MULTILEVEL OPTIMIZATION PROCEDURES
FOR GAS TURBINE DESIGN:
PART II — SOLUTION AND EXAMPLES

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ABSTRACT

The mathematical formulations of two procedures for breaking down a large optimization energy problem into a hierarchy of separated but coupled subproblems were presented and discussed in the first part of this work (Ghiglino and Massardo, 1994). Here their applicability is illustrated by formulation of realistic analysis and design problems in the field of gas turbine systems, both for open and closed cycle plants. The definition of objective functions, design variables, constraints and subproblem analyses is discussed; the results obtained with the sensitivity, derivative procedure are compared with the results obtained with the linearization procedure.

NOMENCLATURE

- As nondimensional specific compressor area
- Ai weighting factor
- C nondimensional cost
- cf friction coefficient
- c, stall margin coefficient
- D diameter
- dh hydraulic diameter
- F objective function
- h, l height and length respectively
- Li recuperator overall length
- m mass flow rate
- Mo Mach number
- N number of tubes, banks, passes
- o throat
- p, Ap pressure, pressure loss respectively
- P power
- Pr Prandtl number
- R radius
- Re Reynolds number
- T temperature
- t thickness
- z blade number
- w relative velocity
- α, β absolute and relative flow angle respectively
- βc compressor pressure ratio
- εt turbine expansion ratio
- e, recuperator effectiveness
- η efficiency

Subscripts

1,2 compressor inlet, outlet respectively
3,4 turbine inlet, outlet respectively
c, t compressor, turbine respectively
ei electrical
TES Thermal Energy Storage

INTRODUCTION

In recent years, considerable research has been devoted to the development of multilevel optimization methods for structural system design. Two principal motives are offered for this effort: the first is that the engineering organizations operate in multilevel and multidiscipline organizational structure. The second motivation is to effectively break the overall optimization task into smaller parts that are more amenable to a solution using numerical optimization methods.

Much of the work, proposed for structural engineering, has followed the concepts presented by Sobieszczanski-Sobieski (1986) and Vanderplaats (1990). Nevertheless, these procedures have not yet been widely adopted in the field of energy systems.

So the main objective of this work is the exploitation of a multilevel optimization approach for gas-turbine design, improving the all-in-one optimization procedures.

In the first part of this work (Ghiglino and Massardo, 1994) the mathematical formulation of two different multilevel optimization procedures was presented and discussed. Here the implementation of both the procedures is described, while the applicability of developed codes is illustrated by formulation of realistic problems in the field of gas turbines (thermoeconomic sensitivity analysis of an open cycle gas-turbine; optimum sensitivity analysis of the design of an axial compressor stage; multilevel optimization of a Closed Brayton Cycle system for space power generation).
CODES FOR THE MULTILEVEL OPTIMIZATION OF ENERGY SYSTEMS

Two codes based on the mathematical formulation of two different multilevel optimization procedures (Ghiglino and Massardo, 1994), have been developed. They are named "SENSITIVITY" and "LINEAR" respectively and can operate on 8086 or RISC computers. Both the codes have been written for two-level optimization problems, however they can be easily modified to include three or more levels.

Two well-known structural problems, already utilized in the field of multilevel optimization (Sobieski 1989; Vanderplaast, 1990), have been utilized to test the codes. The results, presented by Ghiglino (1993), agree well with the already cited published data.

a) code "SENSITIVITY"

The code refers to the sensitivity derivatives procedure described by Ghiglino and Massardo (1994). In this approach the correlation between the system and the subsystems of the multilevel problem is evaluated through the optimum sensitivity analysis. That is the evaluation in the subproblem optimum point of the sensitivity derivatives of the objective function and of the variables in relation to the parameters (coincident with the design variables of the system).

Fig. 1 shows the simplified flow chart of the code for a two-level optimization problem. The program starts with the initialization of the system and subsystems design variables; then the unfeasible starting condition is verified as requested by the correct definition of the subproblem objective functions. If the starting point is feasible the chosen starting data must be modified, until an unfeasible condition is reached.

In the next step the subproblem analyses are carried out. Utilizing an exterior penalty function formulation the subproblem penalty function, coincident with the subproblem objective function, is evaluated and, at the same time the non equality constraints are calculated. The minimization of the subsystem objective function is obtained by utilizing a classical Powell algorithm, taking into account the lateral constraints on the subsystem parameters (they are coincident with the system variables). At this point the lateral active constraints are considered and in the subsystem optimum point their gradients, with those of the penalty function, of the equality constraints and of the subsystem variables, are evaluated. The Hessian of these functions are also calculated, always in the subsystem optimum point, in relation to the subsystem variables, while the mixed Hessian is evaluated in relation to the parameters and the variables of the subsystem.

In the next step of the code a linearity - independence check on the components of the calculated gradient vectors is carried out. If this condition is not verified a Gram test is performed and the nonlinearly independent vector components are eliminated. In this way only the linearly independent components of the gradients and Hessian are utilized in the solution of the system equations (8) of the first part of this work (Ghiglino and Massardo, 1994).

The results are the Lagrange multiplier sensitivity derivatives in relation to the subsystem parameters and similarly the subsystem design variable sensitivity derivatives. Utilizing this information the linearized forms of the subsystem penalty function and variables are calculated and transferred to the system.

Obviously all the previous steps must be repeated for each subsystem presents in the hierarchy tree.

At the end of the subproblem analyses, the system objective function minimum is evaluated by utilizing the Powell algorithm again. In this minimization the influence of system non-equality constraints, of the linearized subsystem penalty functions, of the system lateral constraints and of the limits connected to the subsystem variables linear extrapolation, are taken into account.

In this way the optimum point of the system variables can be obtained. If the system objective function does not reduce its value or the linearized form of the subsystem penalty functions are all less than zero the convergence criterion is satisfied. Otherwise all the procedure is repeated utilizing a new system starting point, coincident with the point calculated in the last iteration.

More details of the code are given by Ghiglino (1993).

b) code "LINEAR"

The code refers to the multilevel optimization procedure where the exchange of the information between the different levels of the hierarchy tree is performed utilizing linear extrapolations of the system and subsystems active constraints.

Fig. 2 shows the simplified flow chart of the code "LINEAR" for a two-level problem. The definition of the problem starting point is the first step of the code, that is the system and subsystem design variables are defined with their numerical values. Then all the constraints of the problem are calculated in this starting point. This allows the feasibility of this point to be verified. If all the constraints are less than zero the starting point is feasible and the calculation continues. If the starting point is unfeasible it is necessary to define another starting point (another set of the design variables).

In the next step, in the first iteration, the gradients of all the system constraints are calculated, while in the subsequent iterations...
the gradients are calculated only for the active system constraints. Obviously the gradients are evaluated at the starting point at the first iteration and in the system optimum point previously calculated in subsequent iterations. Then the linear extrapolation of the system constraints are calculated by utilizing the cited gradients.

In the next step the subsystem analysis starts, the subsystem design variables are chosen with their constraints. The minimization of the physical subsystem objective function in carried out utilizing the Powell algorithm again. In this minimization both subsystem constraints and the linearized forms of the system constraints are considered. In this way the optimum value of the subsystem objective function and of the subsystem design variables are defined. Then the code evaluates the active constraints of the subsystem, they will be utilized in the following part of the procedure. The next step is similar to one already utilized for the calculation of the gradient of the active constraints of the system. Here it is utilized for the calculation of the gradients of the subsystem. Then the linearized form of the active constraints of the subsystem is evaluated utilizing the already cited gradients; the linearized form is centered in the subsystem optimum point.

All the previous steps must be repeated for each subsystem of the hierarchy tree. At the end of the subsystem analysis, the system (top-level) is considered. Utilizing the Powell algorithm the optimum point of the system objective function is calculated; this minimization is subject to the system constraints and to the linearized forms of the subsystem active constraints. At the end of this step the optimum set of the system design variables is available.

The last step is the convergence criterion: the calculation converges if the system objective function does not reduce its value or if all the system constraints are satisfied. Otherwise, a check on the active system constraints is performed and another iteration of the whole procedure is carried out. Apt checks on the subsystem variable modifications are included in the code; more details of the program are given by Ghiglino (1993).

APPLICATIONS

The first example consists of an optimum sensitivity analysis applied to a thermoeconomic gas turbine approach described by Frangopoulos (1988). The thermoeconomic analysis allows the evaluation of the optimum efficiency and cost of the plant to be obtained, utilizing an all - in - one optimization procedure. The objective function to be minimized is the following multiple objective function:

\[ F = A_1 \cdot \dot{C} + A_2 \cdot (1 - \eta) \]

where \( A_1 \) and \( A_2 \) are apt weighting factors for the two nondimensional functions that compose the objective (the plant losses and the plant cost).

In the analysis the design variables are: compressor pressure ratio; turbine inlet temperature; compressor efficiency, turbine efficiency; combustion chamber efficiency. The ambient conditions and the plant power (20 MW) are considered fixed. The thermodynamic and economic characteristics of the plant are obtained using an apt code developed by Ghiglino (1993). The objective function minimization is carried out utilizing the Powell algorithm. Lateral and non - equality constraints are considered in a penalty function approach.

The calculation can be repeated for various couples of the factors \( A_1 \) and \( A_2 \); in this way the effect of the parameter \( A_1 \) is considered utilizing a reoptimization procedure, unfortunately this approach is time consuming, particularly if compared to the optimum sensitivity analysis approach.

Optimum sensitivity analysis can be used to evaluate the influence of each of the multiple objectives on the constrained minimum. Since each objective function weighting factor is a parameter for the problem, the sensitivity of the optimum in relation to the \( A_1 \) factors can be obtained to determine the influence these factors have on the optimum and the trend associated with changes to the \( A_1 \) values.

It is conceivable that it may be possible to use the derivatives in relation to \( A_1 \) for the extrapolation of \( F \) and design variables values to estimate their magnitude for \( A_1 = 1 \) and \( A_2 = 0 \) and viceversa or for intermediate values. Indeed, this would provide estimates of single objective optimization for each of the objectives, at a computational cost not much larger than the cost of a single computation.

Fig.3 shows the objective function behavior versus the weighting factors \( A_1 \) and \( A_2 \); the solid line represents the result obtained with the reoptimization approach, while the dotted line represents those obtained utilizing optimum sensitivity derivatives. The agreement between the results obtained utilizing these different approaches is particularly good in all the field. This fact is due to the high linearity of the analyzed objective function. Fig. 4 shows, as an example, the behavior of a design variable, the compressor pressure ratio, obtained utilizing both the methods. In this case the function is not particularly linear, and the function extrapolation obtained utilizing sensitivity derivatives must be restricted around the starting optimum point utilized to evaluate the derivatives.

Fig.2: Linear analysis flow chart.
described by Massardo (1993), here the design is performed utilizing and the SENSITIVITY and LINEAR codes.

Another example is the application of the optimum sensitivity analysis to the optimum design of an axial compressor stage as described by Massardo and Satta (1990). In this case the objective function is composed of three different and conflicting quantities: the stage efficiency; the stage frontal area; the stage stall margin.

The described codes are both part of the multilevel procedures; to continue in this example it is now necessary to differentiate the description.

a) procedure SENSITIVITY

In this case the system design variable are well distinct from the subsystem design variables. They are (see tab.1): recuperator effectiveness; compressor pressure ratio; compressor inlet temperature; compressor inlet pressure; hot and cold side recuperator pressure losses; gas side gas-cooler pressure losses.

The CBC system has been described in depth by Massardo (1993), and fig.6 shows a Venn diagram of the plant. A two level break down approach has been utilized: the top level coincides with the whole CBC plant (system), the bottom level coincides with the subsystems (compressor, turbine, recuperator, gas cooler). In this example the solar receiver is considered only at the top level utilizing a simplified analysis, while the radiator calculation is included in the gas cooler analysis.

Both the multilevel procedures need apt computer codes to evaluate the system and subsystems performance. The code named DES (Massardo, 1993) is utilized to obtain the whole CBC plant performance without the need to know the detailed geometrical data of the plant components. At the subsystem level the routines that constitute the code WGHT (Massardo, 1993) are utilized to evaluate for any subsystem the performance and the geometry.

From the analysis of all these examples, presented in a complete manner by Ghiglino (1993), it appears that the optimum sensitivity analysis, which is the most important part of the homonymous multilevel procedure, has the potential of providing useful estimates of the results of many single objective optimizations by extrapolating from the results obtained by executing only one optimization with a composite (conflicting) objective function.

The optimum design of a solar space Closed Brayton Cycle (CBC) system is the third example. The all-in-one design has been described by Massardo (1993), here the design is performed utilizing both the multilevel optimization procedures previously described and the SENSITIVITY and LINEAR codes.
The break down also requires that the compressor delivery pressure and temperature values are constant and equal to the values already calculated at the system level. This means that the results obtained with the codes DES and COMPREX must be the same. If a difference exists it must be included in a penalty function of the subsystem. On the contrary if all the constraints are verified the power value is also equal at the two levels, because the mass flow rate is considered a fixed datum of the whole problem.

Likewise, fig. 8 shows the scheme for the turbine subsystem. In this case the quantities transferred from the system level to the subsystem are: turbine efficiency, which is a parameter for all the calculations, turbine delivery pressure, inlet turbine temperature and pressure, mass flow rate, which takes into account the bleed flows (Massardo, 1993), compressor power, net power. Compressor power value is calculated at the system level to avoid the presence of a lateral interaction between the compressor and the turbine subsystems.

The equality constraints are coincident with the net power and the delivery pressure values. These quantities are evaluated at the subsystem level with the code TURBO and compared with their correspondent values at the system level (obtained with the code DES).

The objective function for any subsystem consists in a penalty function based on the equality and non equality active constraints in the analyzed subsystem. The minimization at the subsystem level is therefore coincident with the minimization of the violations of these constraints.

To improve the knowledge of the information exchange between system level and subsystems fig. 9 shows a simplified scheme which referred to the turbine subsystem. It shows clearly the interface between the system and subsystem, the calculation needed to evaluate subsystem performance, geometry and active constraints.
It is important to observe that with the detailed turbine (or compressor, etc.) calculation (code TURBO, COMPREX, etc.), when all the constraints are verified, the designer obtains the detailed data of a single component of the whole plant that satisfies all the data fixed or calculated at the preliminary step (system analysis).

**System:**
- Working fluid characteristics;
- Concentrator excess ratio;
- Bleed flow rate; radiator emissivity; cold plate heat;
- \( \eta_r; \eta_i; \eta_{ai}; (\Delta T) \) _component;
- \( (\Delta p) \) _component;
- \( T_{inlet, compressor}; T_{intake, turbine}; T_{intake,_solar} \)

**Subsystem**
- Cooling fluid characteristics;
- Turbine, compressor, recuperator, gas cooler materials;
- \( (z_{rel})_{turbine}; (z_{rel})_{compressor}; (z_{rel})_{recuperator}; (z_{rel})_{gas cooler} \);
- \( (t_{max})_{compressor}; (t_{max})_{recuperator}; (t_{max})_{gas cooler} \);
- \( G_{ref} / G_{gain}; G_{ref} / G_{gain} \);
- \( N_{source}; N_{aten}; N_{source}; N_{atten} \).

Tab. 2: Fixed parameters.

A similar consideration on the equality constraints and subsystems analysis can be made for the recuperator and the gas cooler. More details on this subject may be found in Ghiglino (1993).

b) Procedure "LINEAR".

In this case, fig. 6 is still valid, because the break down approach is similar to those of the previous procedure. LINEAR, as already stated, utilizes the codes DES, TURBO, COMPREX, RECUP and GASCOOL to evaluate system and subsystems performance.

The system design vector \( \{X\} \) contains now a large number of variables, in fact it also contains the subsystem design variables:

\[
\{X\} = \{\{x\}, \{y\}, \{z\}, \{w\}, \{v\}\} \quad \text{(see tab 1)}.
\]

Vector \( \{x\} \) contains only the system design variables, vector \( \{y\} \) the turbine subsystem variables and so on.

At the system level the objective function is coincident with the function utilized in the procedure SENSITIVITY, suitably augmented by the penalty functions built with the active constraints at the system level and the linearization of the active subsystem constraints.

System and subsystems non equality constraints are the same as in the previous case (see tab 3), but the lateral constraint values on the subsystem variables are reduced to allow the subsystem active constraint extrapolation to be always valid.

As for subsystem objective functions, now they do not coincide with penalty functions, but can be chosen in the field of subsystems performance (efficiency, mass, surface, etc.).

In this example the system objective function is the same for both the procedures and it is coincident with the total surface of the space system (concentrator and radiator surface). In the LINEAR procedure the subsystem objective functions are the component masses.

\[
\begin{align*}
& g_1 < O_1 < g_2 < \ldots < g_i < \beta_{2ab} < g_4 < \ldots < g_j < \beta_{2ab} < g_6 < \ldots \\
& \ldots g_T < R_{2ab} / R_1 < g_4 < \beta_{2ab} / R_{2ab} < g_1 \ldots \\
& \ldots M_{react} > g_{11} \ldots g_{13} < g_{14} < k_n < g_{15} \ldots \\
& g_{16} < D_{asp} / D_{mean} < g_{17} < g_{18} < \beta_{2ab} / D_{mean} < g_{19} \ldots \\
& g_{20} < O_{as} < g_{21} \ldots g_{22} < \beta_{2ab} < g_{23} \ldots \\
& \ldots g_{24} < D_{3} / D_{2} < g_{25} \ldots g_{26} < D_{3} / D_{1} < g_{27} \ldots \\
& g_{28} < D_{asp} / D_{mean} < g_{29} \ldots g_{30} < b_1 / b_2 < g_{31} \ldots \\
& \ldots M_{as} < g_{32} \ldots M_{as} < g_{33} < a_2 < g_{34} \ldots \\
& \ldots g_{35} < W_2 / W_{as} < g_{36} < L_1 / L_2 < g_{37} \ldots \\
& g_{38} < L_1 / L_2 < L_3 < g_{39} < L_1 / L_2 < g_{40} \ldots \\
& g_{41} < L_3 \ldots g_{42} < L_3 \ldots \ldots < g_{43} \ldots \\
& \ldots g_{44} < \Pi_{mean} < g_{45} < g_{51} \ldots \ldots < g_{52} \ldots \\
\end{align*}
\]

Lateral constraints \( (x' < x < x'') \) on design variables.

Tab. 3: Nonequality constraints.

Fig. 6: Venn diagram of a two level decomposition of a solar space Closed Brayton Cycle (CBC).
The system design optimum values are shown in Tab. 4, where the results obtained by Spencer and Bons (1989) are also shown. The results obtained with the use of the two multilevel procedures are very similar, while some differences are evident if they are compared to the results of Spencer and Bons. Probably these differences are due to the off-design approach utilized by the cited authors, as discussed by Massardo (1993).

The difference of the multilevel procedure results is responsible of a small reduction of the cycle efficiency (about 0.3%). This reduction is directly correlated to the subsystem influence on the system optimization.

The calculation of the subsystem performance was obtained without lateral and nonequality constraints violation. This allows a correct design of the component to be obtained.

For the SENSITIVITY procedure the equality constraints management is resulted not easy; some small violations are present in the final calculation (violation of turbine efficiency constraint 0.3%; compressor efficiency 0.29%; compressor pressure ratio 0.19%, etc.).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sensitivity procedure</th>
<th>Linear procedure</th>
<th>Spencer and Bons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$ (kPa)</td>
<td>291.3</td>
<td>289.0</td>
<td>294.10</td>
</tr>
<tr>
<td>$T_1$ (K)</td>
<td>296.7</td>
<td>299.3</td>
<td>294.03</td>
</tr>
<tr>
<td>$\beta_e$ (-)</td>
<td>1.93</td>
<td>1.87</td>
<td>1.8494</td>
</tr>
<tr>
<td>$\varepsilon_m$ (-)</td>
<td>0.974</td>
<td>0.970</td>
<td>0.9874</td>
</tr>
<tr>
<td>$(\Delta p_m)_{w1}$ (kPa)</td>
<td>7.55</td>
<td>6.99</td>
<td>6.65</td>
</tr>
<tr>
<td>$(\Delta p_m)_{w2}$ (kPa)</td>
<td>2.08</td>
<td>2.01</td>
<td>2.03</td>
</tr>
<tr>
<td>$(\Delta p_m)_{w3}$ (kPa)</td>
<td>1.57</td>
<td>1.54</td>
<td>1.53</td>
</tr>
</tbody>
</table>

Tab. 4: System design variables optimum value.
In the LINEAR procedure the definition of narrow lateral constraints on subsystem variables is very important to obtain correct results and to avoid large influence of subsystem optimizations on system calculation.

REFERENCES


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