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CYCLIC FINITE ELEMENT MODELING OF SHROUDED TURBINE BLADES INCLUDING FRICTIONAL CONTACTS

J. Szwedowicz

Thermal Machinery Laboratory,
ABB Turbo Systems Ltd,
CH-5401 Baden / Switzerland
E-mail: jaroslaw.szwedowicz@chtus.mail.abb.com

ABSTRACT

A new cyclic finite element model for shrouded blades is proposed. Using this technique, a contact area between the shrouds is computed with a good accuracy. Imposing a no-slip condition on the computed contact area, eigenfrequencies of the shrouded blades are calculated with very good agreement in relation to measurements. This method is verified with measurements of non-rotating and rotating bladed disc assemblies.

INTRODUCTION

For preventing failure of airfoils due either to flutter or resonance, free standing blades are connected circumferentially by different types of coupling elements. For minimizing the blade vibration amplitudes, a lacing wire is threaded through a hole of the airfoils. Occasionally an unrealistic diameter of the wire cross section may be required due to excessive centrifugal loads. In this case, staggered zigzag pins are arranged to pass through holes in the blade. However, each blade requires two holes, which reduce the airfoil section and may generate significant notch stresses. Therefore, winglets are employed on the longer blades in the low pressure section of a unit. Also, low pressure turbine blades are often coupled by cover bands at the airfoil tip. These bands, produced separately, are attached to the blade tip by riveting. For avoiding problems of rivet creep or accelerated corrosion at the interface, blades with integrally machined shrouds are applied. Moreover, shrouded blades enclose the flow and increase the efficiency of the stage in relation to the coupling elements described above.

Up to the end of eighties, the vibration characteristics of free standing blades have been computed using one-dimensional (1D) approaches, such as analytical, transfer matrix or beam finite element (FE) methods. Root flexibility of the blade is usually defined by a system of resulting springs whose stiffness was evaluated from measurement or elastic theories. Good agreement between the computed and measured lowest frequencies of the free standing blade confirms the reliability of the developed numerical methods. Using a receptance coupling technique or component-mode synthesis, which matches forces and displacements at each connection point, the 1D models were extended to represent disc assemblies. During the last decade, three-dimensional (3D) FE modeling of the disc assembly was mostly applied. Using the wave propagation theory (Thomas, 1974), vibration of the tuned disc assembly is represented efficiently by a single coupled blade with complex constraints imposed on circumferential sides. Eigenfrequencies of the coupled blade are calculated in terms of a nodal diameter number.

In 1D models, oscillations of the blade cross section represent 6 (3 translations and 3 rotations) degrees of freedom (d.o.f). In the definition of an effective coupling between blades, less than 6 d.o.f are applied on the shroud interfaces in the cyclic constraints. Then, an agreement between the measurement and calculation can be improved (e.g. Cottney and Ewins, 1974). Moreover, one-, two-dimensional or shell cyclic finite element models of the shrouded blades cannot approximate fillets between the shroud and airfoil. For the adjustment between measured and computed blade eigenfrequencies, an equivalent density and Young's modulus of the shroud are then used in the numerical analysis (e.g. Mayer, 1987). Geometry of shrouded blades is accurately modeled with 3D finite elements. In 3D models, the interface between the shrouds is usually represented by a few nodes (with 3 translation d.o.f) located on the circumferential sides. Thus, the effective contact area between shrouds has to be defined by imposing cyclic constraints only on the chosen circumferential nodes. Finally, the disc assembly is represented by all blades having relative slip motion and identical normal contact forces between shrouds. Disadvantages of these models can result in a huge number of degrees of freedom in which

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case either a modal transformation or a FE-superelement technique must be applied.

In general, computed eigenfrequencies of the shrouded blades depend on both an adequate mesh of the airfoil and shroud interface. In practical applications for blade numbers between 30 and 120 in the turbine row, these requirements are difficult to fulfil where either the airfoil or the contact grid is usually coarse (e.g. Jacquet-Richardet et al, 1997, Bladh et al, 1998). Most studies try to analyse rubbing effects at shroud interfaces using simple friction contact models (Yang and Menq, 1997). Additionally, the reliability of the computed eigenfrequencies of the shrouded blades has received very little attention.

In this paper, a new cyclic FE model of the shrouded blade including frictional contact is presented. By applying this technique, a contact condition between tuned shrouded blades is computed using the non-linear static analysis. For a no-slip condition imposed on the computed contact area between the shrouds, eigenfrequencies are then calculated.

CYCLIC FINITE ELEMENT MODELS INCLUDING FRICTIONAL CONTACT

Considering identical blades in the disc, the static and dynamic deformations of the whole disc are represented effectively using a single blade with complex boundary conditions. Neglecting dissipation effects, the dynamic equation of the single coupled blade, disconnected from the disc assembly, is given by

$$(1)\qquad [M]\begin{Bmatrix}\{\ddot{q}\}_{left}\\ \{\ddot{q}\}_{inner}\\ \{\ddot{q}\}_{right}\end{Bmatrix} + [K(\Omega)]\begin{Bmatrix}\{q\}_{left}\\ \{q\}_{inner}\\ \{q\}_{right}\end{Bmatrix} = \begin{Bmatrix}\{S\}_{left}\\ \{0\}\\ -\{S\}_{right}\end{Bmatrix},$$

where [M] and $[K(\Omega)]$ is the blade mass and non-linear stiffness matrix in terms of rotational speed Ω, respectively. Vectors $\{q\}$ and $\{\ddot{q}\}$ are nodal displacement and acceleration of the blade vibration, $\{S\}$ is a vector of internal forces. The internal force, deformation and acceleration vectors are divided into three subvectors referring to nodes located inside, and on the left and right side of the sector (nodes a, b, c and A, B, C, respectively, in Fig. 1a). Since the disc assembly of N blades is deformed harmonically around the circumferential direction, complex kinematic constraints are applied between the left and right circumferential nodes, as

$$(2)\qquad \{\chi\}_{right} = \{\chi\}_{left}\, e^{jm\varphi}, \qquad j=\sqrt{-1},$$

$$m=0,1,2,\cdots,\begin{cases}\dfrac{N}{2} & \text{for even N},\\[2mm] \dfrac{N-1}{2} & \text{for odd N}\end{cases}$$

where $\{q\}\equiv\{\chi\}$, $\{\ddot{q}\}\equiv\{\chi\}$ or $\{S\}\equiv\{\chi\}$ and

$$(3)\qquad \varphi = \frac{2\pi}{N}$$

is the circumferential periodicity angle between neighbouring blades.

After substituting equation (2) into (1), the internal forces S disappear and the free vibration equation of the cyclic finite element model of the shrouded blade is obtained by

$$(4)\qquad \left[M\left(e^{jm\varphi}\right)\right]\begin{Bmatrix}\{\ddot{q}\}_{left}\\ \{\ddot{q}\}_{inside}\end{Bmatrix} + \left[K\left(e^{jm\varphi},\Omega\right)\right]\begin{Bmatrix}\{q\}_{left}\\ \{q\}_{inside}\end{Bmatrix} = \{0\},$$

where the blade mass and stiffness matrices depend additionally on nodal diameter number m. For each mode i with nodal diameter m (besides m=0 and m=N/2), two identical eigenfrequencies are computed from equation (4) which refer to two possible mode shapes of the disc assembly. For the static analysis, the inertial term of equation (4) is omitted and m equals 0 is substituted. Considering assembly forces A, centrifugal loads F and flow pressure P acting on the blade, the static equation of the shrouded blade is then given by

$$(5)\qquad [K(\Omega)]\{q\} = \{A\} + \{F(\Omega)\} + \{P\}.$$

A cyclic structure must be exactly symmetric. Thus, the same number of nodes have to represent the left and right circumferential side of the cyclic mesh. Moreover, each node on one circumferential side must have a partner (slave) node on the other circumferential side at the circumferential distance $\varphi=2\pi/N$ with identical axial and radial co-ordinates (Fig. 1a). If these conditions are not met, two eigenfrequencies of the same mode, calculated from equation (4), are no longer identical. These two 'numerically detuned' eigenfrequencies differ from each other in dependency of differences between mesh configurations on the right and left circumferential sides of the cyclic model (Fig. 1b). Detuned cyclic models can usually appear because of meshing problems (so called the free meshing technique, see Fig. 5) or design requirements for asymmetric contact conditions of the shrouded blade.

In this paper, for symmetrical or asymmetrical configuration between the circumferential nodes, a new cyclic finite element model defines contact between shrouded blades (Fig. 1c). For creating this cyclic model, four steps must be done:

1. the finite elements e on one circumferential side are selected (grey elements in Fig. 1b),
2. the selected finite elements are disconnected from the mesh by making a copy of these elements with increased nodal Δn and elemental Δe labels higher than the labels applied in the mesh,

3. the generated finite elements e+Δe are copied by the periodicity angle $\varphi=2\pi/N$ to the opposite circumferential side (Fig. 1c),
4. the selected finite elements e (point 1) are deleted from the mesh.

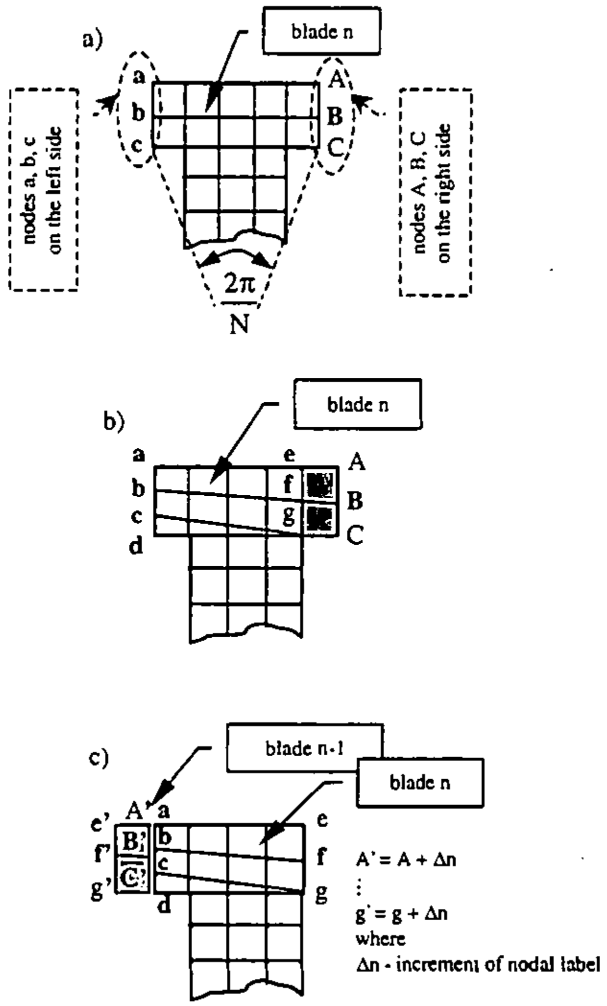

Fig. 1 Upper part of the two-dimensional mesh of the shrouded blade, a) required symmetrical node configuration for the traditional cyclic finite element model of blade n, b) asymmetrical node configuration of blade n, c) proposed cyclic finite element model of the shrouded blade including contact, where N denotes a blade number in the turbine row

Thus, the cyclic model of the coupled blade includes the contact between the shrouds and the required symmetrical circumferential nodes (see node pairs e-e', f-f', g-g' in Fig. 1c) simultaneously. The contact between the shrouds is modeled by either gaps[1] or two-dimensional (2D) contact elements[2]. In general, the 2D contact elements are more accurate and convenient to use in relation to the gap approach. By applying Coulomb frictional forces on the contact elements, an effective contact area between the shrouds is calculated from the non-linear static analysis of the rotating turbine blades (ABAQUS, 1998).

Finally, by imposing a no-slip contact condition on the computed contact area, two orthogonal modes of each analysed eigenfrequency are computed. Alternatively, using the time-domain analysis, vibration responses of the shrouded blade can be simulated for Coulomb forces acting on the computed contact area. This transient vibration analysis may be an alternative or verification solution of the harmonic balance results (Yang and Menq, 1997). For a faster transient simulation, the number of d.o.f of the blade model should be reduced by applying the FE superelement technique.

APPLICATIONS

To illustrate the use of the method in a practical problem, effective contact areas and eigenfrequencies of non-rotating and rotating shrouded disc assemblies are computed and compared to experimental results.

Non-rotating shrouded disc assembly

A disc with 50 rectangular welded bars (30x6 mm) with shroud coupling has been analysed. The bar, as a simple blade model, with a height of 135 mm is staggered by 27° in relation to the disc axial direction. A separate shroud is mounted to the bar tip through two tight pins (a diameter of 7.7 mm) and a radial single M4 screw (Fig. 2). By pre-twisting of 1° of each blade, the disc is assembled. The coupled blade is approximated with parabolic solid finite elements (Fig. 3) where the contact between the shrouds is modeled with 4×10 2D contact elements. Imposing a pre-twisting assembly deformation of 1° on the blade and assuming a friction coefficient of 0.2, the stresses and contact areas (Fig. 4) between the shrouds are computed from the non-linear static analysis. Finally, the blade eigenfrequencies are calculated from the classical (Fig. 1a) and new (Fig. 1c) cyclic models and compared to the measurement (Schaber, 1987) in Fig. 4. For the method proposed here, a very good agreement is obtained between the measured and computed eigenfrequencies (black points and solid line, respectively in Fig. 4). In the disc assembly, strong mistuning effects among blades are expected considering the type of shroud assembly at the blade tip. In the measurement, for the same nodal diameter number, two different eigenfrequencies were occasionally measured due to mistuning effects (for instance, see the 2nd mode with 13th nodal diameter in Fig. 4). Therefore, some experimental eigenfrequencies were not classified in terms of nodal diameter number (Schaber, 1987). The mistuning ratios of the shrouded blades (slight geometry differences among blades) were not estimated in the measurement.

[1] the simplest contact element between two coincided nodes located on opposite meshes

[2] defining contact between faces of elements of the opposite meshes

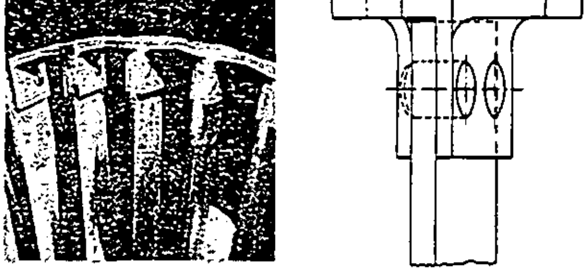

Fig. 2 Analysed disc assembly and details of the shroud assembly at the blade tip (Schaber, 1987)

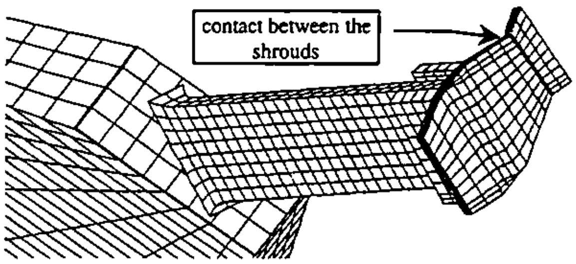

Fig. 3 Cyclic finite element model of the coupled blade including contact between the shrouds

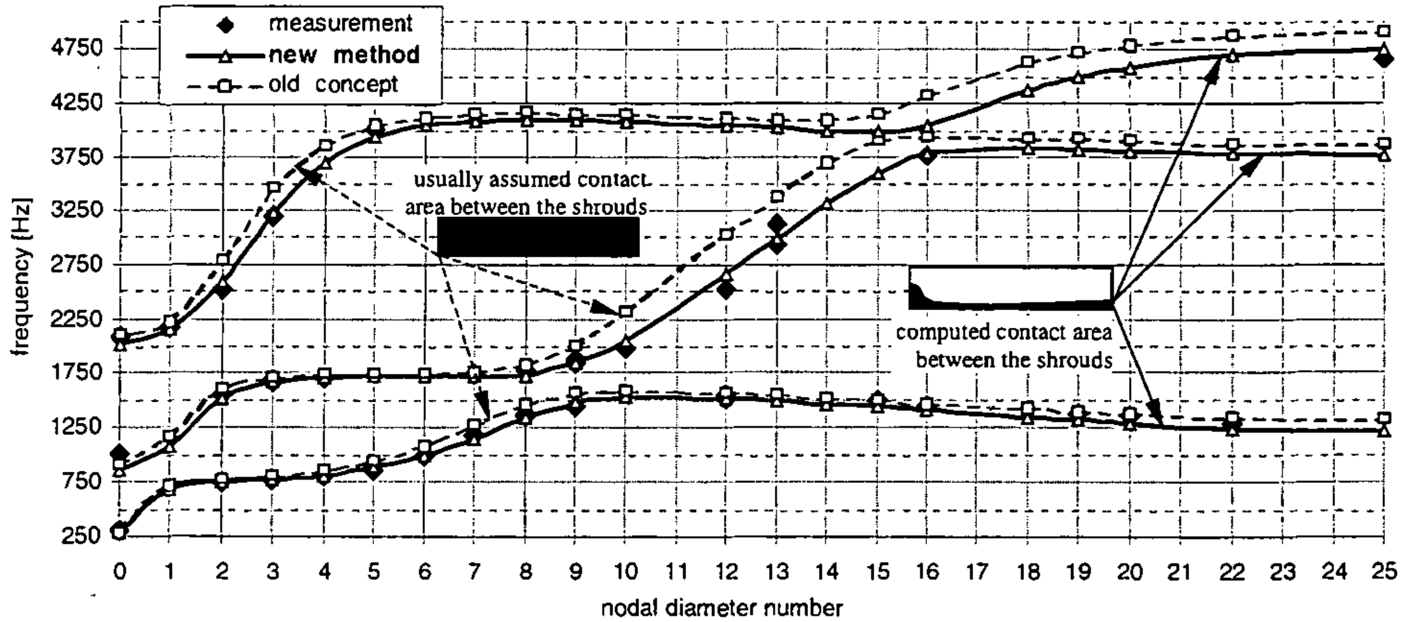

Fig. 4 Blade eigenfrequencies in terms of nodal diameter number computed from the classical (a dashed line) and new (a solid line) cyclic model of the shrouded blade, where the measured eigenfrequencies are indicated by discrete black points and the contact area between the shrouds is marked by black colour. Shroud coupling dominates for the 1^{st}, 2^{nd} and 3^{rd} blade modes between the 4^{th} and 9^{th}, 8^{th} and 15^{th}, 14^{th} and 25^{th} nodal diameter numbers, respectively.

Rotating pre-stressed shrouded steam turbine blade assembly

Seventy rotating steam turbine blades are coupled circumferentially by the shroud. The blade is attached to the rotor by a fir tree. The shroud and airfoil are modeled with tetrahedron parabolic finite elements (so called 'free mesh') and the shank and rotor are approximated by parabolic brick solid finite elements ('mapped' mesh). Both dissimilar meshes are coupled to each other using multi-point constraints (Fig. 5). Some simplifications are applied in the generated FE model, such as: omitted fillet between the airfoil and platform, neglected fir tree and direct attachment of the airfoil shank to the rotor part on radius referring to the position of the upper shoulder of the fir tree (Fig. 5). In the cyclic model, a discontinuity between the shrouds is modeled using 45 two-dimensional contact elements (Fig. 6).

Assuming a friction coefficient of 0.7 (friction coefficients for steel/steel combination: static 0.45-0.80, sliding 0.4-0.7 (Beitz and Küttner, 1983)), shear friction forces act on the contact plane between the shrouds. By pre-twisting of each blade, the turbine stage is assembled. Assembly pre-stresses between the shrouds are induced by a rigid kinematic rotation about the radial direction of the rotor nodes. Additionally, centrifugal forces relating to the nominal rotational speed are imposed on the blade. For assumed ambient temperature of 20°C, thermal stresses are neglected. The contact area between the deformed shrouds is obtained from the non-linear static analysis.

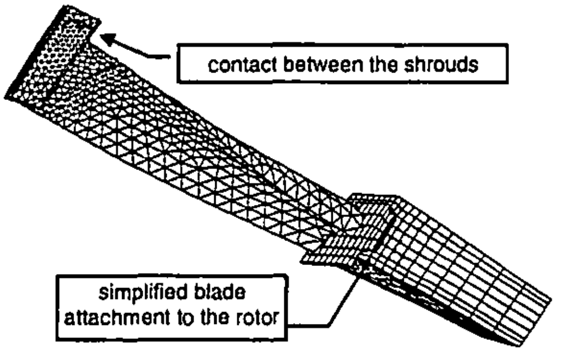

Fig. 5 Cyclic finite element model of the steam turbine blade including the frictional contact between the shrouds

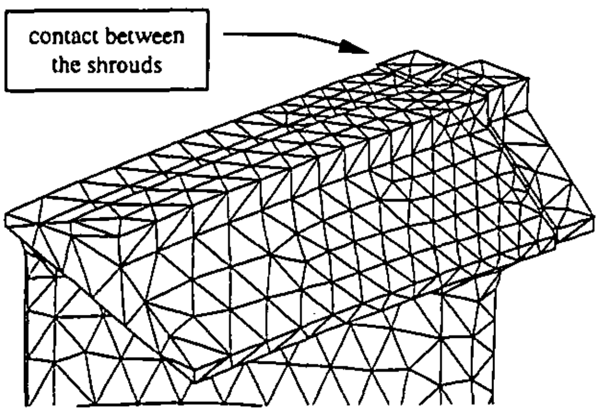

Fig. 6 The deformed configuration between the shrouded blades in the spin pit condition

By imposing a no-slip condition on the calculated contact area, blade eigenfrequencies are computed and compared with the measured ones (a bold solid line in Fig. 7). Also, blade eigenfrequencies are calculated for the usually applied 'full' contact between the shrouds and related to the measurement in Fig. 7. By applying this new cyclic model, the calculated eigenfrequencies are obtained with a very good agreement in relation to the measurement.

CONCLUSIONS

Using this new cyclic finite element model, a contact area between the shrouded blades can be computed with good accuracy. By imposing a no-slip condition on the computed contact area, eigenfrequencies of the coupled blade are calculated with a very good agreement in relation to the measurement. For instance, the spin pit eigenfrequencies of the shrouded turbine blade (Fig. 5) were measured with an uncertainty of about 0.5% by applying an air jet and magnetic excitation of the blades. Using this new technique, the computed eigenfrequencies are found with relative error below 1% for high nodal diameter numbers (black bar in Fig. 8), where shroud coupling dominates in comparison to the rotor coupling. The traditional cyclic solution gives relative errors between 2 and 6% (white bar in Fig. 8) and were not acceptable due to expected excitations. For lower nodal diameter numbers, where the rotor coupling is stronger than the shroud coupling, both cyclic techniques give unsatisfying numerical results in relation to the measurement (see first four bars in Fig. 8). This is due to the neglected fir tree part and the lower segment of the rotor in the FE blade model (Fig. 5).

In the relation to the traditional cyclic FE model, the new concept satisfies very well a numerical reliability of the static and free vibration computations of the shrouded blades as it is quantitative illustrated in Fig. 4.

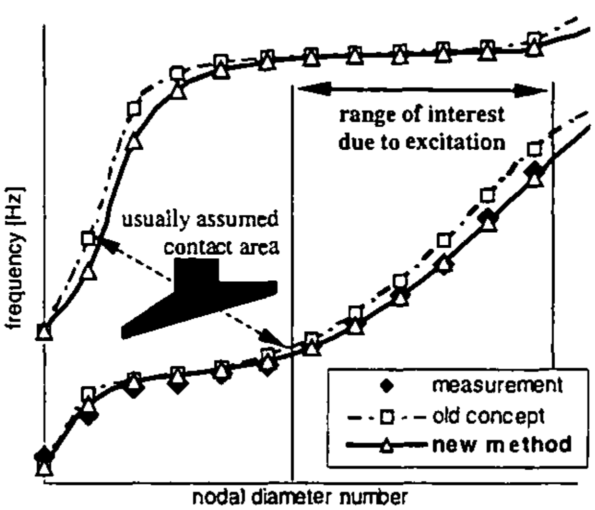

Fig. 7 Calculated and measured (only 1st mode) eigenfrequencies of the shrouded blade in the spin pit measurement

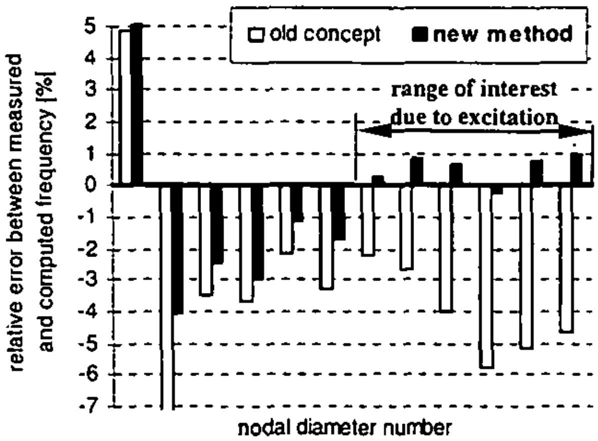

Fig. 8 Relative errors of the computed eigenfrequencies obtained from the traditional (Fig. 1a) and new (Fig. 1c) cyclic finite element models for the turbine blades shown in Fig. 5

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