PASSIVE CONTROL OF FLOW INDUCED VIBRATIONS
BY SPLITTER BLADES

Hsiao-Wei, D. Chiang,* and Sanford Fleeter
School of Mechanical Engineering
Purdue University
West Lafayette, Indiana

ABSTRACT

Splitter blades as a passive control technique for flow induced vibrations is investigated by developing an unsteady aerodynamic model to predict the effect of incorporating splitter blades into the design of an axial flow blade row operating in an incompressible flow field. The splitter blades, positioned circumferentially in the flow passage between two principal blades, introduce aerodynamic and/or combined aerodynamic-structural detuning into the rotor. The unsteady aerodynamic gust response and resulting oscillating cascade unsteady aerodynamics, including steady loading effects, are determined by developing a complete first-order unsteady aerodynamic analysis together with an unsteady aerodynamic influence coefficient technique. The torsion mode flow induced vibrational response of both uniformly spaced tuned rotors and detuned rotors are then predicted by incorporating the unsteady aerodynamic influence coefficients into a single-degree-of-freedom aeroelastic model. This model is then utilized to demonstrate that incorporating splitters into axial flow rotor designs is beneficial with regard to flow induced vibrations.

NOMENCLATURE

- $\bar{p}$ steady surface static pressure, $\frac{P_s}{2}\rho U^2$
- $[CGI]^n$ gust influence coefficient of airfoil $n$
- $[CMI]^n$ motion-induced influence coefficient of airfoil $n$
- $k_1$ reduced frequency, $\omega b U_0$
- $k_2$ transverse gust wave number
- $M_{GR}$ gust unsteady aerodynamic moment
- $M_{MIN}$ motion-induced unsteady aerodynamic moment
- $P$ steady pressure
- $P_u$ unsteady pressure
- $\bar{Q}$ complete flow field
- $Q_0$ steady mean flow
- $U_e$ far field uniform mean flow
- $U_0$ steady airflow surface chordwise velocity
- $V_0$ steady airflow surface normal velocity
- $\theta$ streamwise gust component
- $\psi$ transverse gust component
- $\xi$ complex gust amplitude
- $\gamma \phi$ interblade phase angle
- $\Gamma \phi$ detuned interblade phase angle
- $\phi$ gust direction angle
- $\Phi$ general velocity potential
- $\Phi_0$ steady velocity potential
- $\phi_u$ unsteady velocity potential
- $\phi_{NC}$ circulatory unsteady velocity potential
- $\phi_{NC}$ noncirculatory unsteady velocity potential
- $\Gamma$ steady circulation constant
- $\Gamma_u$ unsteady circulation constant
- $\epsilon$ level of aerodynamic detuning
- $\omega_0$ natural torsional frequency
- $\omega_n$ reference frequency

INTRODUCTION

Axial flow blade rows which can achieve higher pressure ratios without flow separation are required for the development of advanced gas turbine engines which are more compact, lighter weight, and have increased reliability as compared to current technology engines. Thus these engines will feature high solidity, low aspect ratio blade rows, with the number of airfoils and/or the airfoil chord increased. Unfortunately, these adversely affect aerodynamic efficiency, with the additional airfoils and end wall surface areas resulting in increased losses and the increased blockage reducing the flow capacity. In addition, the effect of flow induced vibrations of these advanced blade row designs must be considered.

One approach to minimizing these performance penalties is the use of partial chord blades, termed splitters, between principal full chord blades. In particular, the splitters help to control trailing edge flow separation without the aerodynamic penalties associated with high solidity full chord blade rows, i.e., the splitters enable the aft rotor blade row to have increased flow turning, thereby increasing...
the work transfer. In addition, the splitters do not affect the blade row flow capacity as it is controlled by the entrance region, with the splitters not affecting the entrance region solidity.

Splitter vanes are routinely incorporated into high pressure ratio centrifugal compressor impellers. However, the incorporation of splitters into the design of axial flow rotors is still a research concept. Wennerstrom (1974, 1975) utilized splitters in the study of a 3:1 single stage axial flow rotor. The camberline of the splitters was identical to that of the principal blades, with the full span splitters positioned circumferentially at midspan. Although some other positions were later considered in cascade experiments, the effect of incorporating splitters into the aerodynamics acting on the typical two-dimensional airfoil sections of a blade row is determined by considering: (1) a single principal blade passage with periodic boundary conditions for the conventional tuned uniformly spaced rotor; and (2) two principal blade passages with two passage periodic boundary conditions for the aerodynamically detuned rotor. The flow field is assumed to be linearly comprised of a steady potential mean flow and an unsteady shear flow field. The steady and unsteady potential flow fields are individually described by Laplace equations, with both the steady and unsteady potentials further decomposed into circulatory and noncirculatory components. The steady flow field is independent of the unsteady flow. However, the unsteady flow is coupled to the steady flow field through the unsteady boundary conditions on the airfoil surfaces.

A locally analytical solution is developed in which the discrete algebraic equations which represent the flow field equations are obtained from analytical solutions in individual grid elements, with the complete flow field then obtained by assembling these locally analytical solutions.

Mathematical Model - Tuned Cascade

Figure 1 presents a schematic representation of a thick, cambered airfoil cascade at finite mean flow incidence, \( \alpha_0 \), to the farfield uniform mean flow, \( \dot{U}_{\infty} \), by executing torsion mode oscillations with a superimposed convected two-dimensional harmonic gust. The cascade has a stagger angle of \( \delta \), with the distance between the airfoils along the stagger line and \( \theta \) the inlet blade angle. The stagger angle is defined as the angle between the leading edge locus line and the line which is perpendicular to the airfoil chord. The inlet blade angle is defined as the angle between the line tangent to the camberline and the line which is perpendicular to the leading edge line. The gust amplitude and harmonic frequency are denoted by \( A \) and \( \omega \), with the interblade phase angle \( \beta_k \) specified by the number of gusts to the number of airfoils in the rotor blade row. The harmonic two-dimensional gust with transverse and streamwise components propagates in the direction \( \vec{K} \), where \( k_1 = \omega A \) is the reduced frequency and \( k_2 \) is the transverse wave number, i.e., the transverse component of the gust propagation direction vector, with the gust direction angle defined as \( \gamma \) and \( \gamma = \tan^{-1} \left( \frac{k_2}{k_1} \right) \).

The complete flow field is assumed to be comprised of a steady mean flow and an harmonic unsteady flow field, Equation 1. The unsteady flow field corresponds to the gust unsteady flow field, \( \vec{G} \), or the motion-induced unsteady flow field, \( \vec{M} \).
\( \mathbf{Q}(x,y,t) = \mathbf{Q}_0(x,y) + \mathbf{Q}(x,y) \exp[ikx] \)  

(1)

**Steady Flow Field**

A velocity potential function can be defined for the steady flow of an incompressible inviscid fluid. The complete flow field is then described by the Laplace equation.

\[
\nabla^2 \Phi_0(x,y) = 0
\]

(2)

where \( \mathbf{Q}_0(x,y) = \nabla \Phi_0(x,y) \).

Since the Laplace equation is linear, the velocity potential can be decomposed into noncirculatory and circulatory components \( \Phi_{NC}(x,y) \) and \( \Phi_C(x,y) \).

\[
\Phi_0(x,y) = \Phi_{NC}(x,y) + \Phi_C(x,y)
\]

(3)

where \( \nabla^2 \Phi_{NC} = 0 \) and \( \nabla^2 \Phi_C = 0 \).

To complete the steady flow mathematical model, farfield inlet, farfield exit, airfoil surface, wake dividing streamline and cascade periodic boundary conditions must be specified. The steady farfield inlet flow is uniform, with the mass flow rate specified by the farfield exit boundary conditions. Also, a zero normal velocity is specified on the airfoil surfaces.

The steady velocity potential is discontinuous along the airfoil wake dividing streamline. This discontinuity is satisfied by a continuous noncirculatory velocity potential, with the discontinuity in the circulatory velocity potential equal to the steady circulation, \( \Gamma \). The Kutta condition is also applied, thereby enabling the steady circulation constant to be determined. It is satisfied by requiring the chordwise velocity components on the upper and lower airfoil surfaces to be equal in magnitude at the airfoil trailing edge. In addition, the cascade periodic steady velocity potential boundary conditions are satisfied by requiring both the normal and chordwise velocity components to be continuous between the upper and lower periodic boundaries. Refer to Chiang and Fleeter (1988) for a complete description of the steady flow boundary conditions.

**Unsteady Gust Aerodynamics**

The two-dimensional gust unsteady flow field \( \mathbf{Q}_G \) is determined by decomposing the gust generated unsteady flow field into harmonic rotational \( \mathbf{Q}_R \) and potential \( \mathbf{Q}_P \) components. The rotational gust component is specified consistent with linearized unsteady Euler equations, determined by linearizing the unsteady flow about the steady flow field. The gust is assumed to be convected with the steady mean flow and therefore does not interact with the airfoil cascade. Thus the following solution for the rotational gust is determined by solving the linearized Euler equations in the far upstream where the steady flow field is uniform.

\[
\mathbf{Q}_R = u^r \mathbf{i} + v^r \mathbf{j}
\]

(4)

where \( e^r \) and \( e^v \) are coupled with the ratio of their amplitudes being \( u^r/v^r = -k_2/k_1 \). Also, the solution corresponds exactly to the Sears transverse gust, Sears (1941) when \( k_2 = 0 \). However, this gust solution differs from that used in the Horlock (1968) and Naumann and Yeh (1972) models in which the two components are uncoupled, \( u^r = u^r \exp[ik_1(x-x)] \) and \( v^r = v^r \exp[ik_1(x-x)] \), where \( u^r \) and \( v^r \) denote the individual amplitudes of the two independent gust components and the gust and resulting unsteady aerodynamics are independent of the transverse component of the gust propagation direction vector \( \mathbf{k} = k_1 + ik_2 \).

The potential gust component \( \Phi_G \) is described by a Laplace equation. The solution is determined by decomposing this potential gust component into circulatory and noncirculatory components \( \Phi_{GNC}(x,y) \) and \( \Phi_{GC}(x,y) \), each of which is individually described by a Laplace equation.

\[
\begin{align*}
\Phi_0 &= \frac{\partial \Phi_G}{\partial x} + \frac{\partial \Phi_G}{\partial y} \\
\Phi_{GNC} &= \Phi_{GNC} + \Phi_{GC}
\end{align*}
\]

(5a)

\[
\begin{align*}
\nabla^2 \Phi_{GNC} &= 0 \\
\nabla^2 \Phi_{GC} &= 0
\end{align*}
\]

(5b)

Boundary conditions must be specified in the farfield inlet, farfield exit, airfoil surface, wake dividing streamline and cascade periodic boundary for the gust circulatory and noncirculatory components. The inlet farfield gust velocity potential boundary conditions are obtained by using a Fourier series to satisfy the periodicity condition at the far upstream. The exit farfield gust noncirculatory potential boundary condition is obtained in a manner analogous to that for the inlet. Since the wake does not attenuate in the farfield, the gust circulatory potential boundary equation is obtained by solving the Laplace equation at the farfield exit and satisfying the blade-to-blade periodicity condition, Verdon (1987).

\[
\begin{align*}
\Phi_{GNC, farfield inlet} &= -\left[ \frac{\beta_0}{\beta_0} \right] \frac{\partial \Phi_{GNC}}{\partial n} \\
\Phi_{GC, farfield inlet} &= -\left[ \frac{\beta_0}{\beta_0} \right] \frac{\partial \Phi_{GC}}{\partial n} \\
\Phi_{GNC, farfield exit} &= -\left[ \frac{\beta_0}{\beta_0} \right] \frac{\partial \Phi_{GNC}}{\partial n} \\
\Phi_{GC, farfield exit} &= -\left[ \frac{\beta_0}{\beta_0} \right] \frac{\partial \Phi_{GC}}{\partial n}
\end{align*}
\]

(6a)

\[
\begin{align*}
\Delta \Phi^0 \mathbf{e}^{-ik_1x} \left[ \frac{\mathbf{e}^{ky}}{1 - \mathbf{e}^{(k_1 + i)v Scos\delta}} + \frac{\mathbf{e}^{ky}}{1 - \mathbf{e}^{-i(k_1 + i)v Scos\delta}} \right]
\end{align*}
\]

(6b)

where \( \beta_0 = k_0 \), \( \sin \delta \), and \( \Delta \Phi^0 \) is the unsteady velocity potential discontinuity at farfield exit.

The airfoil surface boundary conditions specify that the normal velocity of the flow field must be equal to that of the airfoil. The gust generated unsteady rotational and potential flow fields are coupled through the boundary conditions on the noncirculatory gust component. In particular, the airfoil cascade is stationary, with the rotational gust convected with the mean steady flow field. Thus the upwash on the airfoil is determined by requiring the normal component of the unsteady flow field to be zero on the airfoil.
The unsteady gust velocity potential is discontinuous along
the airfoil wake dividing streamline. This discontinuity is satisfied
with a continuous noncirculatory velocity potential and a
discontinuous circulatory velocity potential. The unsteady circulation
velocity potential discontinuity is specified by requiring the pressure
to be continuous across the wake and then utilizing the unsteady
Bernoulli equation to relate the unsteady velocity potential and the
pressure. Also specified is the continuity of the noncirculatory
velocity potential across the wake and then utilizing the unsteady
velocity potential discontinuity is specified by requiring the pressure
within the unsteady gust field thereby enabling the unsteady
velocity potential discontinuity to be continuous across the wake streamline.

The unsteady pressure for the motion-dependent model \( P'_M \) is determined from the solution for the steady flow field, the unsteady
velocity potential, and the unsteady Bernoulli equation. The unsteady aerodynamic moment on the reference airfoil is calculated by integrating the unsteady surface pressure difference across the chord.

\[
M_{MR} = \int [P'_M(x,y)dx - P_Gdy]
\]

Motion-Induced Unsteady Aerodynamics

The unsteady flow field associated with the harmonic motion
of the airfoil cascade \( Q_M \) is assumed to be potential and is therefore
described by a Laplace equation. The solution is again determined by decomposing this velocity potential into circulatory and noncirculatory components \( Q_{MC} \) and \( Q_{MNC} \), each of which is
individually described by a Laplace equation.

\[
\nabla^2 \Phi_{MC} = 0; \quad \nabla^2 \Phi_{MNC} = 0
\]

The boundary conditions in the farfield inlet, farfield exit,
wake dividing streamlines, and cascade periodic boundaries as well
as the Kutta condition for the motion-induced circulatory and
noncirculatory components are identical to those for the gust potential
flow. The only boundary condition that changes is that requiring the
normal flow velocity to be equal to the airfoil velocity on the airfoil
surface, the upwash condition.

The upwash on the airfoil for the motion-dependent model \( W_M \) is a function of both the position of the airfoil and the steady
flow field. Thus, this boundary condition couples the unsteady flow
field to the steady aerodynamics. For an airfoil cascade executing
harmonic torsion mode oscillations about an elastic axis location at
\( x_0 \), the upwash on the airfoil is

\[
W_M(x,y) = \alpha \left[ \frac{ik_1[(x - x_0) + y]}{1 + \omega k_1 y} + U_0 + \frac{\partial \Phi}{\partial x} \right]^{1/2}
\]

where \( U_0 = U_0(x,y) \) and \( V_0 = V_0(x,y) \) are the steady airfoil surface velocity components, \( f(x) \) denotes the airfoil profile and \( \alpha \) is the amplitude of the torsional oscillations.

The unsteady pressure on the motion-dependent model \( P'_M \) is determined from the solution for the steady flow field, the
unsteady velocity potential, and the unsteady Bernoulli equation. The unsteady aerodynamic moment on the reference airfoil is calculated by integrating the unsteady surface pressure difference across the chord.

\[
M_{MR} = \int [P'_M(x,y)dx - P'_Mdy]
\]

Locally Analytical Solution

A boundary fitted computation grid generation technique is utilized
for the numerical solution. A Poisson type grid solver is
used to fit a C-type grid around a reference airfoil in the cascade.
This method permits grid points to be specified according to the entire
boundary of the computational plane.

Laplace equations describe the complete flow field including
the unknown velocity potentials \( \Phi_{MC}, \Phi_{MNC}, \Phi_{MC}, \) and \( \Phi'_{MC}. \) In the
transformed \((\xi, \eta)\) coordinate system, the Laplace equation takes on
the following nonhomogeneous form.

\[
\frac{\partial^2 \Phi}{\partial \xi^2} + \alpha \frac{\partial^2 \Phi}{\partial \eta^2} - 2\alpha \frac{\partial \Phi}{\partial \eta} - 2\gamma \frac{\partial \Phi}{\partial \xi^2} = F(\xi, \eta)
\]

where \( \Phi \) is a shorthand method of writing these four velocity
potentials in the transformed plane, \( F(\xi, \eta) \) contains the cross
derivative term \( \partial^2 \Phi / \partial \xi \partial \eta \) and the coefficients \( \alpha, \beta, \) and \( \gamma \) are
functions of the transformed coordinates \( \xi \) and \( \eta \) which are treated as
constants in each individual grid element.

To obtain the analytical solution to the transformed Laplace
equation, it is first rewritten as a homogeneous equation by defining
a new dependent variable \( \Phi(\xi, \eta) \).
where \( \Phi = \Phi \exp \{ \gamma S + \beta \eta \} = \frac{F(\gamma S + \beta \eta)}{2(\gamma^2 + \alpha \beta^2)} \)

The following general solution for \( \Phi \) is determined by separation of variables.

\[
\Phi(x,y) = \left[ A_1 \cos (\lambda x) + A_2 \cos (\lambda y) \right] \left[ B_1 \cos (\mu x) + B_2 \sin (\mu y) \right]
\]

where \( \mu = \sqrt{(\alpha + \beta^2)^2 + \lambda^2}/\alpha \), \( \lambda, \lambda_1, A_1, A_2, B_1, B_2 \) are constants to be determined from the boundary conditions.

Analytical solutions in individual computation grid elements are determined by applying proper boundary conditions to evaluate the unknown constants in this general velocity potential solution. The solution to the global problem is then determined through the applications of the global boundary conditions and the assembly of the locally analytical solutions. The locally analytical method for steady two-dimensional fluid flow and heat transfer problems was initially developed by Chen, Naseri-Neshat and Ho (1981). They have shown that the locally analytical method has several advantages over the finite difference and finite element methods.

### Aerodynamically Detuned Cascade

For the aerodynamically detuned rotor configuration of interest herein, i.e., variable circumferential spacing and chord length, an analogous cascade unsteady aerodynamic model is developed by considering two passages with two passage periodic boundary conditions.

In this model, the rotor will incorporate splitters (short chord airfoils) between each pair of full chord airfoils. As schematically depicted in Figure 2, the splitters are not required to have the same airfoil shape as the full chord airfoils, nor are they restricted to particular circumferential or axial positions between each pair of the full chord airfoils.

There are two distinct passages: (1) a reduced spacing, or increased solidity, passage; and (2) an increased spacing, or reduced solidity, passage. There are also two distinct sets of airfoils, with the two reference airfoils denoted by \( R_0 \) and \( R_1 \). These individual sets of airfoils can be considered as cascades of uniformly spaced airfoils each with twice the spacing of the associated baseline aerodynamically tuned uniformly spaced cascade. The circumferential spacing between these two sets of airfoils, \( S_1 \) and \( S_2 \), is determined by specifying the level of aerodynamic detuning, \( \epsilon \).

\[
S_{1,2} = (1 \pm \epsilon) S
\]

where \( S \) is the spacing of the baseline uniformly spaced cascade, and \( S_1 \) and \( S_2 \) denote the spacings of the detuned cascade.

An interblade phase angle for this aerodynamically detuned cascade configuration can be defined. In particular, each set of airfoils is individually assumed to be executing harmonic torsional oscillations with a constant aerodynamically detuned interblade phase angle, \( \beta_0 \), between adjacent airfoils of each set. Figure 3. Thus, this detuned cascade interblade phase angle is two times that for the corresponding baseline tuned cascade.

\[
\beta = 2\beta_0
\]

where \( \beta_0 \) is the tuned baseline cascade interblade phase angle, defined between adjacent airfoils on the rotor.

For a rotor with \( N \) uniformly spaced blades, Lane (1956) showed that the values of \( \beta_0 \) must satisfy the following condition.

\[
\beta_0 = \frac{2\pi}{N}, \quad r = 0, \pm 1, \pm 2, \ldots, \pm N - 1
\]

where \( \pm \) refers to forward and backward traveling waves, respectively.

### Steady Aerodynamics

The steady aerodynamic model developed for the baseline uniformly spaced cascade can be applied directly to the two reference passage model of the detuned cascade with the addition that the steady potential for the detuned cascade, \( \Phi_0(x,y) \), is decomposed into one noncirculatory \( \Phi_0NC(x,y) \) and two circulatory components, one associated with each of the two reference airfoils, \( \Phi_0CR_0(x,y) \) and \( \Phi_0CR_1(x,y) \).

\[
\Phi_0(x,y) = \Phi_0NC(x,y) + \Phi_0CR_0(x,y) + \Phi_0CR_1(x,y)
\]

where \( \nabla^2 \Phi_0NC = 0; \ \nabla^2 \Phi_0CR_0 = 0; \ \text{and} \ \nabla^2 \Phi_0CR_1 = 0 \).

The circulatory component \( \Phi_0CR_0(x,y) \) is discontinuous along the wake dividing streamline of the reference airfoil \( R_0 \) and is continuous along the wake dividing streamline of the reference airfoil \( R_1 \), while \( \Phi_0CR_1(x,y) \) is continuous along the wake of \( R_1 \) and continuous along the wake of \( R_0 \). The steady potential discontinuity is satisfied by a continuous noncirculatory velocity potential, with the discontinuities in the circulatory components \( \Phi_0CR_0 \) and \( \Phi_0CR_1 \), equal to the steady circulation, \( \Gamma_R \) and \( \Gamma_R \), respectively. Also, the steady circulation constants \( \Gamma_R \) and \( \Gamma_R \) are determined by simultaneously applying the Kutta condition to the two reference airfoils.

### Unsteady Aerodynamics

The unsteady potential for the detuned cascade \( \Phi'(x,y) \) is also decomposed into one noncirculatory \( \Phi'_0NC(x,y) \) and two circulatory components, one associated with each of the two reference airfoils \( \Phi'_0CR_0(x,y) \) and \( \Phi'_0CR_1(x,y) \).

\[
\Phi'(x,y) = \Phi'_0NC(x,y) + \Phi'_0CR_0(x,y) + \Phi'_0CR_1(x,y)
\]

where \( \Phi'_0CR(x,y) \), \( \Phi'_0CR_0 \), and the unsteady circulation constants \( \Gamma_R \) and \( \Gamma_R \), are defined analogous to the corresponding detuned cascade steady aerodynamic quantities.

### Influence Coefficient Technique

The unsteady airfoil surface boundary conditions require that the cascaded airfoils oscillate with equal amplitudes. Also, the interblade phase angle between adjacent nonuniformly spaced airfoils must be specified. Neither of these requirements is appropriate for the aerodynamically detuned cascade. To overcome these limitations, an unsteady aerodynamic influence coefficient technique is utilized.

The unsteady aerodynamic moment acting on the two reference airfoils is expressed in terms of influence coefficients.

\[
M_{R_0,R_1} = W_{R_0}[CG]^{2}_{R_0,R_1} + W_{R_1}[CG]^{2}_{R_1,R_0}
\]

where \( W_{R_0}[CM]^{2}_{R_0,R_1} + W_{R_1}[CM]^{2}_{R_1,R_0} \) is the unsteady aerodynamic influence coefficient technique is utilized.
where \( \alpha_{R0} \) and \( \alpha_{R1} \) are the unknown complex oscillatory displacements for the reference airfoils \( R_0 \) and \( R_1 \), respectively, and \( W_{R,R} \) are the complex gust amplitudes which are related by the detuned interblade phase angle \( \beta_d \) and the level of aerodynamic detuning \( \varepsilon \).

\[
W_{R,R} = W_R \exp[i(1-\varepsilon)\beta_d/2]
\]  

(19)

The influence coefficients \([CG]_{R,R}, [CG]_{R,R}, [CM]_{R,R}, [CM]_{R,R}, [CM]_{R,R} \) are the unsteady aerodynamic moments acting on the two reference airfoils. They are determined by analyzing the two reference passages with the unsteady cascade model developed herein. \([CG]_{R,R}, [CM]_{R,R} \) are determined by considering a unit amplitude gust acting only on the reference airfoil \( R_d \) and unit amplitude motion of only the reference airfoil \( R_0 \) respectively, and reference airfoil \( R_0 \) with no gust and stationary. Note that the gust is modeled only through the airfoil surface boundary conditions. Thus the gust acts either on one or both airfoils. The influence coefficients \([CG]_{R,R}, [CM]_{R,R} \) are obtained in an analogous manner.

**FORCED RESPONSE MODEL**

The equations describing the single-degree-of-freedom torsional motion of the two reference airfoils of the aerodynamically detuned cascade are developed by considering the typical airfoils depicted in Figure 4. The elastic restoring forces are modeled by linear torsional springs at the elastic axis location, with the inertial properties of the airfoils represented by their mass moments of inertia \( I_x \) about the elastic axis. The equations of motion, determined by Lagrange's technique, are

\[
I_{GR0} \ddot{\alpha}_{R0} + (1 + 2i \beta R_0) I_{GR0} \omega^2 \alpha_{R0} = M_{R0}
\]

\[
I_{GR1} \ddot{\alpha}_{R1} + (1 + 2i \beta R_1) I_{GR1} \omega^2 \alpha_{R1} = M_{R1}
\]

where \( I_{GR0} \) and \( I_{GR1} \) are the mass moments of inertia about the elastic axis, \( \beta R_0 \) and \( \beta R_1 \) denote the structural damping coefficients for the reference airfoils, and the undamped natural frequencies are \( \omega_{R0}^2 = K_{GR0}/I_{GR0} \) and \( \omega_{R1}^2 = K_{GR1}/I_{GR1} \).

Considering harmonic time dependence of the reference airfoils and utilizing the total unsteady aerodynamic moments and Equation 19, the equations of motion are written in matrix form.

\[
\begin{bmatrix}
\mu_0 & [CM]_{R0} \\
[CM]_{R1} & \mu_1
\end{bmatrix}
\begin{bmatrix}
\alpha_{R0} \\
\alpha_{R1}
\end{bmatrix}
= \begin{bmatrix}
- W_{R0} \\
- W_{R1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
[CG]_{R0} + e^{i(1-\varepsilon)\beta_d/2} [CG]_{R0} \\
[CG]_{R1} + e^{i(1-\varepsilon)\beta_d/2} [CG]_{R1}
\end{bmatrix}
\]

(21)

where \( \mu_0 = \rho_{R0} \omega_{R0}^2 \) \( \mu_1 = \rho_{R1} \omega_{R1}^2 \) \( \gamma = \frac{\omega_{R0}^2}{\omega_{R1}^2} \)

\[
\begin{bmatrix}
\alpha_{R0} \\
\alpha_{R1}
\end{bmatrix}
= \begin{bmatrix}
\alpha_{R0} \\
\alpha_{R1}
\end{bmatrix}
\]

and \( \omega_0 \) is reference frequency.

The aerodynamically forced response of a tuned or detuned cascade is determined by solving Equation 21 for the airfoil displacements using standard matrix techniques. It should be noted that both the motion dependent oscillating airfoil and the gust response unsteady aerodynamic influence coefficients must be determined for each specified interblade phase angle \( \beta_d \). The resulting airfoil displacement vector solution is

\[
\begin{bmatrix}
\alpha_{R0} \\
\alpha_{R1}
\end{bmatrix}
= \begin{bmatrix}
\alpha_{R0} \\
\alpha_{R1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
[CM]_{R0} \\
[CM]_{R1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
[CG]_{R0} + e^{i(1-\varepsilon)\beta_d/2} [CG]_{R0} \\
[CG]_{R1} + e^{i(1-\varepsilon)\beta_d/2} [CG]_{R1}
\end{bmatrix}
\]

(22)

The effects of structural detuning are included through the frequency terms \( \gamma \alpha_{R0}, \alpha_{R1} \). These terms represent the ratios of the natural frequency to a specified reference frequency. Defining the reference frequency \( \omega_0 \) as the torsion mode natural frequency, a structurally tuned cascade would have \( \omega_{R0}, \omega_{R1} \) equal to unity. For the case when the cascade is structurally detuned, the values of \( \omega_{R0}, \omega_{R1} \) are altered.

In this regard, splitters enable combined aerodynamic and structural detuning to be introduced into a rotor. In particular, the natural torsion mode frequency of an airfoil is a function of its chord length, thickness, and span as well as its material properties. As a result, the splitters may have a higher natural frequency than the full chord airfoils.

The vibrational characteristics of the full chord and splitter airfoils are determined by modeling each airfoil as a thin rectangular cross-section cantilevered slender beam. The torsion mode natural frequency \( \omega_T \) is

\[
\omega_T = \sqrt{\frac{G}{2 \pi^2 \rho_m C L}}
\]

(23)

where \( G \) is the material modulus of rigidity, \( L \) is the airfoil span, \( T \) is the airfoil thickness, \( C \) is the chord length, \( m = \rho_m LTC \) is the mass of the airfoil and \( \rho_m \) is the material density.

Two types of splitters are of interest: (1) splitters with the same thickness-to-chord ratio as the full chord airfoils and (2) splitters with the same thickness as the full chord airfoils.

For the first type, the splitters and full chord airfoils have the same natural frequency when their material properties and spans are the same, with the splitters having a higher natural frequency when the material properties are different or the splitter span is smaller than that of the full chord airfoils. Hence, these splitters incorporate either aerodynamic detuning or combined aerodynamic-structural detuning into a rotor design.

For the second case, the splitters have a higher natural frequency than the full chord airfoils as long as the two sets of airfoils have the same material properties and spans, with the possibility of the splitters and full chord airfoils having the same natural frequency if different materials are utilized or the splitters have increased span. Thus, these splitters can also introduce either aerodynamic detuning or combined aerodynamic-structural detuning into the rotor.

**RESULTS**

To demonstrate the effects of the splitter generated aerodynamic and structural detuning on flow induced vibrational response, the single degree-of-freedom torsional aerelastic model is applied to a baseline twelve bladed rotor and to a rotor with alternate blades replaced with splitters.
Baseline Rotor Configurations

The baseline rotor is comprised of twelve uniform circumferentially spaced blades characterized by a Gostelow cascade geometry with a solidity of 0.83, a stagger angle of 40°, a mean flow incidence angle of 24°, and an airfoil mass ratio μ and radius of gyration tgc of 193.776 and 0.3957 respectively, typical of modern fan blades. The reduced frequency is 1.67, resulting in a stable baseline rotor for all interblade phase angles.

Aerodynamic detuning is introduced into the rotor by means of splitters with the same thickness-to-chord ratio as the full chord airfoils, with this thickness-to-chord ratio varied by ±180°. The aerodynamic forcing function considered is a 45° two-dimensional gust characterized by interblade phase angles β0 of -30°, 0° and 180°, corresponding to detuned interblade phase angles β0 of -60°, 0° and 360°. These values were selected because they are the gust loads which were found to result in the highest amplitudes of response. In this study, the frequency of the gust ω is nondimensionalized by the undamped natural torsional frequency of the baseline airfoils ωn. The rotor response generated by this forcing function is presented in the format of the reference airfoil amplitude of response α normalized by the maximum response of the baseline reference airfoil αn. The amplitudes of response for the three gust interblade phase angles of the baseline reference airfoil as a function of the non-dimensional gust frequency ω/ωn are shown in Figure 5.

Splitted-Rotor Configuration

Splitters are a convenient means of introducing aerodynamic and/or structural detuning into a rotor. Thus, the baseline twelve bladed rotor is detuned by replacing alternate airfoils with splitters with chords that are two-thirds that of the full chord airfoils, with two splitter designs considered. Aerodynamic detuning is introduced into the rotor by means of splitters with the same thickness-to-chord ratio as the full chord airfoils, and thus the same natural frequencies. The aerodynamic detuning results from both the decreased chord of the splitters and the splitter circumferential position between adjacent full chord airfoils. Three circumferential splitter locations are considered: 40%, 50% and 60%. Figure 6 shows examples of these splitted-rotor flow geometries and the associated computational grids.

Combined aerodynamic-structural detuning is accomplished with splitters having the same thickness but a different chord length than the full chord airfoils positioned such that the splitter trailing edge is aligned with that of the full chord airfoils.

Steady Aerodynamic Performance

Steady rotor performance is defined by the chordwise distributions of the airfoil surface steady static pressure coefficient, presented in Figure 7 for the baseline and splittered-rotors. The introduction of the splitters into the rotor has a noticeable effect on the steady loading distribution of the full chord airfoils, with this effect being a function of the splitter circumferential position.

Forced Vibrational Response

For the rotor with splitters having the same thickness-to-chord ratio as the full chord airfoils, the effect of aerodynamic detuning on forced response is shown in Figures 8 through 13.

With the splitters at 40% passage spacing, the aerodynamic detuning results in a significant decrease in the maximum amplitude of response for the backward traveling wave mode (β0 = -30°). For the full chord reference airfoil R0, as shown in Figure 8, the response amplitude for the forward traveling wave mode (β0 = +180°) is decreased as compared to the baseline around α/αn = 0.96 but increased near α/αn = 1.0. The response for the in-phase mode is almost unchanged by this detuning. For the splitter reference airfoil R1, as shown in Figure 9, the response amplitude is decreased slightly for the in-phase mode and exhibits coupling of the two modes for the forward traveling wave mode (β0 = +180°).

With these splitters at 50% passage spacing, as shown in Figures 10 and 11, there is a significant increase in the maximum response amplitude of both reference airfoils as compared to the baseline for the backward traveling gust wave mode. For the other two interblade phase angles (β0 = 0° and 180°), the aerodynamic detuning has an adverse effect on forced response.

Positioning these splitters at 60% passage spacing results in a significant decrease in the response amplitude for a gust characterized by β0 = -30°, as shown in Figures 12 and 13. However, there is an increase in the response amplitude associated with the forward traveling wave mode and a slightly decreased response amplitude in the in-phase mode. Thus, depending on the gust interblade phase angle, this aerodynamic detuning may be either beneficial or detrimental to the rotor forced response.

The effect of forced response of combined aerodynamic and structural detuning generated by these splitters is also considered, with the splitters considered to have a 15% higher natural frequency than the full chord airfoils, Figures 14 through 19. This combined aerodynamic and structural detuning results in decreased amplitudes of response for the full chord airfoils for all modes. Also, there is minimal response of the reference splitters in the range of frequencies near that of the full chord natural frequency. However, for some of the detuned rotor configurations, the splitter responses at the higher frequency is greater than or of the same order as the 180° mode response of the baseline full chord airfoils at the lower frequency.

For the rotor with splitters having the same thickness as the full chord airfoils, the effect of combined aerodynamic and structural detuning on forced responses is shown in Figures 20 and 21, with the effect of aerodynamic detuning presented in Figures 22 and 23. The combined detuning significantly decreases the response for both the backward and in-phase traveling wave gust modes of the full chord reference airfoil R0 and a slightly decreased response in the in-phase mode for the 180° gust mode. Also, this combined aerodynamic and structural detuning results in minimal response of the splitters in the range of frequencies near that of the full chord airfoil natural frequency, but an additional response near the splitter natural frequency. This higher frequency splitter response is not small, being somewhat larger for the forward wave mode and smaller for the other two modes than the full chord airfoil response amplitudes.

SUMMARY AND CONCLUSIONS

A mathematical model has been developed and utilized to predict the effect of incorporating splitter blades on the torsion mode forced response of a rotor operating in an incompressible flow field. The splitter blades, positioned circumferentially in the flow passage between two principal blades, introduce aerodynamic and/or combined aerodynamic-structural detuning into the rotor. The two-dimensional gust response and oscillating cascade unsteady aerodynamics, including steady loading effects were determined by developing a complete first-order unsteady aerodynamic analysis together with an unsteady aerodynamic influence coefficient technique. The torsion mode forced response of both uniformly spaced tuned rotors and detuned splintered rotors were then predicted by incorporating the unsteady aerodynamic influence coefficients into a single-degree-of-freedom aeroelastic model.
The viability of splitters as a passive torsion mode forced response control technique for an aerodynamically loaded rotor operating in an incompressible flow field was then considered, accomplished by applying this model to a baseline twelve bladed rotor. This study demonstrated that the aerodynamic detuning associated with the splitters was sometimes beneficial and other times detrimental with regard to forced response. However, the combined aerodynamic and structural detuning due to the splitters was generally beneficial for the full chord airfoils, with minimal splitter response in the frequency range near to that of the full chord airfoils. Thus, aerodynamic detuning and combined aerodynamic-structural detuning associated with the incorporation of splitters into a rotor are viable passive control mechanisms for flow induced response of rotors.

ACKNOWLEDGEMENTS

This research was sponsored, in part, by the NASA Lewis Research Center.

REFERENCES


Figure 3. Reference airfoils and passages of detuned cascade

Figure 4. Single degree-of-freedom detuned cascade model

Figure 5. Baseline airfoil response for gust with $\beta = -30^\circ$, $180^\circ$ and $0^\circ$

Figure 6. Aerodynamically detuned 12 bladed splittered-rotor flow geometries and computational grids

Figure 7. Steady aerodynamic performance of baseline and splittered rotors
Figure 8. Aerodynamic Detuning Effect on Airfoil R₀ response with splitters with the same T/C as baseline airfoils at 40% circumferential spacing.

Figure 9. Aerodynamic Detuning Effect on Airfoil R₁ response with splitters with the same T/C as baseline airfoils at 40% circumferential spacing.

Figure 10. Aerodynamic Detuning Effect on Airfoil R₀ response with splitters with the same T/C as baseline airfoils at 50% circumferential spacing.

Figure 11. Aerodynamic Detuning Effect on Airfoil R₁ response with splitters with the same T/C as baseline airfoils at 50% circumferential spacing.
Figure 12. Aerodynamic Detuning Effect on Airfoil R₀ response with splitters with the same T/C as baseline airfoils at 60% circumferential spacing.

Figure 13. Aerodynamic Detuning Effect on Airfoil R₁ response with splitters with the same T/C as baseline airfoils at 60% circumferential spacing.

Figure 14. Aerodynamic-Structural Detuning effect on Airfoil R₀ response with splitters with the same T/C as baseline airfoils at 40% circumferential spacing.

Figure 15. Aerodynamic-Structural Detuning effect on Airfoil R₁ response with splitters with the same T/C as baseline airfoils at 40% circumferential spacing.
Figure 16. Aerodynamic-Structural Detuning effect on Airfoil $R_0$ response with splitters with the same T/C as baseline airfoils at 50% circumferential spacing

Figure 17. Aerodynamic-Structural Detuning effect on Airfoil $R_1$ response with splitters with the same T/C as baseline airfoils at 50% circumferential spacing

Figure 18. Aerodynamic-Structural Detuning effect on Airfoil $R_0$ response with splitters with the same T/C as baseline airfoils at 60% circumferential spacing

Figure 19. Aerodynamic-Structural Detuning effect on Airfoil $R_1$ response with splitters with the same T/C as baseline airfoils at 60% circumferential spacing
Figure 20. Aerodynamic-Structural Detuning effect on Airfoil R₀ response with splitters with the same thickness as baseline airfoils at 50% circumferential spacing.

Figure 21. Aerodynamic-Structural Detuning effect on Airfoil R₁ response with splitters with the same thickness as baseline airfoils at 50% circumferential spacing.

Figure 22. Aerodynamic Detuning effect on Airfoil R₀ response with splitters with the same thickness as baseline airfoils at 50% circumferential spacing.

Figure 23. Aerodynamic Detuning effect on Airfoil R₁ response with splitters with the same thickness as baseline airfoils at 50% circumferential spacing.