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A REDUCED ORDER MODEL OF MISTUNING USING A SUBSET OF NOMINAL SYSTEM MODES

BREAK

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ABSTRACT

Reduced order models have been reported in the literature that can be used to predict the harmonic response of mistuned bladed disks. It has been shown that in many cases they exhibit structural fidelity comparable to a finite element analysis of the full bladed disk system while offering a significant improvement in computational efficiency. In these models the blades and disk are treated as distinct substructures.

This paper presents a new, simpler approach for developing reduced order models in which the modes of the mistuned system are represented in terms of a sub-set of nominal system modes. It has the following attributes: the input requirements are relatively easy to generate; it accurately predicts mistuning effects in regions where frequency veering occurs; as the number of degrees of freedom increases it converges to the exact solution; it accurately predicts stresses as well as displacements; and it accurately models the deformation and stresses at the blades' bases.

1. INTRODUCTION

The resonant amplitudes of turbine blades tend to be sensitive to small changes in the blades' properties. In the literature, this problem is often referred to as the *blade mistuning* problem and has been studied extensively by a number of researchers; for examples, refer to Whitehead (1966), Ewins and Han (1984), Fabunmi (1980), Griffin and Hoosac (1984), Sinha and Chin (1988), Wei and Pierre (1988), and Lin and Mignolet (1997). Recently, researchers have been developing reduced order models of mistuned bladed disks (Kruse and Pierre (1996) and Yang and Griffin (1997a)) in which the properties of the reduced order model are calculated directly from finite element models of the blade and disk substructures. The advantage of the reduced order models is that they can accurately represent the true geometry of the bladed disk system while incurring computational costs comparable to those of spring mass models.

In this paper a new approach is presented for developing reduced order models of mistuned bladed disk vibration. This new approach represents the mistuned modes in terms of a limited sum or subset of "nominal" system modes. For brevity the approach will be referred to with the acronym SNM[1] for subset of nominal modes. Nominal system modes are defined as the modes of the system for a nominal blade and disk geometry. The nominal geometry could be one in which every blade and disk sector is identical, i.e., a tuned bladed disk, or it could be one in which the blade and disk sector geometry or the constraints are varied in a prescribed manner. For example, the latter approach could be used to specify a nominal system in which the blades are intentionally mistuned in a repeating pattern such as alternate mistuning. The SNM approach determines the effect of variations from the nominal design, such as those caused by uncontrolled manufacturing variations, on the vibratory response of the system. SNM differs from the earlier reduced order models proposed in the literature in that the blades and disk are treated as a single structure and not as separate substructures with different mathematical representations.

In practice the mistuning in a bladed disk system is often thought of in terms of variations in the frequencies of individual blades. One of the reasons that the earlier reduced order models represented the blades as separate substructures was that it made it easier to use the

[1] A preliminary version of the results reported in this paper was presented at a DoD and industrial review meeting of the GUIde Consortium in August 1998. At that time, the authors of this paper referred to their method as the "Modal Domain Approach" or by the acronym MDA and that designation is still being used by some members of the gas turbine community to refer to what is called the SNM approach in this paper.

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frequencies of individual blades to characterize mistuning in their models. A methodology is presented in this paper which also allows the blade frequencies to be used to determine mistuning in the SNM approach.

The SNM approach is based on a concept developed in a recent paper by the authors, Yang and Griffin (1997b). That paper discusses how the modes of the nominal system can be used to represent the modes of the system if the system's properties slightly change, for example, because of manufacturing variations. If the nominal structure has modes with closely spaced natural frequencies, then the altered structure will also have modes with closely spaced frequencies in the same frequency range. It is these closely spaced modes that are very sensitive to structural changes. The key result from the paper is that the closely spaced modes in the altered system can be approximated as a sum of the closely spaced nominal modes. This approximation results in a reduced order formulation of the problem where the number of degrees of freedom is equal to the number of closely spaced modes. The Yang and Griffin (1997b) paper shows that neglecting nominal modes with more remote natural frequencies results in errors which are inversely proportional to the frequency difference. Thus, if more accuracy is required the error in the SNM reduced order model can be reduced by increasing the number of nominal modes used in the representation.

The SNM approach works very naturally with bladed disk systems because bladed disks tend to have natural frequencies that fall into closely grouped clusters. For example, the lowest frequency modes in a bladed disk often correspond to modes in which the strain energy is stored mostly in the blades. In a tuned bladed disk, this family of modes is often referred to as the first bending family and the closely spaced frequencies correspond to different numbers of nodal diameters in the disk. If the disk is relatively stiff this set of clustered modes may be fairly well isolated. In this case, the SNM approach could accurately represent the first bending family of mistuned modes using only the first bending family of tuned modes. If additional accuracy is needed then several families of tuned modes also could be included in the reduced order model.

The representation of using nominal modes to express the solution is similar to a representation employed by Lin and Mignolet (1997). In their case, they transform the physical domain to a modal domain. The columns of their transformation matrix are the mode shapes of the tuned system when certain coupling terms are large and the mode shapes of the decoupled system when the coupling terms are small. However, the solution procedure that they use is quite different from the one discussed in this paper in that they directly solve for the forced response of the system using an impedance approach and an adaptive perturbation scheme. In the case of SNM, the solution is determined by first calculating the modes of the mistuned system by solving a "modal eigenvalue" problem, Yang and Griffin (1997b). The eigenvalue problem is important in its own right since it is formulated so that it can readily include the motion dependent aerodynamic forces. When the aerodynamic terms are included the eigenvalues determine the aerodynamic damping as well as the frequency and composition of the mistuned modes. Since negative aerodynamic damping can cause flutter the modal eigenvalue calculation provides a computationally efficient method for determining the effect of mistuning on the aerodynamic stability of the stage. If the response of the stage is stable the mistuned modes can be used to express the forced response of the system using modal superposition.

This paper is organized as follows. Section 2 provides the theoretical basis for SNM. In Section 3 a benchmark case is discussed in which the modes and forced response of a mistuned bladed disk are determined using a finite element code, SNM, and for comparison LMCC, a first generation reduced order model from Yang and Griffin (1997a). Some attributes of the new method are summarized in Section 4. Lastly, as part of the SNM methodology, a new procedure has been developed for calculating the mistuning in the reduced order model from the frequencies of the mistuned blades. The procedure is summarized in the Appendix.

2. THEORY

This section presents the mathematical basis of SNM. The first part transforms the equations of motion from the physical domain into one that is in terms of the nominal system modes. The equations are then rewritten in state space form so that motion dependent aerodynamic forces and gyroscopic forces can be included in the modal analysis. The number of degrees of freedom is then reduced by limiting the number of nominal modes used in the representatinn. The modal eigenvalue problem is then described. The modal eigenvalue problem determines mode shapes and complex eigenvalues of the mistuned system. It is a "modal" eigenvalue prohlem in that the mode shapes of the mistuned modes are represented as a weighted sum of nominal modes and the eigenvectors determined in the analysis are the weighting coefficients. The complex eigenvalue determines the natural frequency and aerodynamic damping of corresponding mistuned mode. This analysis determines the stability of the system since if the aerodynamic damping is negative then the mode may flutter. Expressions are then given for the forced response of the system in terms of the modes.

2.1 Transformation from Structural Domain to Modal Domain

Consider a mistuned bladed disk which rotates at a constant speed through a fluid that has a sinusoidal disturbance in space. By assuming a harmonic, steady state response, the equation of motion of the system can be written as

$$\left[K^o + \Delta K + i\,\omega(C+G) - \omega^2\left(M^o + \Delta M\right)\right]\bar{u} = \bar{f}_e + \bar{f}_m \qquad (1)$$

where K^o and M^o are the stiffness and mass matrices for a perfect system of the nominal design, ΔK and ΔM are the variations in the stiffness and mass matrices, **C** and **G** the damping and gyroscopic matrices, $\bar{u}$ the amplitudes, $\bar{f}_e$ the prescribed excitation forces, $\bar{f}_m$ the motion dependent aeroelastic forces, and ω the excitation frequency.

Consider also the modes $\bar{\phi}_j^o$ and natural frequencies ν_j^o of the nominal system which satisfy

$$K^o \Phi^o = M^o \Phi^o \Lambda^o \quad (2)$$

where the nominal modal and eigenvalue matrices Φ^o and Λ^o have the form

$$\Phi^o = \left[\vec{\phi}_1^o \, \vec{\phi}_2^o \cdots \vec{\phi}_N^o\right] \quad (3)$$

$$\Lambda^o = \mathrm{diag}\,({v_1^o}^2, {v_2^o}^2, \cdots, {v_N^o}^2) \quad (4)$$

and N is the number of degrees of freedom of the system. Typically, Φ^o and Λ^o are calculated using a finite element analysis of the nominal system. The nominal modal matrix Φ^o can be either real (a standing wave) or complex (a traveling wave) depending on whether it is computed using a full bladed disk model or a sector model. It is assumed in this paper that Φ^o is complex since the traveling wave coordinate system is consistent with the approach typically used for the aerodynamic forces.[2]

Since the $\vec{\phi}_j^o$ form a complete basis, the amplitude vector $\bar{u}$ can be expressed as a weighted sum of nominal modes, i.e.,

$$\bar{u} = \Phi^o \vec{\alpha} \quad (5)$$

where

$$\vec{\alpha} = [\alpha_1 \alpha_2 \cdots \alpha_N]^T \quad (6)$$

with α_j determining the amount that the j-th nominal mode $\vec{\phi}_j^o$ contributes to the response. Furthermore, the motion dependent aeroelastic forces $\vec{f}_m$ can be written as

$$\vec{f}_m = P^o \vec{\alpha} \quad (7)$$

where

$$P^o = \left[\vec{p}(\vec{\phi}_1^o)\, \vec{p}(\vec{\phi}_2^o) \cdots \vec{p}(\vec{\phi}_N^o)\right] \quad (8)$$

and $\vec{p}(\vec{\phi}_j^o)$ is the vector of unsteady aeroelastic forces caused by a unit amplitude vibration of the system in the mode $\vec{\phi}_j^o$.

Substituting (5) and (7) in (1) and pre-multiplying by ${\Phi^o}^H$, the Hermitian of Φ^o, the equation of motion of the mistuned system in terms of the nominal modes is,

$$\left[\hat{K}^o + \Delta\hat{K} + i\omega\left(\hat{C} + \hat{G}\right) - \omega^2\left(\hat{M}^o + \Delta\hat{M}\right) + \hat{Z}_a\right]\vec{\alpha} = \hat{\vec{f}}_e \quad (9)$$

where $\hat{K}^o$ and $\hat{M}^o$ are the diagonal modal stiffness and mass matrices of the nominal system, $\Delta\hat{K}$ and $\Delta\hat{M}$ the variations in the modal stiffness and mass matrices,

$$\Delta\hat{K} = {\Phi^o}^H \Delta K \Phi^o \quad (10)$$

$$\Delta\hat{M} = {\Phi^o}^H \Delta M \Phi^o \quad (11)$$

$\hat{C}$ and $\hat{G}$ the modal damping and gyroscopic matrices,

$$\hat{C} = {\Phi^o}^H C \Phi^o \quad (12)$$

$$\hat{G} = {\Phi^o}^H G \Phi^o \quad (13)$$

$\hat{Z}_a$ the modal aerodynamic impedance matrix,

$$\hat{Z}_a = -{\Phi^o}^H P^o \quad (14)$$

$\hat{\vec{f}}_e$ the modal excitation force,

$$\hat{\vec{f}}_e = {\Phi^o}^H \vec{f}_e \quad (15)$$

and the vector of weighting coefficients $\vec{\alpha}$ determines the amount of each nominal mode in the response.

Suppose that the modal aerodynamic impedance $\hat{Z}_a$ can be written in the following form (Hall et al., 1995)

$$\hat{Z}_a = \hat{K}_a + i\omega\hat{C}_a - \omega^2\hat{M}_a \quad (16)$$

where $\hat{K}_a$, $\hat{C}_a$, and $\hat{M}_a$ are the modal aerodynamic stiffness, damping, and inertia matrices that are, in general, complex and non-Hermitian. The modal equation of motion (9) can then be solved using standard methods by casting it in its state space form, i.e.,

$$(-A + i\omega B)\bar{y} = \bar{q} \quad (17)$$

where

$$A = \begin{bmatrix} 0 & I \\ -\left(\hat{K}^o + \Delta\hat{K} + \hat{K}_a\right) & -\left(\hat{C} + \hat{G} + \hat{C}_a\right) \end{bmatrix} \quad (18)$$

[2] Using a bladed disk sector model with cyclic symmetric boundary conditions to compute complex modes is more efficient and has less numerical problems since the eigenvalues are more isolated.

$$B=\begin{bmatrix} I & 0 \\ 0 & \hat{M}^{o}+\Delta\hat{M}+\hat{M}_a \end{bmatrix} \tag{19}$$

$$\bar{y}=\begin{Bmatrix} \bar{\alpha} \\ i\omega\bar{\alpha} \end{Bmatrix} \tag{20}$$

$$\bar{q}=\begin{Bmatrix} \bar{0} \\ \hat{\bar{f}}_e \end{Bmatrix} \tag{21}$$

Equation (17) is the state space form of the equation of motion of the mistuned system in terms of nominal modes.

2.2 Reduced Set of Modes

Thus far, the modal equation of motion (17) does not show any particular advantage over the original equation of motion (1) in terms of computational costs, since both of them have the same number of degrees of freedom. Equation (17) could be even worse since the coefficient matrices **A** and **B** are not sparse matrices. However, as discussed in Section 1, it is known from an earlier study (Yang and Griffin (1997a)) that, given small variations in the system's properties (i.e., small $\hat{K}_a$, $\hat{C}_a$, $\hat{M}_a$, $\hat{G}$, $\Delta\hat{K}$, and $\Delta\hat{M}$), a mode nf the structure will significantly change its shape only if the neighboring modes have frequencies which are very close. In this case, the resulting mistuned mode can be well approximated as a linear combination of the closely spaced nominal modes. Neglecting nominal modes with more remote natural frequencies results in errors which are inversely proportional to the frequency difference. Based on this understanding, it is then possible to use nnly a subset of the nnminal modes to estimate the associated mistuned modes of interest.

Suppose $\bar{\phi}^{o}_{s+1}$, $\bar{\phi}^{o}_{s+2}$, ..., and $\bar{\phi}^{o}_{s+n}$ are the subset of n nominal modes to be considered. Equation (17) then implies that the reduced order equatinn of motion in terms of a subset of nominal system modes is

$$(-a+i\omega b)\bar{y}=\bar{q} \tag{22}$$

where a and b are the reduced order matrices of **A** and **B** with dimensions 2n by 2n. The choice of n will be discussed in Section 3. The accuracy of a solution can be studied by examining how the results converge as n is increased. Clearly, the error goes to zero in the limit since the method provides an exact solution when all of the nominal modes are included in the representation.

2.3 Free Vibration (Flutter) Analysis

For free vibration, (22) implies a modal eigenvalue problem of the form,

$$\left(-a+i\nu_j b\right)\bar{r}_j=\bar{0} \qquad j=1,2,\cdots,2n \tag{23}$$

where ν_j is the j-th eigenvalue and $\bar{r}_j$ is the associated, state space right eigenvector of the mistuned system. Equation (23) can be solved using standard numerical routines. The eigenvalues are, in general, complex, i.e.,

$$\nu_j=\nu_{R,j}+i\nu_{I,j} \tag{24}$$

When $\nu_{I,j}>0$, the j-th mistuned mode has positive damping and is stable. When $\nu_{I,j}<0$, the mode has negative damping and the system flutters since the transient response will grow exponentially as time increases.

2.4 Forced Response Analysis

Assuming that $\nu_{I,j}>0$ for all $\bar{r}_j$, the solution tn the forced response problem (22) can be written as a linear combination of the state space eigenvectors $\bar{r}_j$, i.e.,

$$\bar{y}=\sum_{j=1}^{2n}\beta_j\bar{r}_j \tag{25}$$

where β_j is the amplitude of the mistuned mode $\bar{r}_j$ induced by the harmonic excitation. Second, substitute (25) in (22) and utilize mode orthogonality to decouple the equations. A simple expression for β_j can then be written as

$$\beta_j=\frac{\bar{l}_j^{T}\,\bar{q}}{\bar{l}_j^{T}\,b\,\bar{r}_j\left(\nu_j-\omega\right)}\,i \tag{26}$$

where $\bar{l}_j$ is the j-th left eigenvector of the eigenvalue problem associated with (23) which satisfies

$$\bar{l}_j^{T}\left(-a+i\nu_j b\right)=\bar{0}^{T} \tag{27}$$

Given expression (26) for β_j, the physical amplitude $\bar{u}$ can be calculated using (5), (20), and (25), i.e.,

$$\bar{u}=\Phi^{o}R_d\bar{\beta} \tag{28}$$

where

$$R_d=\left[\bar{r}_{d1}\,\bar{r}_{d2}\cdots\bar{r}_{d2n}\right] \tag{29}$$

and $\bar{r}_{dj}$ is the displacement part of the state space eigenvector $\bar{r}_j$.

3. NUMERICAL RESULTS

A computer program SNM (Subset of Nominal Modes) was written to implement the theory presented in the last section. A flow diagram of the SNM is shown in Fig. 1. Note that the structure of the program is relatively simple and, as a result, the code was much easier to develop than the first generation program LMCC (Yang and Griffin (1997a)) which required separate representations for the various substructures.

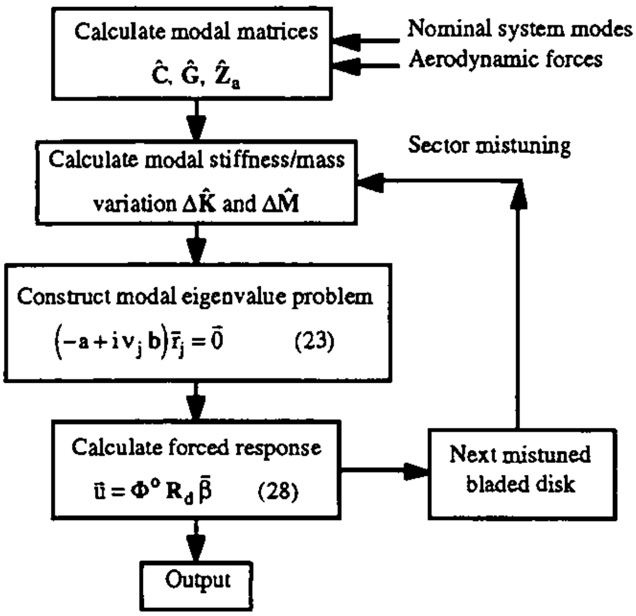


Fig. 1: Flow chart for the SNM algorithm

3.1 Benchmark Test Problem and Tuned System Results

In this section SNM is used to compute the dynamic response of the bladed disk system, Fig. 2, originally analyzed by LMCC. The geometry of the test problem was chosen to represent a structure with low aspect ratio, plate-like blades. A coarse model of plate elements was used so that the entire system would oot have too many degrees of freedom. This made it possible to run a benchmark, finite element analysis of the entire bladed disk without too much difficulty.

In this case the "nominal" system is "tuned", i.e., every blade, disk sector is identical and blade frequency mistuning is introduced by changing the elastic modulus from one blade to the next. An efficient procedure for calculating the change in the modal stiffness matrix, $\Delta\hat{K}$ in equation (10), from the frequencies of the mistuned blades and the stiffness matrix of a nominal blade is giveo in the Appendix. In this example, the motion dependeot aeroelastic forces, the gyroscopic forces, and ΔM in (1) are zero.

The geometry of the test problem is the same as the "thin disk" analyzed in the earlier work. When the disk is thin the disk modes tend to interact with the blade modes and more regions of frequency veering occur. Regions of frequency veering are wanted since they provide a more severe test of the accuracy of reduced order models, Yang and Griffin (1997a). The natural frequencies of the tuned system are shown in Fig. 3 as a function of the phase angle parameter, j, where the relative phase between adjacent sectors is equal to $2\pi j/N_b$ and N_b is the number of blades. Frequency veering is apparent in the way in which the frequencies tend to merge and then veer away from each other.

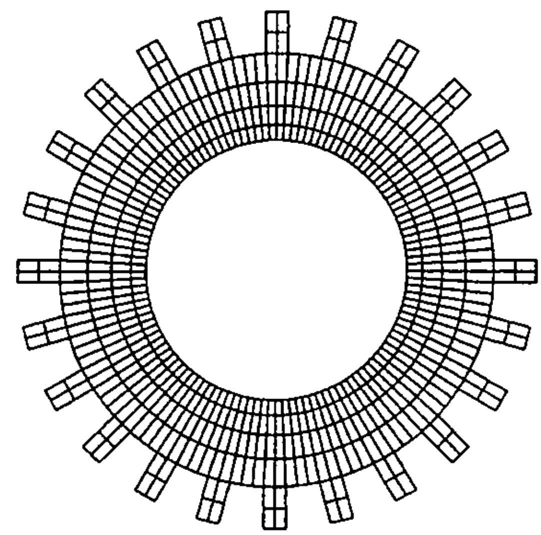


Fig. 2: Finite element model of bladed disk system

The SNM approach works well in bladed disk systems because the natural frequencies tend to occur in closely spaced clusters. This is because the modes tend to belong to families that share certaio characteristics. In many systems, the families can be readily separated into blade and disk modes. A family of *blade modes* are system modes in which the strain energy is stored primarily in the blades. Typically, a family of blade modes have frequencies that fall into a narrow range that is close to a frequeocy that the blade would have if it were fully coostraioed at its base. Thus, blade modes are further classified by type, e.g., a "first bending" family or a "first torsion" family of modes. For example, in Fig. 3, the lowest frequency modes for j equal to 6 through 12 belong to the first bending family of modes. *Disk modes* are system modes that have their strain energy stnred primarily in the disk. Often the frequencies of disk modes increase significantly as the phase angle parameter, j, increases. An example of a set of frequencies that belong to a family of disk modes would be the second highest frequencies for j equal to 6 through 11 in Fig. 3. Families of disk modes can be classified by the number of nodal circles. The lower frequency families have fewer nodal circles.

The bladed disk system used in this paper has a very flexible disk and, as a result, the frequencies of the disk mode families appear to cross over the frequencies of the blade modes. For example, the lowest frequency modes for j equal to 0 through 3 are disk modes whereas the lowest frequency modes for j equal to 6 through 12 are blade modes. Since for j equal to 4 and 5 the lowest frequency mode has to transition from being a disk mode to a blade mode, the actual mode shapes in the transition region tend to be a combination of modes from the two families. This phenomenon is ofteo referred to as frequency or curve "veering" in the literature, e.g., Perkins and Mote (1986), since a more careful analysis of the situation would show that the frequencies do not cross but, in fact, "veer" away from each other.[3]

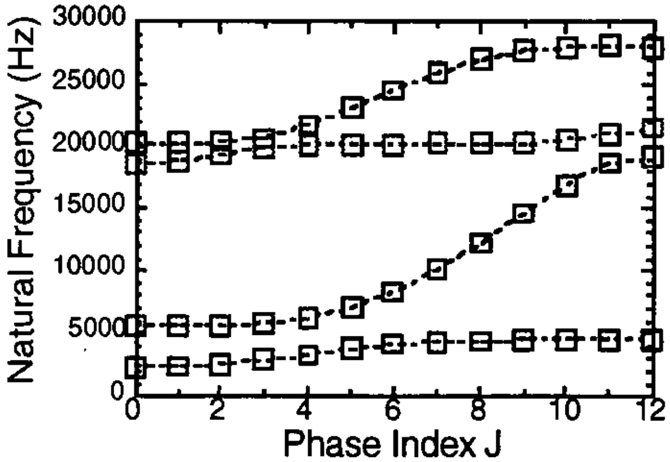


Fig. 3: Natural frequencies of tuned bladed disk

In an earlier paper on reduced order models, Yang and Griffin (1997a), the authors compared the exact natural frequencies of the tuned system with the natural frequencies of the tuned system calculated with their linear, mistuning computer code, LMCC. Their error was related to the number of blade modes and disk modes used to represent the substructures and the approximation in assuming that the bases of the blades are limited to rigid body type motions. This type of frequency comparison is not relevant in the approach introduced here since the exact, tuned system modes are now used as the basis of the representation. As a result, the new approach automatically gives exactly the same natural frequencies as the finite element model when the mistuning is set to zero in the reduced order model. The current method reduces the number of degrees of freedom by limiting the number of tuned system modes. Thus, the type of error that is introduced does not affect the frequencies of the tuned system modes that are included in the representation, but instead is apparent in the absence of frequencies that are not included in the representation.

Another property of SNM is that it exactly represents the modes of the tuned system when the mistuning goes to zero. This is especially important in regions of frequency veering. For example, consider the tuned system modes associated with the 4th highest frequency for j = 3 in Fig. 3. Because this frequency is very close to the 3rd highest frequency for j = 3, the associated mode shapes (it is a repeated frequency) are more difficult to determine since any error in the reduced order model will be amplified by the closeness of the two sets of modes. The sensitivity of the tuned modes to small modeling errors is due to the same mathematical problem that causes mistuning, i.e., small variations in the structural model can cause large changes in the mode shapes when the tuned system modes are close together. Here, however, the small variations in the structural model result from errors in the reduced order model, the changes in the mode shapes are displayed in the radial rather than the circumferential direction, and the tuned system modes are close together because of veering.

Thus, one of the reasons that LMCC had difficulty in predicting the mistuned response of the system in regions of frequency veering (Yang and Griffin (1997a)) is that it had difficulty in accurately predicting the modes of the tuned system. This is not a source of error in the SNM reduced order model since SNM automatically gives the same results as the finite element method when mistuning goes to zero.

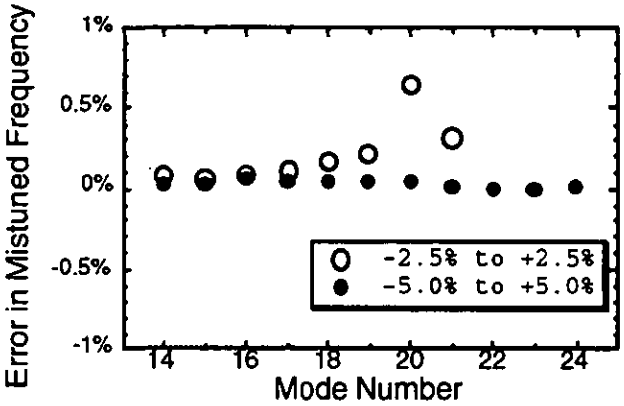
[3] Frequency veering can be seen in this example by using cyclic symmetric boundary constraints on a single blade, disk sector when calculating the system's natural frequency. In such an analysis, the phase difference across the sector can be taken as a continuous variable. One then finds that the frequencies do not cross but veer away from each other as the phase difference varies.

3.2 Mistuned System Results

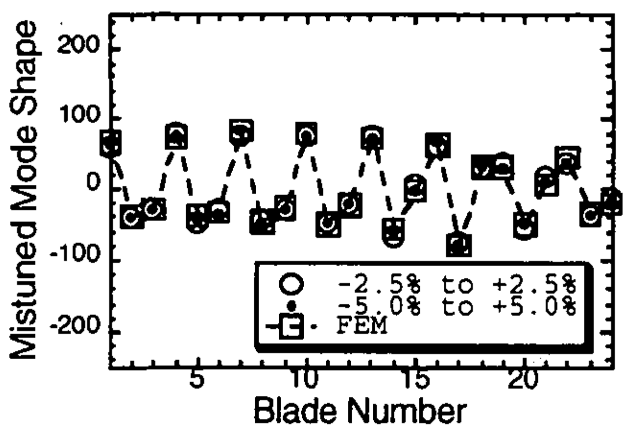
The accuracy of SNM depends on the number of degrees of freedom used to represent the structure. The number of degrees of freedom in SNM is the same as the number of tuned system modes used in the representation. An issue explored in this section is how the number of tuned system modes used in SNM affects the accuracy of the solution to the forced response problem. This issue can be divided into two parts. The first is how the number of tuned system modes affects the estimate of the mistuned system modes. The second issue is how many mistuned system modes are needed to determine forced response.

Mistuned modes. Since the frequencies of the bladed disk are only slightly perturbed when the system is mistuned, the mistuned modes also can be classified as belonging to same types of families as the tuned disk. For example, in the mistuned system the lowest frequency modes also belong to the first bending and the zero nodal circle mode families. As discussed in sections 1 and 2, the paper on modal interaction by Yang and Griffin (1997b) shows that the degree to which tuned system modes participate in forming the modes of the mistuned system is inversely proportional to the differences in their natural frequencies. Thus, the lowest frequency mistuned modes are comprised primarily of the first two families of tuned modes. This point is illustrated by the results shown in Fig. 4.

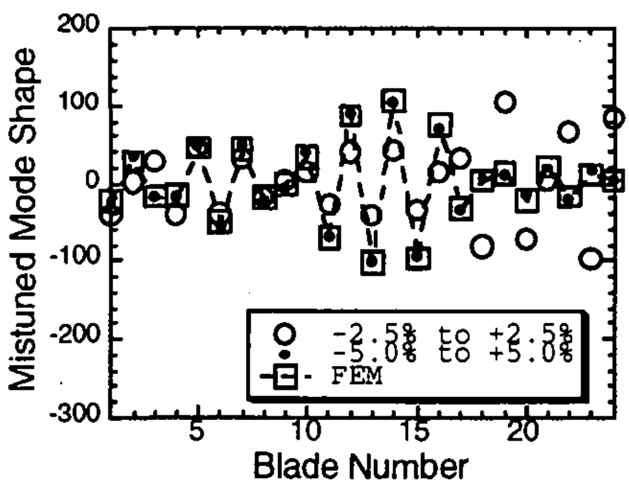
In Fig. 4(a), the estimated errors in the natural frequencies of the mistuned system are shown for a representative case from the first bending family of modes. The criteria used to determine which tuned modes to include in the reduced order model was to include all modes with natural frequencies in a certain range, i.e., $[f_{ave}(1-p), f_{ave}(1+p)]$. In Fig. 4(a) the average frequency, f_{ave}, was set equal to the lowest frequency mode for j equal to 8 (Fig. 3) and p was taken as 2.5% and then increased to 5.0%. The number of frequencies shown in plots is the same as the number of modes used in the model. Thus, Fig. 4(a) indicates that the number of tuned modes in the model increased from 8 to 11 when p increased from 2.5% to 5.0%. The



(a) Estimated frequency error



(b) Mode shape of the 16th mode



(c) Mode shape of the 20th mode

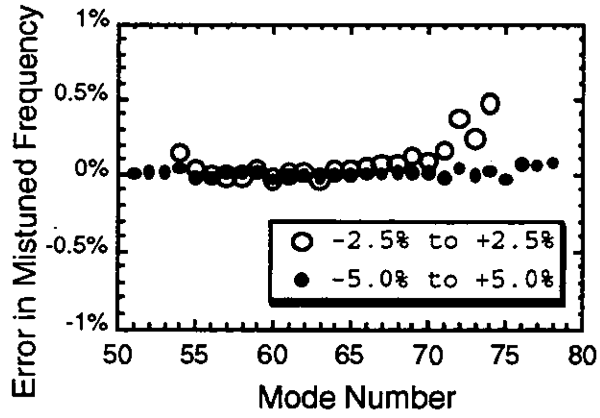
Fig. 4: Representative results for the first bending family of modes

estimated error is defined as $(f_j^{fem} - f_j^{SNM})/f_j^{fem}$ where f_j^{fem} is the j-th frequency calculated by a finite element analysis of a full mistuned bladed disk and f_j^{SNM} is the corresponding value calculated from the SNM model. The error is only an estimate since the eigenvalue solver in the finite element program may have significant errors when trying to calculate the modes and natural frequencies of a system in which the frequencies are very close together.[4] As can be seen the estimated error is reduced when p is increased to 5% and more modes are used in SNM. A mistuned mode corresponding to the 16th natural frequency is depicted as a function of circumferential position in Fig. 4(b). Note that both reduced order models, p equal to 2.5% and 5%, provide a reasonably accurate estimate of the mode shape. In this case the mistuned mode has a natural frequency that is in the center of the frequency range of tuned modes included in the representation. A second example of a mode shape, the 20th mode, is shown in Fig. 4(c). In this case, the mistuned mode has a natural frequency near the top of the range of frequencies included in the representation. Clearly, the error in the mode shape has been significantly reduced by increasing p from 2.5% to 5% in this case.

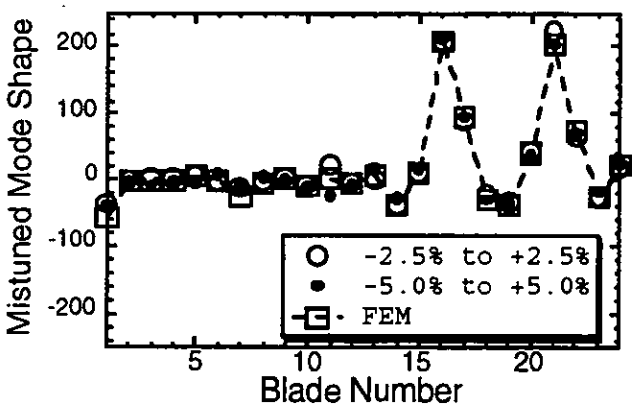
A more severe test of the SNM model are mistuned modes in the higher frequency, veering region near j equal to 4 in Fig. 3. A plot of estimated frequency error is shown for these modes in Fig. 5(a) for p equal to 2.5% and 5.0%. Again, the estimated error is reduced as more tuned system modes are included in the SNM model. A mode with a natural frequency near the center of the frequencies of the tuned modes used in the reduced order model is shown in Fig. 5(b). Again, both the 2.5% and 5% models provide a reasonably accurate representation of the mistuned mode. The results for a mode that has a natural frequency near the top of the range of frequencies included in the representation is shown in Fig. 5(c). Again, the error in the mode shape has been significantly reduced by increasing p from 2.5% to 5%. Note that the 5% model has only 28 degrees of freedom and, yet, gives very accurate results even in the veering region.

Forced response. A second issue is how many mistuned modes are needed to accurately estimate the forced response when the system is excited at a resonant frequency? Standard modal analysis indicates that the degree to which a mode contributes to the steady state, harmonic response of a system depends on three factors: the amount of damping, the difference between the mode's resonant frequency and the excitation frequency, and the generalized force exciting the mode. Typically, bladed disks are very lightly damped. As a result, when the excitation frequency is at a resonant frequency the mode that is excited will have a very high response. The contributions of other modes will die off in inverse proportion to the frequency difference. As a result, other families of modes that have distinctly different frequencies have little effect on the forced response of a particular mistuned family.

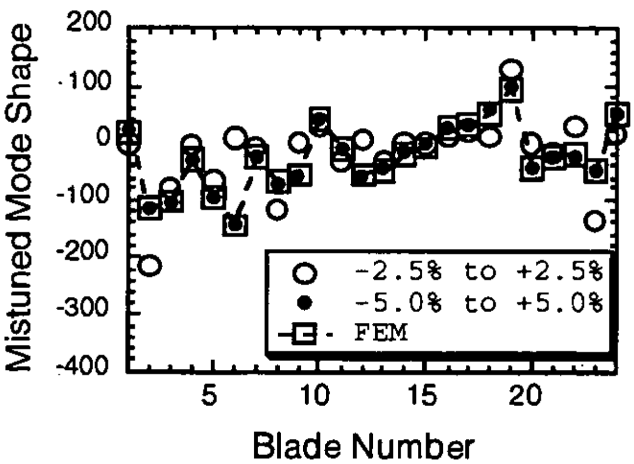
[4] This is usually not a problem for tuned system calculations since analysts use cyclic symmetry boundary conditions to isolate modes.



(a) Estimated frequency error



(b) Mode shape of the 56th mode



(c) Mode shape of the 72nd mode

Fig. 5: Representative modal results for the veering region

Additionally, the generalized force exciting a mode can also play an important role in determining if a mode will respond to a particular engine order excitation. For example, for low frequency, tuned modes the generalized force exciting the mode is zero unless the engine order of the excitation equals the number of nodal diameters (the parameter j of Fig. 3) in the mode. When a system is weakly mistuned the mistuned modes often tend to exhibit a similar spatial oscillatory behavior. Typically, the higher the frequency of a mode in a family and the higher the engine order of the excitation the more spatial oscillations (and nodal lines) they will have around the wheel. The generalized force acting on a mode is proportional to the dot produci of the mode shape and the vector representing the distribution of forces on the system. Consequently, in many cases a low order engine excitatinn tends to produce larger generalized forces on the lower frequency modes in a family, whereas, high engine order excitations tend to excite the higher frequency modes.

For these reasons, the number of mistuned modes needed to represent the forced response of the mistuned system is not very large. For systems in which the mode families are relatively isolated it would typically need to include only a single family of modes. In the case of veering when two families have a significant amount of interaction it would typically need to include only those two families of modes.

The forced response of the test problem was calculated using a modal summation, equation (28), in which the mistuned modes were determined by using SNM as well as the direct finite element method.[5] The forced response was also calculated using LMCC. For the low frequency modes both SNM and LMCC gave very satisfactory results. One difference was that LMCC results tend to exhibit a slight frequency shift[6] which SNM does not have.

A more interesting case was an excitation of modes in the veering region, i.e., a 4th engine order excitation in the frequency range of 19,000 to 21,000 Hz. The tip amplitudes of all twenty-four blades are plotted as a function of the excitation frequency for this case in Fig. 6. The finite element results, Fig. 6(a), is based on summing the response of 90 mistuned modes. The SNM model, Fig. 6(b), has 28 degrees of freedom (28 tuned modes) whereas the LMCC model, Fig. 6(c), has 144 degrees of freednm. Clearly, SNM mnre accurately captures the forced response of the mistuned bladed disk than LMCC. Note that no attempt was made to "calibrate" either the SNM or the LMCC model. As reported in Yang and Griffin (1997a), the accuracy of the LMCC model can be improved by artificially adjusting the attachment stiffness in the blade model so that the resonant frequencies of the tuned LMCC model more closely match the natural frequencies of the finite element model in the veering region. The attractinn of the SNM approach is that no modifications of the model are required to get accurate results in this case.

[5]The damping ratio in each mistuned mode was assumed to be 0.25% of critical damping.

[6]The frequency increase in LMCC results has been attributed to the increase in stiffness associated with the constraint that the blade's base undergoes rigid body type motions.

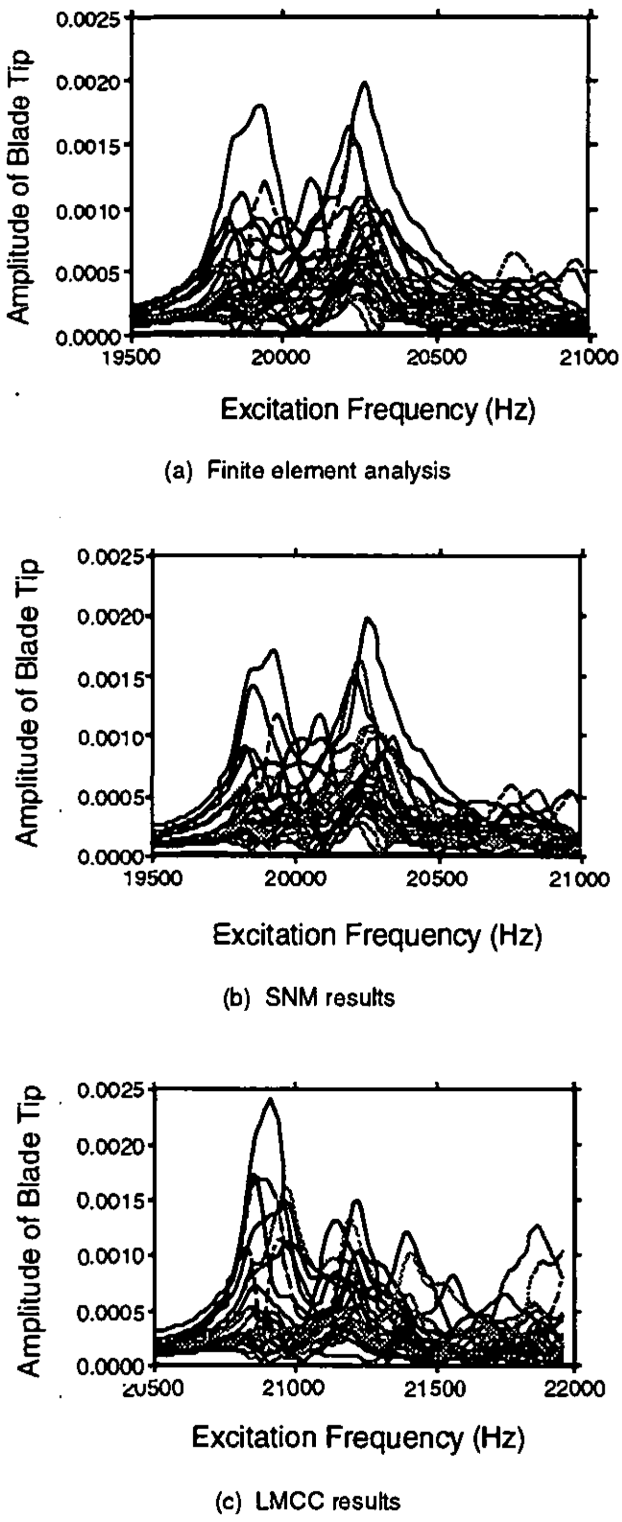


Fig. 6: Representative forced response amplitudes in the veering region

3.3 Numerical Efficiency

The SNM approach is more efficient than the earlier LMCC algorithm for two reasons. The first is that it requires fewer degrees of freedom to accurately represent the system. If N_b is the number of blades then LMCC reduces the number of independent degrees of freedom in the system to 6 N_b, the number of rigid body motions at the base of the blades. As discussed previously, SNM typically requires at most one or two times N_b modes to accurately represent the same system. Thus, the number of degrees of freedom is usually a factor of three nr more smaller in SNM than in LMCC.

A second reason that SNM it is more efficient than LMCC is that it calculates the forced response in a different manner. SNM calculates forced response by summing the mistuned modes, equation (28). The modes are found by solving a reduced order eigenvalue problem. LMCC solves a 6 N_b system of equations at each excitation frequency. In the simulatinns reported in this paper SNM was approximately 30 times faster than LMCC.

The relative efficiency of SNM or LMCC when compared with a direct finite element solution of a full mistuned bladed disk depends on the application. In developing either reduced order model there is the initial computational overhead of determining the tuned system modes that are used as input. Since the tuned system modes need to be calculated only one time, the real computational advantage in using reduced order models occurs when simulating a large number of mistuned bladed disks either to generate the statistical response of the mistuned system or for optimization purposes. In Yang and Griffin (1997a), the authors estimate that for a 100 bladed disk simulation LMCC would be one to two orders of magnitude faster than direct finite element simulation.

4. CONCLUSIONS

A new, reduced order model of mistuned bladed disks has been developed in which the modes of the mistuned system are represented in terms of a subset of nominal system modes, SNM. The SNM approach has a number of attractive features.

1. The input data are relatively easy to generate. For example, only a finite element analysis of a single blade, disk sector is needed to generate the tuned system modes as input to SNM. In addition, since only nominal modes are used as a basis for the representation, it is necessary to determine the motion dependent aeroelastic forces only for these modes. In contrast, in LMCC the aerodynamic influence cnefficients also have to be calculated for rigid body blade motions since they are also used to represent the system's response.

2. The mathematical formulation exactly yields the finite element solution as the amount of mistuning goes to zero. This is especially important when frequency veering occurs since other reduced order models can have difficulty in predicting the tuned system modes and natural frequencies in this case.

3. As the number of degrees of freedom in the SNM model increases the results from a mistuned system calculatinn

converges to the exact solution. Consequently, the accuracy of the SNM model can be checked by observing how the results converge as more degrees of freedom are used in the SNM model. This is especially important in bladed disks with a large number of degrees of freedom in the finite element model since it is difficult to run a finite element analysis of a full mistuned bladed disk as a benchmark.

4. The fact that the SNM approach first solves for the eigenvalues and modes of the mistuned system is appealing because:

   — It provides a very natural formulation for solving the mistuned aerodynamic eigenvalue problem (equation (23)) to determine how mistuning affects flutter.

   — It is easy to calculate the stresses in the bladed disk. This is because the mistuned modes are calculated as a weighted sum of nominal system modes. The modal stresses of the mistuned modes are the same weighted sum of the nominal system's modal stresses.

   — It is computationally very efficient since the forced response is expressed as a sum of mistuned modes that are independent of the excitation frequency (at least over the limited speed range associated with a particular engine order crossing[7]).

5. The SNM approach automatically captures the deformation at the base of the blade and the manner in which it transfers dynamic forces to neighboring blades. In contrast, LMCC imposes rigid interfaces between the blades and disk that only transmit non-selfequilibrated forces and moments. This increase in accuracy should be important when analyzing the effect of mistuning on the response of higher frequency blade mode families in which the dynamic forces tend to be more selfequilibrated.

6. The SNM approach uses "nominal modes" to represent the response and the nominal system modes do not have to be tuned modes. For example, the system cnuld have some type of imposed periodicity which may be taken into account in defining the nominal modes. The periodicity could be from periodic boundary constraints or imposed manufacturing variations. In this case, the nominal modes would be calculated from a single periodic sector that could include multiple blades. In fact, the modes of a completely mistuned system could be used as nominal modes and SNM used to determine the effect of additional random mistuning on the system's response. In this case the nominal modes would have to be calculated once from a finite element model of the full, mistuned stage.

An important area of research that still needs to be addressed is one of system identification. Typically, in the past only a limited amount of information, the blades' frequencies, have been used to characterize mistuning. Reduced order models make it possible to more precisely model the effects of real physical variations in a computatiooally efficient manner. Thus, the issues are: what physical variations are of primary importance, how can they be determined experimentally in an efficient manner, and how can they be efficiently translated into input that can be used in the reduced order models?

[7]If necessary, it would be relatively easy to include the effect of engine speed changes in the analysis either by calculating the nominal modes at different engine speeds and interpolating or by including the changes as prescribed perturbations in equation (1).

5. ACKNOWLEDGMENTS

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APPENDIX: EFFICIENT CALCULATION OF $\Delta\hat{K}$

If a cyclic symmetric structure is chosen as the nominal system, the computation of the modal stiffness variation $\Delta\hat{K}$, modal mass variation $\Delta\hat{M}$, and modal aerodynamic impedance $\hat{Z}_a$ can be simplified by taking into account its cyclic symmetry. This appendix will illustrate how to calculate $\Delta\hat{K}$ only since $\Delta\hat{M}$ and $\hat{Z}_a$ can be computed in a similar fashion.

Suppose the nominal system is of rotational periodicity m, the modal matrix of the nominal system Φ^o can be written as

$$\Phi^o = \begin{bmatrix} \Phi_1^o \\ \vdots \\ \Phi_m^o \end{bmatrix} \qquad \text{(A-1)}$$

where Φ_j^o is the modal matrix of the j-th cyclic symmetric sector of the nominal system. Since the mode shapes of all sectors are the same except for phase differences, the sector modal matrices can be expressed as

$$\Phi_j^o = \Phi_1^o E^{j-1} \qquad j = 1, \cdots, m \qquad \text{(A-2)}$$

where

$$E = \text{diag}(e_s) \qquad \text{(A-3)}$$

$$e_s = \exp\left(i\frac{2\pi s}{m}\right) \qquad s = 0, \cdots, m-1 \qquad \text{(A-4)}$$

Furthermore, the stiffness variation of the whole structure ΔK can be written in terms of those of the individual sectors ΔK_j

$$\Delta K = \text{diag}(\Delta K_j) \qquad j = 1, \cdots, m \qquad \text{(A-5)}^{8}$$

Substituting (A-2) and (A-5) in (10), the modal stiffness variation $\Delta\hat{K}$ becomes

$$\Delta\hat{K} = \sum_{j=1}^{m} \left(\Phi_1^o E^{j-1}\right)^H \Delta K_j \Phi_1^o E^{j-1} \qquad \text{(A-6)}$$

Suppose the stiffness variation of a sector can be characterized by q parameters

$$\Delta K_j = \sum_{r=1}^{q} \Delta c_j^{(r)} K^{(r)} \qquad j = 1, \cdots, m \qquad \text{(A-7)}$$

then, substituting (A-7) in (A-6) gives

$$\Delta\hat{K} = \sum_{j=1}^{m} \sum_{r=1}^{q} \Delta c_j^{(r)} \left(E^{j-1}\right)^H \hat{K}^{(r)} E^{j-1} \qquad \text{(A-8)}$$

where

$$\hat{K}^{(r)} = \Phi_1^{oH} K^{(r)} \Phi_1^o \qquad r = 1, \cdots, q \qquad \text{(A-9)}$$

In the case of blade frequency mistuning as studied in this paper, q is equal to 1, $\Delta c_j^{(1)}$ is the percentage change of the Young's Modulus of the j-th blade (the blade's frequency is proportional to the square root of Young's Modulus), and

$$K^{(1)} = \begin{bmatrix} K_b & 0 \\ 0 & 0 \end{bmatrix} \qquad \text{(A-10)}$$

where K_b is the stiffness matrix of the nominal blade.

It should be noticed that $\hat{K}^{(r)}$ needs to be calculated only once before running Monte Carlo simulations and the mistuning for each new bladed disk is incorporated by changing $\Delta c_j^{(1)}$. Therefore, using (A-8) to calculate $\Delta\hat{K}$ for each additional bladed disk requires very low computational cost.

[8] It is assumed here that the coupling stiffness between sectors does not change. This formulation can be extended to include that effect if it proved to be important.