \omega_{R_1} \end{bmatrix}$$

$$-W_{R_0} \begin{bmatrix} [CG]_{R_0}^0 + 2(1-\epsilon)\beta_d/2 \omega_{R_0} \omega_{R_1} \\ [CG]_{R_1}^0 + 2(1-\epsilon)\beta_d/2 \omega_{R_0} \omega_{R_1} \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_{R_0} \\ \alpha_{R_1} \end{bmatrix}$$ \hspace{1cm} (22)

The effects of structural detuning are included through the frequency terms $\gamma R_{kR_k}$. These terms represent the ratios of the natural frequency to a specified reference frequency. Defining the reference frequency $\omega_0$ as the torsion mode natural frequency, a structurally tuned cascade would have $\omega_{R_k} = \omega_0$. For the case when the cascade is structurally detuned, the values of $\omega_{R_k}$ are altered.

To demonstrate the effects of the splitter generated aerodynamic and structural detuning on flow induced vibrational response, the single degree-of-freedom torsional aerelastic model is applied to a baseline twelve bladed rotor and to a rotor with alternate blades replaced with splitters.
Baseline Rotor Configurations

The baseline rotor is comprised of twelve uniform circumferentially spaced blades characterized by a Gostelow cascade geometry with a solidity of 0.83, a stagger angle of 40°, a mean flow incidence angle of 24°, and an airfoil mass ratio μ and radius of gyration τ of 193.776 and 0.3057 respectively, typical of modern fan blades. The reduced frequency is 1.67, resulting in a stable baseline rotor for all interblade phase angles.

The aerodynamic forcing function considered is a 45° two-dimensional gust characterized by interblade phase angles β0 of -30°, 0° and 180°, corresponding to detuned interblade phase angles β0 of 60°, 0° and 360°. These values were selected because they are the gust loads which were found to result in the highest amplitudes of response. In this study, the frequency of the gust ω is nondimensionalized by the undamped natural torsional frequency of the baseline airfoils ωn. The rotor response generated by this forcing function is presented in the format of the reference airfoil amplitude of response normalized by the maximum response of the baseline reference airfoil ωn. The amplitudes of response for the three gust interblade phase angles of the baseline reference airfoil as a function of the nondimensional gust frequency ω/ωn are shown in Figure 5.

Splitters-Rotor Configuration

Splitters are a convenient means of introducing aerodynamic and/or structural detuning into a rotor. Thus, the baseline twelve bladed rotor is detuned by replacing alternate airfoils with splitters with chords that are two-thirds that of the full chord airfoils, with two splitter designs considered.

Aerodynamic detuning is introduced into the rotor by means of splitters with the same thickness-to-chord ratio as the full chord airfoils, and thus the same natural frequencies. The aerodynamic detuning results from both the decreased chord of the splitters and the splitter circumferential position between adjacent full chord airfoils. Three circumferential splitter locations are considered: 40%, 50% and/or structural detuning into a rotor. Thus, the baseline twelve bladed rotor is detuned by replacing alternate airfoils with splitters with chords that are two-thirds that of the full chord airfoils, with two splitter designs considered.

Aerodynamic forcing function considered is a 45° two-dimensional gust characterized by interblade phase angles β0 of -30°, 0° and 180°, corresponding to detuned interblade phase angles β0 of 60°, 0° and 360°. These values were selected because they are the gust loads which were found to result in the highest amplitudes of response. In this study, the frequency of the gust ω is nondimensionalized by the undamped natural torsional frequency of the baseline airfoils ωn. The rotor response generated by this forcing function is presented in the format of the reference airfoil amplitude of response normalized by the maximum response of the baseline reference airfoil ωn. The amplitudes of response for the three gust interblade phase angles of the baseline reference airfoil as a function of the nondimensional gust frequency ω/ωn are shown in Figure 5.

For the rotor with splitters having the same thickness as the full chord airfoils, the effect of combined aerodynamic and structural detuning on forced responses is shown in Figures 10 and 21, with the effect of aerodynamic detuning presented in Figures 22 and 23. The combined detuning significantly decreases the response for both the backward and in-phase traveling wave gust modes of the full chord reference airfoil, but a slightly increased response amplitude in the response for the 180° gust mode. Also, this combined aerodynamic and structural detuning results in minimal response of the splitters in the range of frequencies near that of the full chord airfoil natural frequency, but an additional response near the splitter natural frequency. This higher frequency splitter response is not small, being somewhat larger for the forward wave mode and smaller for the other two modes than the full chord airfoil response amplitudes. When only the aerodynamic detuning generated by these thick splitters is considered, the response of the full chord airfoils in the backward traveling wave mode is decreased. However, this aerodynamic detuning alone has little effect on the full chord airfoil response for the other two gust modes. Also, with only aerodynamic detuning, the splitter response is decreased as compared to the case with the combined aerodynamic and structural detuning. It should be noted that in this case the splitter resonant response frequency is near to that of the baseline airfoils.

SUMMARY AND CONCLUSIONS

A mathematical model has been developed and utilized to predict the effect of incorporating splitter blades on the torsion mode forced response of a rotor operating in an incompressible flow field. The splitter blades, positioned circumferentially in the flow passage between two principal blades, introduce aerodynamic and/or combined aerodynamic-structural detuning into the rotor. The two-dimensional gust response and oscillating cascade unsteady aerodynamics, including steady loading effects were determined by developing a complete first-order unsteady aerodynamic analysis together with an unsteady aerodynamic influence coefficient technique. The torsion mode forced response of both uniformly spaced tuned rotors and detuned splattered rotors were then predicted by incorporating the unsteady aerodynamic influence coefficients into a single-degree-of-freedom aeroelastic model.
The viability of splitters as a passive torsion mode forced response control technique for an aerodynamically loaded rotor operating in an incompressible flow field was then considered, accomplished by applying this model to a baseline twelve bladed rotor. This study demonstrated that the aerodynamic detuning associated with the splitters was sometimes beneficial and other times detrimental with regard to forced response. However, the combined aerodynamic and structural detuning due to the splitters was generally beneficial for the full chord airfoils, with minimal splitter response in the frequency range near to that of the full chord airfoils. Thus, aerodynamic detuning and combined aerodynamic-structural detuning associated with the incorporation of splitters into a rotor are viable passive control mechanisms for flow induced response of rotors.

ACKNOWLEDGEMENTS

This research was sponsored, in part, by the NASA Lewis Research Center.

REFERENCES


Figure 1. Cascade and flow geometry

Figure 2. Aerodynamically detuned cascade with splitters
Figure 3. Reference airfoils and passages of detuned cascade

Figure 4. Single degree-of-freedom detuned cascade model

Figure 5. Baseline airfoil response for gust with $\beta_o = -30^\circ$, $180^\circ$ and $0^\circ$

Figure 6. Aerodynamically detuned 12 bladed splitters-rotor flow geometries and computational grids

Figure 7. Steady aerodynamic performance of baseline and splittersed rotors
Figure 8. Aerodynamic Detuning Effect on Airfoil R₀ response with splitters with the same T/C as baseline airfoils at 40% circumferential spacing.

Figure 10. Aerodynamic Detuning Effect on Airfoil R₀ response with splitters with the same T/C as baseline airfoils at 50% circumferential spacing.

Figure 9. Aerodynamic Detuning Effect on Airfoil R₁ response with splitters with the same T/C as baseline airfoils at 40% circumferential spacing.

Figure 11. Aerodynamic Detuning Effect on Airfoil R₁ response with splitters with the same T/C as baseline airfoils at 50% circumferential spacing.
Figure 12. Aerodynamic Detuning Effect on Airfoil R₀ response with splitters with the same T/C as baseline airfoils at 60% circumferential spacing

Figure 13. Aerodynamic Detuning Effect on Airfoil R₁ response with splitters with the same T/C as baseline airfoils at 60% circumferential spacing

Figure 14. Aerodynamic-Structural Detuning effect on Airfoil R₀ response with splitters with the same T/C as baseline airfoils at 40% circumferential spacing

Figure 15. Aerodynamic-Structural Detuning effect on Airfoil R₁ response with splitters with the same T/C as baseline airfoils at 40% circumferential spacing
Figure 16. Aerodynamic-Structural Detuning effect on Airfoil $R_0$ response with splitters with the same T/C as baseline airfoils at 50% circumferential spacing

Figure 18. Aerodynamic-Structural Detuning effect on Airfoil $R_0$ response with splitters with the same T/C as baseline airfoils at 60% circumferential spacing

Figure 17. Aerodynamic-Structural Detuning effect on Airfoil $R_1$ response with splitters with the same T/C as baseline airfoils at 50% circumferential spacing

Figure 19. Aerodynamic-Structural Detuning effect on Airfoil $R_1$ response with splitters with the same T/C as baseline airfoils at 60% circumferential spacing
Figure 20. Aerodynamic-Structural Detuning effect on Airfoil R₀ response with splitters with the same thickness as baseline airfoils at 50% circumferential spacing

Figure 21. Aerodynamic-Structural Detuning effect on Airfoil R₁ response with splitters with the same thickness as baseline airfoils at 50% circumferential spacing

Figure 22. Aerodynamic Detuning effect on Airfoil R₀ response with splitters with the same thickness as baseline airfoils at 50% circumferential spacing

Figure 23. Aerodynamic Detuning effect on Airfoil R₁ response with splitters with the same thickness as baseline airfoils at 50% circumferential spacing