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Cooling Air Flow Characteristics in Gas Turbine Components

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An analytical model for the prediction of cooling air flow characteristics (mass flow rate and internal pressure distribution) in gas turbine components is discussed. The model addresses a number of basic flow elements typical to gas turbine components such as orifices, frictional passages, labyrinth seals, etc. Static bench test measurements of the flow characteristics were in good agreement with the analysis. For the turbine blade, the concept of equivalent pressure ratio is introduced and shown to be useful for predicting (1) the cooling air flow rate through the rotor blade at engine conditions from the static rig and (2) cooling air leakage rate at the rotor serration at engine conditions. This method shows excellent agreement with a detailed analytical model at various rotor speeds. A flow calibration procedure preserving flow similarity for blades and rotor assemblies is recommended.

NOMENCLATURE

A	Area, M^2
A	Matrix A
B	Matrix B
C	Constant in Equation (2)
C^m	Constant in Equation (5)
C_D	Discharge Coefficient
D_h	Hydraulic diameter M
f	Frictional Loss Coefficient
G	Flow Stream Mass Velocity, (WA), Kg/M^2 sec
gc	Proportionality Factor in Newton's Second Law
K_r	Pressure Change Coefficient Due to Area Variation
K_1	Pressure Loss Coefficient
K_f	Constant Defined in Equations (8), (9) and (12)
K	Flow Loss Constant
L	Flow Element Length, M
N	Number of Knives in Labyrinth, Number of Internal Nodes
P	Static Pressure, Pa
P_r	Pressure Ratio
A	Pressure Due to Centrifugal Force, P_a
AP	Pressure Increase Matrix Iteration, P_a
Q	Volumetric Flow Rate, M^3/Sec
R_n	Air Gas Constant
P	Average Exit Radius, M
r	Radius, M
T	Temperature, K
V	Velocity, M/Sec
w	Mass Flow Rate, Kg/Sec
v	Kinematic Viscosity
	Rotation Speed, Radian/Sec
a	Ratio of Free-Flow Area to Frontal Area
	Thermal Capacity Ratio
p	Density
p	Dynamic Viscosity

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SUBSCRIPTS

0 Total, Equivalent

1 Location 1

2 Location 2

b Flow Bend, disc bore

d Downstream

E Exit

i Node i, disc rim

I Inlet

j Node j

u Upstream

eq Equivalent

in Inlet

SUPERCRIPTS

m Iteration index

INTRODUCTION

With today's level of fuel prices and anticipated fossil fuel shortages in the future, turbine engineers face a challenge to improve the efficiency of the engine. One of the methods for improving the turbine efficiency is to increase the turbine inlet temperature. However, higher gas temperatures will require more sophisticated cooling schemes for achieving acceptable component life with the same material. To achieve the goal, accurate estimates of both local gas side and coolant side heat flux loads are essential during the component design phase. Much emphasis has been on the gas side heat transfer characteristics. As the cooling scheme becomes more complex, a better understanding of the internal pressure distribution and heat transfer coefficient is necessary to yield accurate predictions of hot section metal temperature, especially when the gas side heat transfer coefficient depends on the cooling air injection rate into the main gas stream. For a turbine blade, the effect of turbine rotation on the cooling air complicates the prediction. It would be convenient to develop a simple technique for determining the cooling air flow rate of turbine blades at engine conditions from the static rig flow without using detailed computer simulation on each turbine blade. This paper presents (1) an analytical model for predicting the cooling airflow characteristics (e.g., mass flow rate, internal pressure and temperature distributions) in gas turbine hardware, (2) the concept of equivalent pressure ratio to account for the centrifugal effect in a rotating condition, and (3) recommended flow calibration procedures for turbine blades and blade/disc assembly.

The analytical model lends itself to an effective numerical technique to yield the solution for the flow characteristics because the gas turbine flow network is modeled as a variety of passage elements. Otherwise, non-linearity of the basic governing equations will consume a lot of computation time when the size of the flow network becomes big. Bench tests were done to measure blade flow characteristics. The comparison of data with analytical results is presented. The theory and application of equivalent pressure ratio are discussed in detail. The paper concludes with recommended flow calibration procedures for gas turbine components to accurately estimate the cooling air flow rate at engine conditions.

ANALYTICAL MODEL

Basic Equation

The static pressure loss of a flow passage shown in Figure 1 can be expressed one dimensionally as:

$$\frac{P_2 - P_I}{\rho} = \frac{K_r V^2}{2gc} + \frac{K_l V^2}{2gc} \quad \text{ppb} \qquad (1)$$

The first term on the right hand side is the pressure change due to the flow area variation from the upstream to the downstream, a reversible quantity. The second term is the pressure loss associated with the wall friction and flow separation, both irreversible processes in the passage. The third term is the increase of the body force on the flow passage assuming the exit of the passage at a larger radius than the entrance.

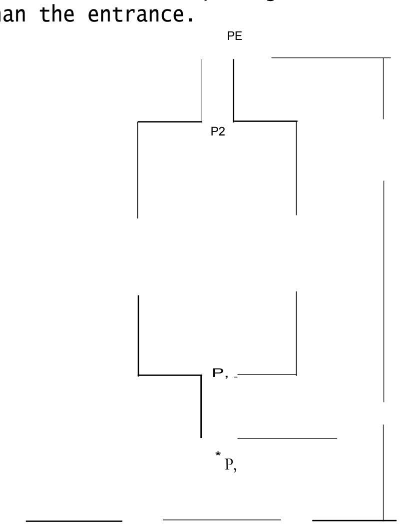

Figure 1 Model of Rotational Flow Passage

The parameters K_r and K_l depend on a number of flow characteristics of the passage, such as equivalent cross-sectional area, passage length, hydraulic diameter, passage configuration, surface roughness, entrance configuration, Reynolds number etc.

The present day turbine stator and blade are cooled by both convective and surface film cooling. The cooling air is supplied by a high pressure source and discharged to one or a number of locations on the airfoil surfaces with different exit pressures. The intricate cooling scheme can be decomposed into a number of basic flow elements with proper correction factors to account for their interaction. The correction factors can be developed by comparing the analytical and experimental results.

The constants K_r, K_1 in equation (1) for a number of flow elements frequeftly used in the flow simulation are shown in Table I. There are flow configurations in which the flow and pressure relationship is directly expressed in the form of:

$$w = \frac{C\, p_0 A}{(R_n T_o)^{1/2}} \qquad (2)$$

where C is a function of pressure ratio. It is more convenient to Keep this form during the numerical analysis. Examples of the constant C are shown in Table II.

Numerical Method

The cooling air passage in gas turbine components can be subdivided into a number of flow passage elements connected in series or parallel to establish a flow network with one or more sources and sinks. To facilitate the numerical analysis, points (nodes) are located at the entrance and the exit of each element and are assigned with reference numbers. The number of nodal points would depend on the magnitude of each flow element pressure drop and the desired accuracy of simulating the flow characteristics in the passage. If neccessary the flow parameters of each element could be properly adjusted to account for the effects of the upstream flow configuration.

After completing the subdivision of the flow system, the flow parameter $K_{iu} = (K^r + K_l)$ between two-end nodes, i and j, can be calculated. The coolant flow rate, w_{ij} from nodes i to j is

$$w_{ij} = A_{ij} \left(\frac{g_c}{R_n}\right)^{1/2} \left(p_i^2 \quad p_i^2\right)^{1/2} \qquad (3)$$

if the body force is excluded temporarily for the convenience of discussion. Equation (3) indicates that the coolant mass flow w_{ij} of each element depends non-linearly on its upstream and downstream pressures $(p_i$ and $p_j)$. The coolant mass flow balance for any internal node i must satisfy the flow continuity condition, i.e.

$$\sum_{j=1}^{N} w_{ij} = 0 \qquad (4)$$

The internal pressures of all nodes cannot be readily obtained by the substitution of equations (3) into (4) due to non-linearity. Any attempt to linearize equation (4) will not be successful because of the impossibility to provide a reasonably good guess on all internal pressures to approximately satisfy the continuity equation. The best alternative for the problem is using the iteration method. When the iteration method is employed to determine the internal pressures and the mass flow rates, an initial guess on these quantities will result in mass flow inbalances at all internal nodes. The objective then becomes the determination of internal pressure increase/decrease at each iteration such that the mass inbalances are reduced gradually at internal nodes. That is, for ith node:

$$C^m \sum_{j=1}^{N} w_{ij} \quad \left(\frac{\partial w_{ij}}{dp_i}\right) \Delta p_i^m + \sum_{\substack{j=i \\ j \neq i}} \left(\frac{\partial w_{ij}}{dp_j} \Delta p_j^m\right) = 0 \qquad (5)$$

where m is indexed with each iteration and is not an exponent. The constant Cm (1) on the first term will be adjusted internally during each iteration such that the numerical instability is avoided and a faster convergency is achieved. Equation (5) can be rewritten into a matrix form as

$$\underset{\sim}{A} \cdot \Delta P^m = \underset{\sim}{B} \qquad (6)$$

The elements in $\underset{\sim}{A}$ and B are computed on the basis of the pressure P^m of the previous iteration, and subsequently,

$$p^m = p^{m-1} \quad \Delta p^m \qquad (7)$$

The above operation will continue until the convergence criterion on the mass inbalance is reached.

Body Forces

The body force on the cooling air of a turbine blade/disc is the centrifugal force generated due to the turbine rotation. The governing equation of a forced vortex pumping is:

$$\frac{\gamma}{\gamma - 1}\left(1 - \left(\frac{p_i}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right) = \frac{\omega^2 \left(r_o^2 - r_i^2\right)}{2 g_c R_n T} K_f^2 \qquad (8)$$

where K_c = 1.0 for a turbine blade, because the cooling air in the blade flows at the same local tangential velocity as the turbine.

The average velocity of cooling air between a turbine disc and sealing plate is a fraction of the local tangential velocity of the disc because the air is not adjacent to the disc and cannot achieve the same velocity as the disc by viscous shear forces. Daily et al (Reference 8) developed an empirical relationship, Equation (9), for the core rotation. The radial pressure depends on the centrifugal forces of core rotation and the radial diffuser effect on the radial flow of cooling air. The contribution of the radial diffusion is negligible. Therefore, the pressure change, induced by the rotation, in the flow passage of a turbine blade and disc becomes independent of passage geometry, the frictional characteristics, and the coolant mass flow. As a result the pressure can be calculated and accounted for in equation (1) prior to computing equation (3). The effect of body forces is significant for gas turbines.

$$K_f = \frac{0.5}{12.75\left(\frac{Q}{\omega r_o^{2}}\right)^{.2}\left(\frac{r_o}{r_i}\right)^{2.6}} \qquad (9)$$

Heat Transfer

Heat transfer effects on the flow characteristics should also be included in the computation. A number of heat transfer coefficient formulas available in the literature can be incorporated into a computerized numerical analysis. The temperature increase at each element is calculated from the computed heat transfer coefficient and the assumed wall temperature. The effects of a preswirl nozzle on the upstream pressure and the relative total temperature can be included in the analysis. The rotation of the turbine blade will incur a temperature change on the coolant because of the work done on and by the coolant. Euler's turbine equation is used to compute temperature increase and decrease.

EXPERIMENTAL VERIFICATION

To enhance the understanding of flow behavior in the gas turbine cooled component and to improve its prediction accuracy, a number of Lycoming turbine hardware were instrumented with pressure taps along the cooling flow passage and bench flowed at a series of pressure ratios to generate static pressure distribution and mass flow rate data. Figure 2 shows a typical instrumented blade. Figure 3 shows a schematic of the test blade and approximate pressure tap location. Figure 4 shows a schematic of the test rig. All pressure taps were adapted to the data system scanivalve and read by a single transducer during the test scans to assure consistency in the readings. A scan was taken when the regulated plenum pressure became stabilized. Test data of air flow rate versus pressure ratio is presented in Figure 5. There is an excellent correlation in flow rate between the experiment and the analysis. Data displaying internal pressure distribution is presented in Figure 6 as compared with analytical results. The test internal pressures shown are the average of pressure and suction sides. The discrepancy in the static pressure is relatively small compared to the total pressure drop in the blade.

Figure 2 Lycoming H.P. Blade Instrumented with Pressure taps.

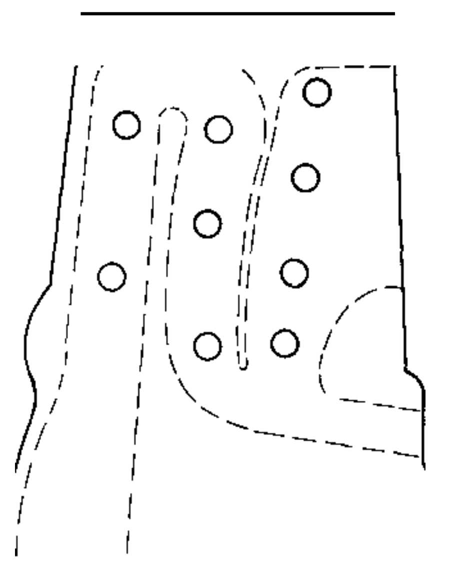

Figure 3 Location of Pressure taps.

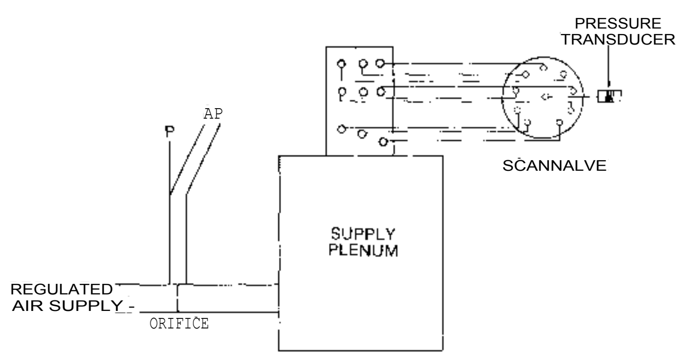

Figure 4 Schematic of Test Rig.

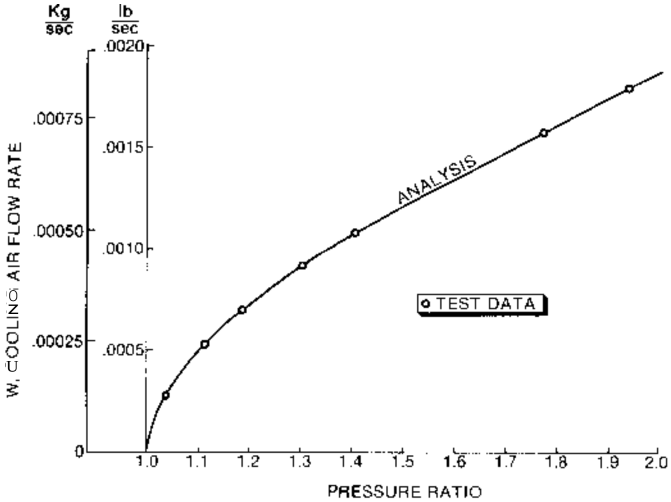

Figure 5 Comparison of Analytical and Experimental Mass Flow vs Pressure Ratio Results.

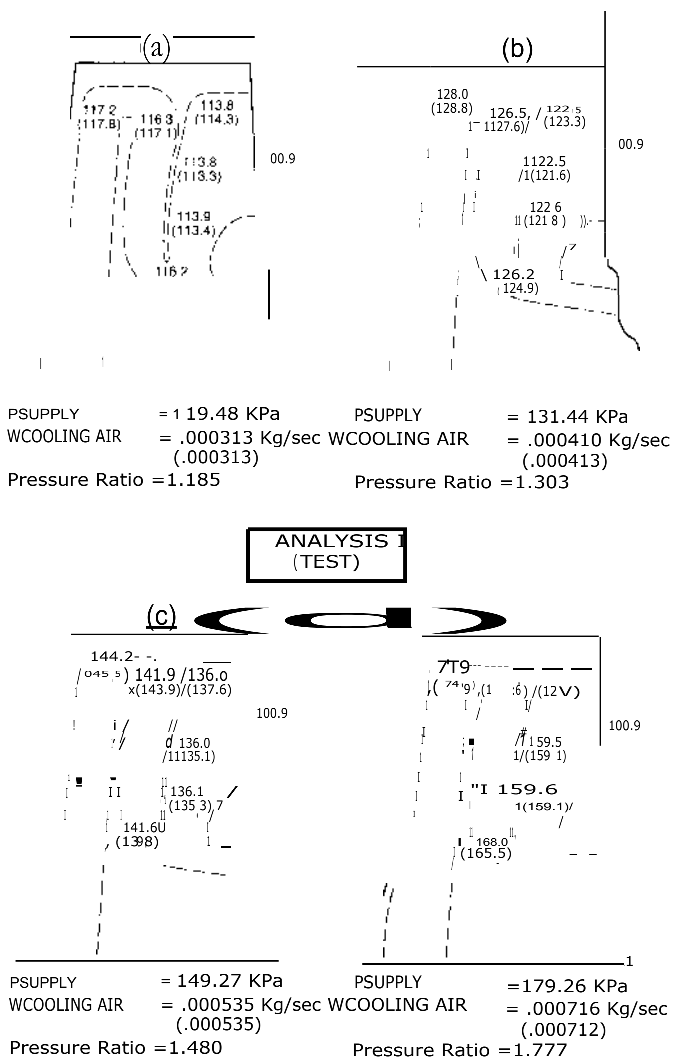

Figure 6 Comparison of Analytical and Experimental Flow Characteristics at Four Pressure Ratios.

EQUIVALENT PRESSURE RATIO

A turbine blade is normally flow calibrated in a static rig for reasons of cost saving, convenience and quickness in the calibration. An estimation of the cooling airflow rate at engine conditions is then required from the data of the rig flow calibration. During the estimation the effect of turbine rotation must be included in conjunction with the correction on pressure and temperature. This section presents a simple concept and technique for considering all three parameters simultaneously during the estimation.

Turbine rotation introduces centrifugal forces on the cooling air and results in higher cooling airflow. The magnitude of increase depends on the rotor speed and geometric factors. As a result the cooling airflow of a turbine blade at part power condition differs from those at full power. When analyzing the engine performance at various powers, one could rely on the analytical simulation of the blade at the static rig condition and then extend the model to include the centrifugal force effect. However, at the various engine conditions the mass flow rate of an actual blade will differ from the design mass flow rate because of the manufacturing tolerance. The extrapolation process becomes more complicated and costly in computation if a computer simulation method is employed for every batch of manufactured blades. The simplified method of assuming a constant ratio between mass flow rates at all engine conditions and the static rig condition is not correct because the ratio varies with the rotor speed. Futhermore, the coolant leakage through the blade platform and serration is under different magnitude of centrifugal and shear pumping. The simplified constant ratio assumption for the cooling air flow rate of an turbine blade to static rig conditions is definitly not applicable to leakage conditions. The leakage calibrated at the static condition does not relate to the leakage at the engine condition in a consistent pattern since the flow geometry associated with the leakage has altered at the blade attachment due to turbine rotation. Therefore, it is desirable to have an algebraic formula to account for the turbine rotation in the engine performance analysis. The concept of equivalent pressure ratio is developed to provide an effective and accurate technique of estimating the cooling airflow rate from the static rig flow calibration.

Simplified Equation

For a simple flow passage shown in Figure 1 with w=0, the governing equation for the system under the static mode is

$$P_I - P_E = (K \quad K_1) \frac{G_1^2}{2 \rho_1 g_c} \qquad (10)$$

on the basis of Equation (1). For the same flow configuration under the rotating mode, the governing equation becomes

$$P_I - P_E + \Delta p_b = (K_r + K_1) \frac{G_2^2}{2 \rho_2 g_c} \qquad (11)$$

By comparing equation (10) with (11), one notes that if the downstream pressure is assumed constant, the effect of centrifugal force on the coolant mass flow rate is equivalent to an additional entrance (supply) pressure in the static mode because the centrifugal force is independent of the passage characteristics. The net pressure induced by the forced pumping on a blade/disc can be derived by conducting a binominal expansion on the left hand side of equation (8) and neglecting higher order terms. That is:

$$\frac{P_b}{P_i} - 1 = \frac{\omega^2 \left(r_o^2 - r_i^2\right) K}{T\, 2 g_c R_n} \qquad (12)$$

It indicates that the pumping depends on the upstream pressure, rotational speed and radial locations of the entrance and the exit. For a turbine blade with a slot exit, the overall effective pumping can be determined by analytical integration on equation (12) across the slot height. The integration yields an equation similar to (12) for the overall pumping effect, but with an equivalent outer radius of

$$\bar{r}_o^2 = \frac{1}{3}\,\frac{r_2^3 - r_1^3}{r_2 - r_1} \qquad (13)$$

The above equation can also apply for a blade with evenly spaced exit holes.

Applying equations (11) and (12), the equivalent supply pressure for a turbine blade becomes

$$P_{eq} = P_{in}\left(1 + \frac{\omega^2 \left(\bar{r}_o^2 - r_i^2\right)}{T\, 2 g_c R_n}\right) \qquad (14)$$

The same principle also applies to the turbine blade/disc assembly. For a turbine blade/disc assembly, the equivalent inlet pressure at the disc inner location becomes:

$$P_{eq} = P_{in}\left[1 + K_f \frac{\omega^2}{T}\,\frac{r_i^2 - r_b^2}{2 g_c R_n}\right]\left[1 + \frac{\omega^2}{T}\,\frac{\bar{r}_o^2 - r_i^2}{2 g_c R_n}\right] \qquad (15)$$

To verify the above concept, the following two steps were taken:

1. Conduct flow experiments on turbine blades to measure the airflow and internal pressure distribution in a static rig at a number of pressure ratios and develop an analytical model for the blades to correlate with measured results.

2. Apply the analytical model to the engine condition because the centrifugal force is independent of the passage characteristics. A number of turbine rotating speeds were considered, including "zero" rotation.

A typical result is presented hereafter to demonstrate the technique of utilizing equivalent pressure ratio for the cooling airflow prediction of turbine blades.

The cooling airflow rates of the turbine blade shown in Figure 5 are presented in Figure 7 for the engine condition. The curve of "zero" rotation speed determined from Figure 5 by assuming $\frac{W\sqrt{T_o}}{P_o}$ is constant at each pressure ratio. $\frac{W\sqrt{T_o}}{P_o}$ can be treated as constant since the Reynolds Number effect on the pressure drop is very small compared to the total pressure loss in the blade. Substitution of geometric values, turbine rotation speed and mean air temperature into equation (14) yields

$P_{eq} = P_{actual} \times 1.216$ for $w = 4850$ rad/sec
$P_{eq} = P_{actual} \times 1.11$ for $w = 4180$ rad/sec

This means that the equivalent pressure ratio, P_{eq} / P_{inlet}, is 1.216. This is the pressure ratio which would result in the same mass flow as if there were just rotational pumping (w = 4850 rad/sec) (i.e., no static pressure ratio across the blade). For a static pressure ratio of 1.2 across the blade, the cooling airflow rate at w = 4850 rad/sec is equal to the flow rate at the equivalent pressure ratio of 1.2 x 1.216 1.46 and w = 0. The same procedure applied to a rotational speed of w= 4189 rad/sec and static pressure ratio of 1.2 yields an equivalent pressure ratio of 1.33.

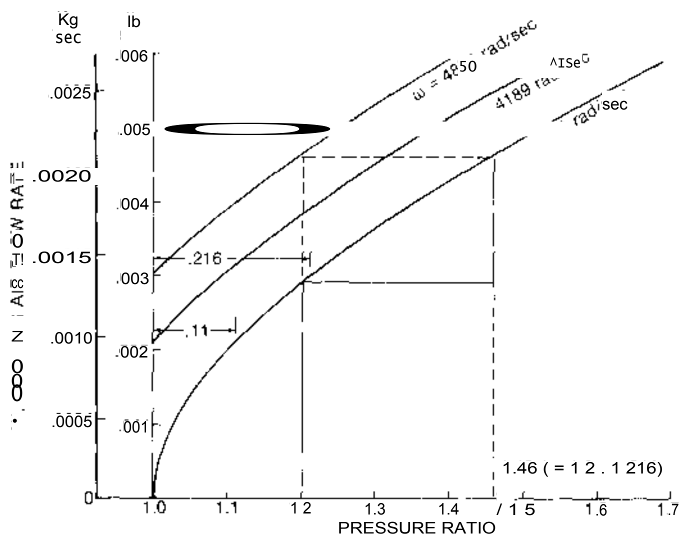

Figure 7 Graph Demonstrating Equivalent Pressure Ratio.

FLOW CALIBRATION PROCEDURE FOR A TURBINE

It is clear from the concept of equivalent pressure that the cooling air flow through a gas turbine blade depends on the turbine rotating speed. To properly account for this variation, the following calibration procedures for a turbine blade similar to Figure 2 are recommended.

1. Select cooling air temperature and discharge (exit) pressure such that Reynolds Number and

pressure coefficient of the rig condition is equal to those of the engine condition. The similarity condition is:

$$\left(\frac{P}{T^{2}}\right)_{engine} = \left(\frac{P}{T^{1/2}}\right)_{rig} \qquad (16)$$

This step can be ignored when the pressure drops in flow passages dependent on Reynolds Number are very small compared to the overall pressure drop.

2. Calibrate the blade at three supply pressure ratios: the actual ratio, the equivalent pressure ratio of;

$$P_{r_{eq}} = P_r \left| 1 \quad \frac{w}{T} \frac{r_o^2 - r_i^2}{2 g_c R_n} \right. \qquad (17)$$

and mean ratio between the actual and the equivalent. Should the interest be on the maximum cooling airflow rate, only the equivalent pressure ratio is chosen.

3. Compute the cooling airflow at the engine condition based on the equation:

$$\left(\frac{W T^{1/2}}{P}\right)_{engine} = \left(\frac{W T^{1/2}}{P}\right)_{rig} \qquad (18)$$

Then a formula can be developed to specify the cooling airflow as a function of turbine speed. The air leakage of a turbine blade/disc assembly cannot be calibrated in a static rig. Therefore, either of the following calibration procedures must be used to determine the cooling airflow rate at engine condition:

1. Flow calibrate all blades following the above procedure prior to assembling except at an equivalent pressure of equation (15) and then sum up with the analytical leakage computed from the serration and platform leakage areas in which a lower equivalent pressure must be used to account for the smaller pumping effect.

2. Flow calibrate all blades as in procedure 1 to determine the cooling airflow rate in terms of pressure ratio or turbine speed, and then calibrate the assembly in a dynamic mode (at a much lower rotation speed than the engine) similar to the blade. Subsequently, the leakage rate can be determined with measured data. The superposition method can then be used to determine the total cooling airflow as a function of engine rotating speed.

CONCLUSIONS

An analytical model has been developed to simulate cooling flow networks of gas turbine cooled components. The matrix solution scheme and the iteration method have resultedin an accurate convergence on the internal pressure distribution, flow rate, and temperature distribution. Analytical results of internal pressure distribution and airflow rate have correlated very well with the measurements in static rig experiments on turbine hardware. It indicates that published flow characteristics on the basic elements such as friction factor, entrance and expansion losses, etc. can be employed directly to accurately predict the cooling airflow rate and internal pressure distribution. Therefore, the study of the coolant side flow characteristics can concentrate on the basic flow elements instead of each new design configuration.

The concept of equivalent pressure ratio to include the centrifugal effect on the cooling air in a rotating component has been shown useful to quickly and accurately predict the cooling airflow at engine conditions from the static rig calibration. Flow calibration procedures for gas turbine components are recommended to satisfy the similarity condition to accurately estimate the cooling airflow rate at engine conditions.

TABLE I BASIC FLOW ELEMENTS SUITABLE TO EQUATION 1

FLOW ELEMENTS	K_R	K_L	FLOW AREA	COMMENTS
ABRUPT CONTRACTION	$(1 - a)^2$	K_c	MINIMUM AREA	K_c =LOSS COEFFIEIENT SEE REFERENCES 1,2,3
ABRUPT EXPANSION	$(1 - a)^2$	K_e	MINIMUM AREA	K_e =LOSS COEFFIEIENT SEE REFERENCES 1,2,3
FRICTIONAL PASSAGE	$(a_d^2 - a_u^2)$	$4f \frac{l}{h}$	AVERAGE AREA OF PASSAGE	SEE REFERENCES 1,4,5
BENDING LOSS	1	K_b	MINIMUM BEND AREA	K_b =BEND LOSS COEFFICIENT SEE REFERENCE 7

TABLE II BASIC FLOW ELEMENTS SUITABLE TO EQUATION 2

FLOW ELEMENT	C	COMMENTS
LABYRINTH SEAL	$5.68\,K \left\| \dfrac{1 - \left(\frac{P_2}{P_{01}}\right)^2}{N - \ln\left(\frac{P_2}{P_{01}}\right)} \right\|$	SEAL EQUATION AFTER REFERENCE 6
FLOW ORIFICE	$C_D \left\| 2g_c \dfrac{\gamma}{\gamma - 1} \left[\left(\frac{P_2}{P_{01}}\right)^{2/\gamma} - \left(\frac{P_2}{P_{01}}\right)^{\frac{\gamma+1}{\gamma}} \right] \right\|^{1/2}$	ISENTROPIC ORIFICE EQUATION

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