ORIGIN OF SPLIT RESONANCE AND BACKWARD WHIRL IN A SIMPLE ROTOR

Knox T. Millsaps
Curtis E. Vejvoda
Naval Postgraduate School
Department of Mechanical Engineering
Monterey, CA 93943, USA

ABSTRACT
Split resonance and backward whirl in a simple rotor were investigated both analytically and experimentally. A two degree-of-freedom rotor model was developed to simulate the steady state, lateral vibration characteristics of a simply supported, single disk rotor. This model included the effects of direct and cross coupled, linear damping and stiffness. The computer model was used to quantify the influence of bearing characteristics on rotordynamic response. In the absence of gyroscopic effects, split resonance is due to separate and distinct natural frequencies in the two orthogonal lateral directions created by non-equal direct stiffnesses. Backward whirl can occur between these two frequencies if the direct damping is sufficiently low. The model was able to predict the observed response of a simply supported rotor, including split resonance and backward whirl. The cause of the asymmetric direct stiffnesses in the experimental apparatus, which created split resonance and backward whirl was investigated. In particular, the influence of geometric imperfections in the plain bearing sleeve, gravitational forces, degree of imbalance and bearing support stiffness asymmetries were isolated using the experimental apparatus. It was determined that the bearing asymmetry was caused by the gravitational influence. However, larger imbalances increased the asymmetry and large damping was able to suppress backward whirl.

NOMENCLATURE

\( c_{ij}, c_{ij} \) Direct and cross damping coefficients. 
\[ \text{[Nm/s]} \text{ or } \text{[lbin/s]} \]

\( k_{ii}, k_{ij} \) Direct and cross stiffness coefficients. 
\[ \text{[N/m]} \text{ or } \text{[lbf/in]} \]

\( e \) Base of natural logarithm

\( \text{em} \) Residual unbalance of rotor. 
\[ \text{[kgm]} \text{ or } \text{[lb m in]} \]

\( M \) Modal mass of rotor. [kg] or [lbm]

\( t \) Time. [sec]

\( x, y, z \) Cartesian coordinates. [m] or [in]

\( \omega \) Angular velocity of shaft. [rads/sec]

\( i = \sqrt{-1} \)

1. INTRODUCTION
Classical rotordynamic theory [1] predicts that the whirl orbits of a long, slender shaft due to residual unbalance, consist of circular processional motion of the center of the shaft about a line extending from one bearing center to the other. The whirl direction and speed are identically the same as the spin direction and the rotational speed of the shaft respectively. This is referred to as forward synchronous whirl. As the speed of the rotor is increased from zero slowly, the circular orbits should increase in magnitude from zero until the first lateral critical speed is reached - the resonant speed. The magnitude of the response is limited only by damping. Further increases in rotational speed cause the whirl amplitude to decrease, asymptotically approaching some finite whirl orbit amplitude at higher speeds. As the rotor transitions through critical speed, the phase between the eccentric mass and the displacement goes from zero to \( -\pi \) radians. That is, it goes from heavy-side out to heavy-side in.

The possibility of synchronous whirling motion proceeding in the opposite direction as spin, which is commonly referred to as backward whirl, retrograde precession or reverse whirl, has been noted. In particular, Den Hartog [2] has explained that it is possible for it to occur, but indicates that it is rarely seen in operating machines.

However, it has been noted by several authors, including Loewy [3], Ehrich [4], Rao [5] and Vance [6], that backward rotor whirl is to be expected in cases where there is large asymmetry in the bearing direct stiffnesses. The various authors present models which can simulate backward whirl.
Another cause of split resonance and backward whirl is gyroscopic forces generated by disks, especially overhung ones.

It is also possible for backward whirl to occur at a shaft natural frequency due to an instability mechanism. This generally occurs at sub-synchronous speeds.

In a recent experimental investigation by Simei [7], a simply supported, centrally located single disk mounted on a long rotor, supported by sleeve bearings, clearly exhibited backward whirl. It occurred between two distinct resonances, one in the horizontal direction and one in the vertical direction, in a system that appeared to have symmetry.

The purpose of this investigation was to determine the physical mechanisms that cause split resonance and backward whirl in a simple rotor system in the absence of gyroscopic forces. To do this, both analytical and experimental methods were used. A simple two degree-of-freedom, linear analytical model, including direct and cross stiffnesses and damping, was developed to assess which parameters control the split resonance and backward whirl. While it was found that many parameters control whether backward whirl will occur, a difference in direct stiffnesses in two mutually orthogonal directions is the single dominating parameter. The model was used to back out the required rotordynamic coefficients to match the experimentally obtained orbits.

The major focus was then to determine what created the asymmetries in the rotor system that was nominally symmetric. Several possibilities were investigated experimentally, including imbalances in the circularity of the sleeve bearing, the mounting structure of the bearing support, gravity, magnitude of residual imbalance and damping. Each of the proposed deviations was isolated and its impact on the rotor behavior was determined. The experimental program will be described along with several auxiliary measurements that were made to isolate the sources of these phenomena.

It was determined that gravity is responsible for creating a greater bearing direct stiffness in the vertical direction than in the horizontal direction. However, the magnitude of imbalance and the damping also had an influence on whether backward whirl occurred.

2. DESCRIPTION OF EXPERIMENTAL FACILITY

The experimental facility consists of a rotor kit mounted on an isolation table and a PC based data acquisition system. A Photograph of it is shown in Figure 1.

The Bently-Nevada rotor kit consists of a base plate on which is mounted a controllable speed motor, two adjustable bearing support pedestals and another support for holding (x-y) eddy-current proximeters. The shaft, which is driven by the motor through a flexible coupling, is supported by two plain sleeve bearings and has an adjustable disk to which small masses can be attached. The nominal characteristics of the rotor for the conditions to be presented in this paper are given in Table 1. For further details consult [7].

3. SPLT RESONANCE & BACKWARD WHIRL

The orbits measured in the experimental facility for the simple rotor were quite different than would be expected from classical rotodynamic theory as outlined in the Introduction.

In particular, while the orbits started out nearly circular in the clockwise direction (same as the spin direction) at speeds that were low relative to the critical speed, they quickly became elongated at higher speeds, as shown in Figure 2. As the first critical speed (in the horizontal direction) was approached, the "orbit" collapsed into a line. As the speed was further increased, the line expanded to an orbit again, but the direction of the whirl had changed to CCW; the rotor was whirling in the opposite direction as the spin. This is backward whirl. The orbits in this speed range are given in Figure 3. With further increases in speed, the major axis of the orbits rotated CW and collapsed again to a line near the second critical speed (in the vertical direction). At higher speeds, forward whirl (CW) occurred again. The orbits became smaller and more circular as shown in Figure 4.

4. ANALYTICAL MODEL AND PREDICTIONS.

4.1 Model Development

In order to determine the bearing forces that are required to create the complex response as described above, a simple two degree-of-freedom model for a rotor spinning at constant speed was developed. The model replaces the rotor with a compressed disk. Implicit in this model is that both bearings are identical and the orbit motion at one axial location is related to that at any other location by a modal shape factor. The validity of this assumption was confirmed with the experimental apparatus.

Figure 5 shows a schematic of the rotor model with the associated forces. The coupled equations of motion are given by

\[
\begin{bmatrix}
M & 0 \\
0 & M
\end{bmatrix}\begin{bmatrix}
x'(t) \\
y'(t)
\end{bmatrix} = \begin{bmatrix}
x''(t) \\
y''(t)
\end{bmatrix} + \begin{bmatrix}
f_x \sin t \\
f_y \sin t
\end{bmatrix} \times \omega^2, \quad \omega = \text{constant}
\]

where \(M\) is the modal mass of the disk and shaft assembly, the \(c\)'s and \(k\)'s are the linear damping and stiffness coefficients, \(x\) and \(y\) are the horizontal and vertical coordinates respectively, and the right hand side is the forcing due to residual imbalance. One can assume a solution of the form

\[
\begin{bmatrix}
x(t) \\
y(t)
\end{bmatrix} = \begin{bmatrix}
x_0 \\
y_0
\end{bmatrix} e^{\alpha t}
\]

Substituting (2) into (1), one can solve algebraically for a solution vector

\[
\begin{bmatrix}
x_0 \\
y_0
\end{bmatrix} = \begin{bmatrix}
x_m \\
y_m
\end{bmatrix} \alpha^2, \quad \omega^2 = \begin{bmatrix}
x_m \\
y_m
\end{bmatrix} \begin{bmatrix}
x''_m \\
y''_m
\end{bmatrix}^{-1} \left\{\begin{bmatrix}
x_i \\
y_i
\end{bmatrix} e^{\alpha t}\right\}
\]

4.2 General Predictions:

Split Resonance and Backward Whirl

This model was used to parametrically investigate the impact of the various rotordynamic coefficients on the orbits. Specific emphasis was placed on discovering which parameters control split resonance and backward whirl. It was only necessary to use different values of the direct stiffnesses, \(k_{xx}\) and \(k_{yy}\), to obtain the general behavior that was observed. The direct damping also played a role, in that at sufficiently high values of damping the backward whirl was
suppressed. However, the split resonance still occurred at these elevated damping levels. Cross stiffness and damping were not needed to explain the phenomena and were not capable of creating it. However, they do slightly change the shape and rotational orientation of the orbits. They may be used for fine turning the model results to match specific experimental results.

4.3 Matching Experimental Results.

Combinations of rotordynamic coefficients were sought that could reproduce the orbits obtained in the experiment. The matching was done largely by trial and error. A single set of coefficients was obtained that is capable of closely reproducing the orbits obtained experimentally from zero speed up to nearly twice the first bending critical speed. Above this speed range, multiple loop orbits and other complex nonlinear phenomena occurred that cannot be reproduced with this linear model. While it appears that the solution set was unique, no proof of this was made. A formal parameter identification procedure to extract coefficients based on experimentally obtained orbits is theoretically possible and is being pursued. It is possible to use slightly different coefficients in different speed ranges, since the rotordynamic coefficients are functions of shaft speed. However, since the emphasis was on behavior near the critical speeds where split resonance and backward whirl occurred, the coefficients were matched to reproduce the response in this speed range. To match the response throughout the entire range required variations in rotordynamic coefficients of less than 10%.

The mass of the model disk, M, was set to be the modal mass of the shaft/disk combination using the experimentally obtained shape function. The direct stiffness coefficients, \( k_{xx} \) and \( k_{yy} \), were chosen to match the horizontal and vertical natural frequencies of the rotating shaft. These were very close to what was expected based on the modal stiffness of the shaft and available stiffness data on sleeve bearings. The direct damping coefficients were selected to match the maximum response amplitude.

While it was necessary to have different direct stiffness coefficients, it was not necessary to use asymmetric direct damping coefficients. Slight differences in \( c_{xx} \) and \( c_{yy} \) do not substantially alter the orbit response and therefore symmetric damping was chosen for the matching condition. No cross coefficients were needed to match the orbits.

Figure 6 shows the model results over a range of speeds near the split resonance with the value of the rotordynamic coefficients used to replicate the experimental orbits. Figure 7 shows the experimentally obtained orbits for comparison. The match is quite good. However, the model yields "cleaner" orbits as one might expect.

4. CAUSE OF DIRECT STIFFNESS ASYMMETRY

Having determined from the model that split resonance and backward whirl are a direct consequence of the difference in direct stiffnesses in two orthogonal directions, a physical reason for this difference in the hardware was sought. Several candidate causes for the asymmetry were hypothesized and experimentally isolated. Non-circularity of the bearings, the influence of gravity, degree of imbalance and bearing support asymmetries were all investigated as to their influence on the rotor orbits.

First, however, preliminary structural response tests were conducted to determine if two distinct natural frequencies could be detected in the non-rotating system and if they were the same as those at which the maximum x and y response occurred in the rotating system. Tests were done to determine lateral natural frequencies. Both proximeters and accelerometers were used. Nearly linear motion could be accomplished with a careful hit with a modal hammer. The x and y natural frequencies (which were 46.60 hz and 47.75 hz respectively) corresponded to the ones obtained for the maximum x and y response to within 2%. They were slightly lower, which is consistent with the parameter matching which suggested that the direct stiffnesses increase very slightly with rotor speed.

As mentioned in the Introduction, split resonance and backward whirl can occur due to gyroscopic coupling. To examine whether gyroscopic forces were important, the central disk was moved about its nominally centered location. No changes in the orbits were observed for small changes in the disk location. For larger excitations the general behavior was unchanged but the resonant frequencies were increased as expected.

Non-Circularity of the Bearings

Since one bearing from the initial test was measured and found to be slightly non-circular, it was thought that one possible cause of the different \( k_{xx}, k_{yy} \) could be geometric imperfections in the bearings. The nominal clearance was about 0.001" on a 0.375" shaft. Therefore a non-circularity of 1/1000 creates a gap non-uniformity of 50%.

Several new pairs of spherical bearing sleeve inserts were machined with a range of eccentricities. A parametric investigation, using various amounts of imperfections and clocking orientation of both bearings with respect to the vertical was made. While the new bearings did affect the orbits near the split resonance and backward whirl speeds somewhat, they appeared to neither create nor eliminate the general phenomena of split resonance or backward whirl. Striking differences in orbits were observed near twice the first critical speeds from the imperfect bearings. A rich array of complex orbits were created near these speeds [8].

The uniformity of surface finishes inside the bearing were also considered. While this did have an influence on the super critical behavior, again near twice the natural frequency, no substantial difference in the orbits near the critical speeds was observed.

Bearing Support Asymmetries

The support structures holding the bearings are geometrically non-symmetric as can be seen in Figure 1. Several augmented supports were tried to assess their impact. No discernible change in orbits was detected when changing the supports. The reason is that the bearing compliance was much greater than that of the support structure. In systems where the bearing supports are more compliant relative to the bearings, a different conclusion seems likely.

Influence of Gravity

The influence of gravity was considered to be one likely cause of stiffness asymmetry, because the principle directions
of maximum and minimum stiffness were aligned in the horizontal and vertical (gravity) planes.

In order to isolate the influence of gravity the rotor was mounted in three different orientations on the table.

1. Flat, with the gravity vector acting in the y-direction.
2. On the side, with the gravity acting in the x-direction.
3. On end, with gravity acting down the shaft (z-direction).

Orbits were recorded in all three orientations, for various degrees of imbalance forcing through the entire speed range. For cases with over 2.84 gram-cm of rotor imbalance the split resonance and backward whirl was observed for both the flat and the on-side cases.

The orbits for these two cases were very similar except for a 90 degree rotation. When the original x-y probes were interchanged the same orbits are observed. This means that in both cases the higher frequency and therefore the higher direct stiffness was in the direction of gravity. This provides strong evidence that gravity causes the difference in k_{xx} and k_{yy}.

Finally, the rotor was placed on end where the gravity was acting down the shaft. No distinct split resonance nor backward whirl was observed for any conditions. Concern has been expressed about the possibility of rotordynamic instability with the essentially unloaded bearings. No direct evidence of this was observed in the speed range tested. Cascade plots revealed no vibrational energy at system natural frequencies, which is a typical sign of such problems.

The mechanism by which gravity changes the direct stiffness is not understood in general. Obviously, when the rotor is sitting at the bottom of the bearing, the direct stiffness downward (-y) is quite high compared with the +x direction where the rotor will roll or slide up the bearing. However, in the +y direction there is no static resistance to movement, except the weight of the rotor. The y-direction dead-band creates much uncertainty on how to calculate the differences in the two direct stiffnesses. Further complicating the matter is the participation of any lubricating forces. The type of boundary forces will depend strongly on the lubrication regime along with other parameters. A detailed exploration of this is certainly warranted, but is beyond the scope of the present investigation.

Degree of Imbalance.

Different imbalance weights were placed in the rotor disk to access the impact of the synchronous forcing magnitude on rotor response. Masses from 0 to 1.6 grams were placed in the disk at a radius of 2.84 cm. The different weights did create very distinct differences in the type of orbits created near the critical speeds. In particular, the higher the forcing the more a rotor tended to have a distinct split resonance and go into backward whirl. The difference in the direct stiffnesses seems to be accentuated at the higher imbalances. Also, the amount of imbalance changed the response near twice the first critical speed dramatically.

Linear theory predicts that the response of a linear system is proportional to the forcing amplitude. Here the coefficients are amplitude dependent. While this system is clearly nonlinear, this does not seriously limit the applicability of the linear model in a sense. It is able to predict the important underlying physics. The system behaves like a linear system at a given forcing amplitude over a wide speed range. It can be considered a quasi-linear system near the critical speed.

Away from the critical region, the response was very nearly proportional to the forcing amplitude, as linear theory predicts, except at near twice the natural frequency.

Damping

Finally the influence of damping was investigated experimentally. As predicted by the model, sufficiently high values of damping eliminated the backward whirl. The damping was altered by adding damping material to the mounting assembly. At a given level of imbalance, the backward whirl could be suppressed by increasing the damping as predicted in the model. At the elevated damping backward whirl could again be produced when a greater imbalance was added. This is a qualitative result.

5. CONCLUSIONS

The simple, 2-DOF, linear model is capable of predicting the complex orbits that were observed in the neighborhood of the critical speeds, including the split resonance and backward whirl. The model predicts that differences in the direct stiffness coefficients will create backward whirl between the two critical speeds, if the direct damping is sufficiently low.

The model was able to match the experimental rotor orbits closely over a wide range of speeds.

Gravity appears to be the source of the different values of direct stiffness. The vertical (in the direction of gravity) stiffness was higher, regardless of the orientation of the rotor base. The actual mechanism which causes this has not been determined. Secondary parameters which influenced whether backward whirl occurred, along with the split resonance, were the degree of imbalance and the damping. Larger imbalances led to increased differences in the direct stiffnesses which is a nonlinear effect.

6. ACKNOWLEDGMENTS

The authors wish to acknowledge the help of Mr. D. Bently and Dr. A. Muszynska of Bentey-Nevada Corporation and the reviewers who made valuable suggestions. This work was sponsored by the Naval Sea Systems Command (SEA-03X).

7. REFERENCES

Total length of shaft: 45.7 cm
Distance between bearings: 35.6 cm
Diameter of shaft: 0.9525 cm
Shaft material: 4140 LA Steel
Nominal bearing clearance: 0.00254 cm
Size of disk: 7.62 dia X 2.45 cm
Mass of disk: 0.8090 kg
Disk Position: Centered - 17.8 cm
Length of bearing: 0.6350 cm
Bearing type: Plain oilyte bronze

Table 1. Properties of rotor
Figure 2. Experimental orbits for low speeds.

Figure 3. Experimental orbits near critical speed.

Figure 4. Experimental orbits for high speeds.

Figure 5. Theoretical model of Rotor.

Figure 6. Model orbits to match Figure 7.

Figure 7. Experimental rotor orbits near critical.