AN ASSESSMENT OF WEIGHTED-LEAST-SQUARES BASED GAS PATH ANALYSIS

David L. Doel
Advanced Engineering Technologies Department
GE Aircraft Engines
Evendale, Ohio

ABSTRACT
Manufacturers of gas turbines have searched for three decades for a reliable way to use gas path measurements to determine the health of jet engine components. They have been hindered in this pursuit by the quality of the measurements used to carry out the analysis. Engine manufacturers have chosen weighted-least-squares techniques to reduce the inaccuracy caused by sensor error. While these algorithms are clearly an improvement over the previous generation of gas path analysis programs, they still fail in many situations. This paper describes some of the failures and explores their relationship to the underlying analysis technique. It also describes difficulties in implementing a gas path analysis program. The paper concludes with an appraisal of weighted-least-squares based gas path analysis.

NOMENCLATURE

Equations Included in This Paper
E(...) expected value operator
H matrix of influence coefficients, defining the effect of the engine state variables on the measurements
J weighted-least-squares objective function (to be minimized)
M covariance matrix for state variables
n number of state variables
Q covariance matrix of measurement deviations derived from a large sample of test data
p number of independent measurements
R covariance matrix for measurement errors
x state vector composed of the characteristics of the engine expected to vary with time
x₀ weighted-least-squares solution for x
z measurement vector comprised of the independent measurements on which the analysis is based
v vector of measurement noise
v₀ weighted-least-squares solution for measurement noise

Superscripts
⁻¹ matrix inverse
T matrix transpose

TEMPER Parameters
EGT Exhaust Gas Temperature (also called Control Temperature)
FAN denotes the engine fan in TEMPER results
FN net thrust
HPC denotes the High Pressure Compressor in TEMPER results
HPT denotes the High Pressure Turbine in TEMPER results
HPT R denotes the High Pressure Turbine Rotor in TEMPER results
HPTS denotes the High Pressure Turbine Stator in TEMPER results
LPC denotes the Low Pressure Compressor in TEMPER results
LPT denotes the Low Pressure Turbine in TEMPER results
MEAS denotes EGT Measurement error in TEMPER results
N12 fan rotation speed
N25 gas generator rotation speed
P1A engine inlet total pressure
P49 high pressure turbine discharge total pressure
P5 low pressure turbine discharge total pressure
PS1A engine inlet static pressure (used to compute air flow)

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**INTRODUCTION**

Gas path analysis is used to decide which gas turbine components are responsible for an observed performance deficit. The deficit may be a loss of performance during revenue service flights, or the failure to achieve performance guarantees in an overhaul acceptance run. In either case, the goal of gas path analysis is to help the user select a work scope to regain performance.

Engine sensors, used as input to gas path analysis, include temperature probes and total and static pressure probes in the gas stream, plus fuel flow and rotor speeds. In the test cell, thrust and total engine airflow are also available (Figure 1). Pressure and temperature sensors are nearly always single element probes. They often deviate substantially from plane average temperature or pressure.

Weighted-least-squares has been the predominant technology for gas path analysis for at least a decade. All major jet engine manufacturers offer gas path analysis programs based on this technique (Urban & Volponi, 1992, Barwell, 1987, Doel, 1992). Each manufacturer has needed to augment the basic weighted-least-squares algorithm to achieve effective results. Rolls Royce offers a "concentrator" module to emphasize module problems. GE provides "fault logic" to improve the diagnosis of large component or measurement faults. Hamilton Standard provides a "large measurement error recovery algorithm" (Volponi, 1982).

Users need gas path analysis most when there are large deviations in engine performance or large measurement errors. These large departures from normal behavior usually require the special features (concentrator, fault logic) for a successful analysis. Thus, the variations on the weighted-least-squares theme are vital to the perceived success of the gas path analysis algorithm.

This paper examines the experience with one gas path analysis algorithm (TEMPER\(^1\)) to identify successes and problems of the weighted-least-squares approach\(^2\). There is no sure way to determine whether an analysis is correct. The best evaluation of the analysis is derived from the success or failure of the work scope developed from its results (and from other considerations). The evaluation is, therefore, subjective.

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\(^{1}\) TEMPER for "Turbine Engine Module Performance Estimation Routine"

\(^{2}\) A mathematical description of the weighted-least-squares algorithm, as used in TEMPER, is provided in Appendix A.
A TURBINE PROBLEM CORRECTLY DIAGNOSED, OR WAS IT?

Test Cell TEMPER (Doel 1992) was introduced during the early 1980's. The initial release relied on the weighted-least-squares algorithm (Bryson & Ho 1975). Users quickly identified a requirement to improve results for engines with large performance deviations or measurement errors. The TEMPER fault logic was added to address this need.

Almost immediately, the fault logic produced a success story for the TEMPER program. An engine, removed for exceeding its EGT limit, was run in-bound to determine the cause for the low performance. A TEMPER analysis of the data (Figure 2) showed the high-pressure-turbine to have deteriorated by 3 percent in efficiency. It also indicated that the high-pressure-turbine flow function was almost 3 percent larger than nominal.

Inspection of the turbine confirmed the damage. The turbine rotor was replaced. No repair was performed on the first stage nozzle of the turbine. This work scope should restore the turbine efficiency without appreciably changing the turbine flow function. When the acceptance data were analyzed (Figure 3), the turbine efficiency had indeed been restored to a normal level while the turbine flow function was still 3 percent above nominal.

This TEMPER success story was due entirely to the newly-added fault logic. Without the fault logic, TEMPER would have assigned roughly half of the problem to the turbine. The remainder would have been attributed to other components and to measurement error. This attenuation of results, typical of the weighted-least-squares algorithm, would have jeopardized the user's ability to seize on the high-pressure-turbine as the primary source of the low performance.

Less than a month after this successful result, the TEMPER fault logic was modified as part of the normal development process. The newer version of the fault logic used "solution probability" to choose among candidate faults where the previous version had used "residual error." Solution probability accounts for the "degrees of freedom" used by the fault and the residual error.

3 The TEMPER solution also suggests the possibility of a large error in the Exhaust Gas Temperature (EGT) measurement. This is often observed in combination with significant turbine deterioration. The apparent temperature error often disappears when the turbine is repaired. The cause for this behavior is not known, but it has been observed often enough that experienced users assume the error will be corrected when the faulty hardware is repaired.

4 Newer versions of TEMPER would do better because of lessons learned over the intervening years. The results would, however, still be fuzzier than the crisp enunciation provided by the fault logic.
This change was needed to eliminate an inherent advantage given to faults having more than one element. The turbine fault has both turbine efficiency and turbine flow function to absorb residual error. Conversely, a measurement fault, such as fuel flow measurement error has only a single element to absorb the residual. When the solutions are compared using residual error, the multiple-fault solutions have an advantage over the single-fault solutions. Using solution probability eliminates the advantage.

After implementing the changes to the fault logic, we ran the data from the engine with the turbine problem through the new program. The out-bound results were unchanged. However, the in-bound solution was completely different. TEMPER now attributed the problem to increased turbine cooling instead of the combination of turbine efficiency and turbine flow function.

This outcome is not surprising. The combination of increased turbine flow function and decreased turbine efficiency, in a one-to-one ratio, cannot be distinguished from a turbine cooling increase, given the available condition monitoring measurements. The turbine performance fault uses two degrees of freedom while the turbine cooling fault uses only one. The turbine performance fault lowered the residual error a little more than the turbine cooling flow fault, but this advantage was overcome by the additional degree of freedom.

To address this new problem, TEMPER now provides the two best fault solutions, whenever their probabilities are within five percent of one another. This places a burden on the user to choose the correct result. In the case described, the user can inspect the turbine to see if damage is present. If so, the performance deficit is blamed on the turbine - if not, turbine cooling is suspected.

**BASE LINES**

Ideally, gas path analysis would report absolute performance for engine modules (efficiency and flow capacity). Targets for the components' performance could be developed from production acceptance runs, or from acceptance runs of well-overhauled engines. The analyst could use the differences between the engine's performance and the targets to decide which components need re-work.

This ideal is yet to be reached because of limitations in the measurements. Measurements available in overhaul or in service are usually single element probes. Placement of these probes is dictated more by convenience and ruggedness than by effectiveness. Thus, the probes have unknown biases relative to the plane-average temperatures and pressures needed to calculate component efficiency and flow capacity.
To address the sensor problems, gas path analysis programs provide relative rather than absolute answers. Test cell programs express performance of the modules relative to an average of other engines tested in the same cell. On-wing programs compare performance of the modules to an average of newly installed engines.

The engine's cycle model would seem to be a candidate reference for the gas path analysis. This is not practical, however, because the cycle model represents only plane average temperatures and pressures. Thus, "baselines" are added to the engine cycle model to adjust its predictions to be compatible with actual measurements.

TEMPER works best with a baseline representing expected values of test cell or on-wing measurements. This expected-value baseline gives proper meaning to the solution residual, permitting recognition of abnormal engine behavior. The expected-value baseline may not, however, be an appropriate reference point for citing results. Output from the gas path analysis program is better referred to production or well-overhauled engine performance levels. This gives a more accurate reading of the module's improvement potential to the user.

Development of an expected-value baseline is a tedious, time-consuming effort. The developer assembles data from as many engines as possible. The data are carefully reviewed to eliminate outliers. Data that pass the screening are processed to compute the deviations of the measurements from the cycle model. To complete the expected-value baselines, these deviations are averaged (or curve-fit versus power level).

Each test cell or engine model needs a unique expected-value baseline. Changes to test cell cowlings can change the baseline, as can changes to the engine configuration (e.g., control deck). It is often necessary to develop several baselines for a single test cell. Engine manufacturers frequently have several engine models and as many as a hundred airline customers. Therefore, an engine manufacturer may need to develop hundreds of separate baselines. Changes to the test cells, or other events, dictate regular updates to the baselines.

A new engine model presents an even greater difficulty. There is no data from which to generate baselines. Either the user must await gathering of sufficient data for the generation of baselines, or some other technique must be developed for providing the initial baselines.

An approach that has met with some success is to predict the baseline from production acceptance data and cell correlation results (for the specific engine model and cell). This approach relies on an estimate of the average quality of the overhauled components. It also depends on the use of a single back-to-back cycle model and cell correlation test. The single test is subject to significant measurement errors that are propagated into the baselines.

The single, most important feature of gas path analysis programs is their ability to cope with large deviations (for example the "fault logic" in TEMPER). This ability depends on having a good prediction for the behavior of the engine - that is on having good expected-value baselines. For new engine models these do not exist. For mature engine models their existence depends on good communication between the test cell owner and the baseline developer, and upon the diligence of the baseline developer. Thus, baselines represent a major risk to the effectiveness of current gas path analysis algorithms.

STATISTICS AND WEIGHTED-LEAST-SQUARES RESPONSE

Ordinarily, the TEMPER solution is derived using the pure weighted-least-squares algorithm. The solution from a pure weighted-least-squares algorithm is a linear function of the differences between the measurements and their predicted values (Doel, 1992). The gain matrix, used to compute the solution, is determined by the influence coefficients (also called partial derivatives) and by the assumed variances for the state variables and measurement errors. This gain matrix can be computed without knowledge of the measurements. Thus, it is possible to examine the response of a weighted-least-squares algorithm to specific faults.

Table 1 shows the response of a representative gas path analysis program to a pure turbine efficiency fault. The input to the algorithm of Table 1 represents an engine that agrees with the baseline engine in every respect except that its turbine efficiency is one percent better than the baseline engine. Table 1 also provides the ideal result for the algorithm (which mirrors the incoming turbine efficiency problem). The ideal result is unattainable because there are fewer equations (one for each independent measurement) than unknowns (one for each independent measurement plus one for each state variable).

Table 1 records that slightly more than 60% of the incoming fault is correctly assigned to turbine efficiency. The remainder of the fault is attributed to other components and to measurement errors. High pressure compressor efficiency, low pressure compressor efficiency, fuel flow measurement error, and high pressure turbine discharge temperature measurement error are all identified as problems by the algorithm.

The influence coefficients, H, are deterministic for an engine model, but the statistical assumptions for state variables and measurement errors are not. They may be adjusted to tune the response of the algorithm. Increasing the assumed variance of the turbine efficiency while holding all other variances fixed will improve the result for a turbine efficiency problem. But, it will interfere with the ability of the algorithm to diagnose other faults properly. Also, an increase in the assumed variance of the turbine efficiency will increase its response to other faults.

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### TABLE 1 WEIGHTED-LEAST-SQUARES SOLUTION FOR PURE TURBAN EFFICIENCY FAULT (+1%)

<table>
<thead>
<tr>
<th>Weighted-Least-Squares Parameter</th>
<th>Result</th>
<th>Ideal Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fan Flow Capacity</td>
<td>-0.002</td>
<td>0.0</td>
</tr>
<tr>
<td>Fan Efficiency</td>
<td>-0.046</td>
<td>0.0</td>
</tr>
<tr>
<td>Low Pressure Compressor Flow Capacity</td>
<td>0.092</td>
<td>0.0</td>
</tr>
<tr>
<td>Low Pressure Compressor Efficiency</td>
<td>0.206</td>
<td>0.0</td>
</tr>
<tr>
<td>High Pressure Compressor Flow Capacity</td>
<td>0.049</td>
<td>0.0</td>
</tr>
<tr>
<td>High Pressure Compressor Efficiency</td>
<td>0.274</td>
<td>0.0</td>
</tr>
<tr>
<td>High Pressure Turbine Flow Function</td>
<td>-0.088</td>
<td>0.0</td>
</tr>
<tr>
<td>High Pressure Turbine Efficiency</td>
<td>0.609</td>
<td>1.0</td>
</tr>
<tr>
<td>Low Pressure Turbine Flow Function</td>
<td>0.081</td>
<td>0.0</td>
</tr>
<tr>
<td>Fan Tip Discharge Static Pressure Measurement Error</td>
<td>-0.004</td>
<td>0.0</td>
</tr>
<tr>
<td>Low Pressure Compressor Discharge Static Pressure Measurement Error</td>
<td>-0.013</td>
<td>0.0</td>
</tr>
<tr>
<td>Low Pressure Compressor Discharge Temperature Measurement Error</td>
<td>-0.003</td>
<td>0.0</td>
</tr>
<tr>
<td>Core Speed Measurement Error</td>
<td>0.040</td>
<td>0.0</td>
</tr>
<tr>
<td>High Pressure Compressor Discharge Static Pressure Measurement Error</td>
<td>-0.009</td>
<td>0.0</td>
</tr>
<tr>
<td>High Pressure Compressor Discharge Temperature Measurement Error</td>
<td>0.051</td>
<td>0.0</td>
</tr>
<tr>
<td>Fuel Flow Measurement Error</td>
<td>-0.202</td>
<td>0.0</td>
</tr>
<tr>
<td>High Pressure Turbine Discharge Pressure Measurement Error</td>
<td>-0.096</td>
<td>0.0</td>
</tr>
<tr>
<td>High Pressure Turbine Discharge Temperature Measurement Error</td>
<td>-1.442</td>
<td>0.0</td>
</tr>
<tr>
<td>Thrust Measurement Error</td>
<td>-0.027</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 2 gives the response of the Table 1 algorithm to all faults. Entries in Table 2 represent the part of the solution that correctly responds to the input problem. For example, the last entry in the table indicates that the weighted-least-squares analysis of a +1% thrust measurement error (with no other deviation) gives an inferred thrust error of +0.559%. The remainder of the thrust measurement error is distributed to other measurement errors and to component faults. A table, similar to Table 1, could be produced to show the algorithm's assignment of the thrust measurement error.

Some Table 2 entries are surprisingly small. The high pressure turbine discharge temperature measurement response is less than 40% and other responses are even smaller. These values may be tuned by adjusting the assumptions for measurement error and component standard deviations. If the standard deviation for the high pressure turbine discharge temperature measurement error were increased, its response also would increase. However, this would lower the response of the algorithm to other measurement errors and to state variables.

A large sample of data can be used to establish constraints on the standard deviations. The equation defining these constraints is:

$$ Q = R + HMH^T $$

where:

- **Q** is the covariance matrix of measurement deviations derived from a large sample of test data
- **R** is the covariance matrix for the measurement errors
- **H** is the matrix of influence coefficients, defining the effect of the state variables on the measurements
- **M** is the covariance matrix for state variables

This equation sets limits on **R** and **M**, but does not uniquely define them. The developer of the gas path analysis program must make assumptions about the form of **R** and **M** (for example that certain covariances will be zero) and then use the equation to select values consistent with observations. There is some freedom of interpretation, especially regarding the state variables.

Table 2 suggests that the mean response for faults and measurement errors is about 50%, given the available sensors. This means that half of the solution, on average, will be erroneous. The only way to improve average response is to increase the number of sensors in proportion to the number of state variables being estimated.

Table 2 and Table 1 do not include low pressure turbine efficiency as one of the state variables. When TEMPER was developed, low pressure turbine efficiency was omitted because there was not sufficient instrumentation to distinguish it from the fan. Low pressure turbine efficiency is only the most obvious state variable missing from the analysis. Other faults that could have been included are combustor efficiency and pressure drop, cooling and leakage flows, nozzle performance parameters and setting parameter measurement errors (for instance, inlet temperature and pressure).

In every case, there is not adequate instrumentation to determine these parameters, even if there were no sensor error. If they were included as state variables without providing additional sensors, the average response would be lowered further.

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8 It might also be possible to select a better measurement set that does a better job of separating the faults of interest.

9 While these variables are not included in the basic Weighted-Least-Squares analysis in TEMPER, they are possible choices for the fault logic. However, as pointed out earlier, the fault logic can select the wrong fault because of the unavailability of sensors to distinguish between faults.
When these un-modeled variables change, their changes are interpreted as other state variables or measurement errors. For instance, turbine cooling flow change will be interpreted as a change to the turbine efficiency and flow function. These errors add to the error in the weighted least squares analysis. The only way to improve the situation is to add sensors that distinguish the un-modeled faults from those currently included in the gas path analysis.

![FIGURE 4 MONITORING SENSORS AT COMPRESSOR DISCHARGE](Image)

**SINGLE ELEMENT SENSORS AND DETERIORATION**

Most of the gas path temperatures and pressures are single element probes. The flow fields have large pressure and temperature gradients that vary with power level and operating condition. This is why baselines are required.

One of the underlying assumptions in gas path analysis is that the sensors respond appropriately to component deterioration. Thus, the temperature and pressure probes surrounding the high pressure compressor are expected to provide an accurate indication of the compressor's efficiency as it changes.

Figure 4 shows a sketch of a jet engine compressor. There is a single exit temperature probe for the compressor and a single static pressure probe. The temperature sensor is immediately behind the last stage, nearly flush with the wall surface to minimize the chance for damage if it should fail. The static pressure probe is located in the transition duct to the combustor.

There are many possible sources for compressor deterioration. Engine transients can result in rubs between the rotor blades and the shrouds or between the stators and the rotor land, increasing clearances. Salt, sand or other materials can be ingested by the engine causing erosion of the compressor blades. Pollutants can cause corrosion of the blades that also will alter the surface finish. Rubs can result in splatter on the blades. Foreign or domestic object damage can bend or even break off compressor blades. All of these mechanisms result in reduced compressor efficiency.

Damage can be sustained uniformly throughout the compressor, or can be limited to some stages. It can be uniform around the circumference or can be confined to smaller sectors.

It is difficult to imagine that a single element thermocouple and a single static pressure probe can reflect these damage mechanisms appropriately. This is especially true when there is no testing to establish the optimal placement for these single element probes. Design and placement of gas path sensors must be emphasized in future research if there is to be much improvement in gas path analysis.

**CONTINUOUS VERSUS DISCRETE ANALYSIS**

Diagnosis is a classification problem - its goal is to determine the faulted element(s) of a system that fails to meet its expected performance. This is true whether the object of the analysis is a sick child or a deteriorated jet engine. The objective in either
case is to identify a course of action that will eliminate the problem.

Weighted-least-squares, in contrast, is a quantification tool. It estimates the efficiency and flow capacity of the components and the measurement errors. The results of the analysis are continuous variables - there is no discrete classification of the result. The burden of deciding whether a deviation is significant (warranting maintenance) falls to the user.

This incompatibility of the tool to the problem may seem trivial - even desirable. Shouldn't users decide the threshold for maintenance, rather than the software? Perhaps they should. But the program is evaluated on the success of the chosen maintenance, not the accuracy of the component results and measurement errors. Thus, the user interpretation becomes an essential part of the algorithm.

Different users will reach different conclusions from the same results. Their decisions are often based on a limited understanding of the weighted-least-squares algorithm. For instance, they may not be aware that an indicated EGT measurement error, in concert with an indicated problem in the high pressure turbine, may be solely the result of the efficiency problem. This may lead them to replace the EGT harness in an attempt to regain the performance associated with the measurement error. This will usually be fruitless, and the failure will be attributed to the algorithm.

The variance of user interpretation may be more significant than the problems in the weighted-least-squares algorithm itself. A better gas path analysis tool might take advantage of information such as maintenance history and borescope results in addition to the weighted-least-squares results to develop specific maintenance recommendations. The results should be presented in such a way that users can develop their own conclusions. Often, the results of the program would be used as presented. Several developing technologies (expert systems, neural networks, model-based reasoning) might provide the framework for this type of advance. There now appears to be very limited activity in this area.

SUMMARY

There are several difficulties associated with gas path analysis based on the weighted-least-squares algorithm. The algorithm is ineffective without augmentation (for instance, TEMPER's fault logic). The algorithm, by itself, significantly underestimates a true deviation, attributing half or more of the real problem to other faults and measurement errors.

Input needs for the algorithm are a significant problem. Baselines and statistical inputs require extensive data analysis and careful judgment. Accurate baselines are critical to the augmentation strategies. Also, the data needed to generate the baselines and statistics is not available when the engine is introduced to service. Thus, an alternate, less accurate approach must be used initially or gas path analysis capability must await acquisition of sufficient data to generate baselines and statistics.

There are significant engine components that are not "observable" via the current gas path analysis algorithms. For example, combustor performance, high pressure turbine performance, turbine cooling and overboard leaks are indistinguishable given existing sensor packages.

Single element sensors cannot be expected to respond consistently to the variety of mechanisms causing component deterioration.

None of these problems is likely to be solved by a different algorithm. The blame should be given to the sensors. No algorithm will consistently interpret poor sensor data. Weighted-least-squares settles for the "most probable" solution.

For gas path analysis to succeed, manufacturers will need to supply enough sensors so that all expected component faults are observable. If sensor problems are also likely, redundant measurements will be needed to be able to diagnose the sensors. The sensors must be designed to give a more reliable indication of the plane average pressures and temperatures. Otherwise, baseline generation will continue to limit the effectiveness of the algorithms, especially during early service. Design rules are needed for placement of sensors so component degradation will be reflected in measured temperatures and pressures. These rules must consider the variety of deterioration mechanisms for each component.

There is a problem to be blamed on the weighted-least-squares algorithm, augmented or not. The engine diagnosis needs to be of a form to help the user to identify the appropriate corrective actions. It should not require the user to be expert on the inner functioning of the algorithm to select the correct maintenance plan. If weighted-least-squares is to be used as the core of the gas path analysis, a shell should be provided to convert the results to discrete action recommendations.

The promise of gas path analysis continues to be viewed from afar. If we are to reach that promise, the next major advance must come from the sensor designers. Without advance in sensor technology, further modification of the algorithm can only produce small gains.

REFERENCES


APPENDIX A  MATHEMATICAL BASIS OF
STATISTICAL GAS PATH ANALYSIS

To determine engine component performance from revenue service data, sensor error must be considered. In TEMPER, weighted-least-squares satisfies this need. This appendix gives a brief description of the weighted-least-squares algorithm. For more detail the reader should consult Doel (1992), Bryson and Ho (1975) or Gelb (1974).

TEMPER uses a model of the measurement process of the form:

\[ z = Hx + v \]  \hspace{1cm} (1)

In this equation:
- \( z \) is the measurement vector comprised of the independent \(^p\) measurements on which the analysis is based (e.g., core speed, fuel flow). \( z \) has dimension \( p \).
- \( x \) is the "state vector," composed of the characteristics of the engine expected to vary with time (e.g., compressor efficiency, turbine flow function). The attributes, not expected to change with time (such as nozzle area), are excluded from \( x \). \( x \) has dimension \( n \).
- \( H \) is a \( p \times n \) matrix describing the effect of the state variables upon the measurements. In the jet engine condition monitoring example, \( H \) is derived from a cycle model.
- \( v \) is the vector of measurement noise. Measurement bias is included in the model.

Several assumptions are made to simplify the mathematics. The measurement error is assumed to be Gaussian, with zero mean. \( R \) is defined to be the covariance matrix of the measurement error:

\[ R = E(vv^T) \]  \hspace{1cm} (2)

where \( E \) is the expected value operator. The diagonal elements of \( R \) represent the variances of the measurement errors. The off-diagonal elements, representing the covariance of the measurement errors, are assumed to be zero.

The state variables, \( x \), are also assumed to be Gaussian. Without loss of generality, their mean can be taken to be zero. \( M \) is defined to be the covariance matrix for the state vector:

\[ M = E(xx^T) \]  \hspace{1cm} (3)

Some off-diagonal elements of \( M \) are expected to be non-zero. For example, any mechanism that changes the flow capacity of the fan will almost certainly affect the fan efficiency. However, most off-diagonal elements of \( M \) are taken to be zero.

The measurement error, \( v \), is assumed to be statistically independent of the engine state, \( x \):

\[ E(xv^T) = 0 \]  \hspace{1cm} (4)

With these assumptions, the objective of the weighted least squares analysis is to obtain a "best estimate" of the state of the system from a set of measurements, \( z \). The assumptions allow definition of a Bayesian probability model for the system. A probability density function is defined for the likelihood that the set of measurements, \( z \), originated from an initial state vector, \( x \). The designation for this probability density function is \( P(x|z) \). It can be shown (Gelb, 1974) that \( P(x|z) \) is a decreasing monotonic function of the quadratic form, \( J \):

\[ J = \frac{1}{2} \{ x^TM^{-1}x + (z - Hx)^T R^{-1} (z - Hx) \} \]  \hspace{1cm} (5)

Minimizing \( J \) with respect to \( x \) yields the state vector estimate with highest conditional probability. The minimum \( J \) is found by requiring that \( dJ \) be zero for arbitrary \( dx^T \). Solving for the state estimate, \( x_0 \), gives:

\[ x_0 = (M^{-1}H^TR^{-1}H)^{-1} H^TR^{-1} z \]  \hspace{1cm} (6)

Corresponding to the estimate of the engine state, \( x_0 \), is an estimate of the true measurement vector, \( z_0 \). This is computed using equation 1.

\[ z_0 = H x_0 \]  \hspace{1cm} (7)

This is combined with equation 6 to compute the estimated measurement error, \( v_0 \):

\[ v_0 = [I - H (M^{-1} + H^TR^{-1}H)^{-1} H^TR^{-1}] z \]  \hspace{1cm} (7)

\[ \text{This can be achieved by defining } x \text{ and } z \text{ as deviations from a nominal condition. Thus } z \text{ is not the raw measurement value, } \]  \hspace{1cm} (7)

\[ \text{but is instead the deviation of the measurement from a nominal reference condition, and } x \text{ represents the deviation of the state variables from that same nominal point.} \]  \hspace{1cm} (7)

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The corresponding solution residual, $J_0$, is obtained by substituting $x_o$ into equation 5. It is shown (Bryson and Ho, 1975) that the expected value for $J_0$ is $p/2$ where $p$ is the number of independent measurements. The actual value of $J_0$ will depend on how well the model is able to fit the observed data. As $J_0$ increases, it is less likely that the assumed statistical model (expressed by the engine model, $H$, and the covariance matrices, $M$ and $R$) is correct for the particular data sample. When $J_0$ is much greater than $p/2$, it is reasonable to assume that one or more of the statistical assumptions is incorrect.

The solution residual, $J_0$, provides the means to recognize that a specific case is far from nominal conditions. TEMPER assumes that $J_0$ follows the Chi-Squared distribution. The TEMPER fault logic is invoked whenever the residual exceeds the 95% confidence limit (Figure A-1).

When the fault logic is invoked, a weighted least squares analysis is performed to evaluate each possible hardware fault or sensor error. To evaluate a specific sensor error fault, its standard deviation is increased by a factor of 100. If the large deviation was due to an abnormally large error in the sensor, the solution residual should be reduced to a reasonable level. A similar technique is used for state variable faults (their standard deviations are also increased by a factor of 100). The technique is even applied to faults not included in the original analysis (such as turbine cooling flow or fan speed measurement error). After testing all possible engine faults and measurement errors, the best solution (based on probability) is accepted if its probability exceeds a minimum value (currently 25%).

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14 Actually, $2J_o$ is assumed to obey the Chi-Squared distribution with $p$ degrees of freedom. We are not aware of a proof for this assertion.