ABSTRACT

The paper deals with the use of friction among bladed segments to reduce vibrations in turbomachinery. The non-linearity of the phenomenon is included in the analysis after that the detailed finite element model of the segment has been condensed. In the solution search, both numerical integration and harmonic balance method are used. The latter is here extended to the case of slip among parts in relative motion. By means of a sample case it is shown that the simplified harmonic solution can lead to the same results given by time-consuming numerical integration. Some insight on the slipping phenomenon between segments is obtained.

INTRODUCTION

To reduce vibrations in turbomachinery blade arrays, one of the most popular technical solutions takes advantage of the friction developed between mating surfaces of adjacent blades. This working condition has been studied by several researchers.

An important reference is represented by the papers presented by Griffin et al. [1, 5, 6, 7, 8]. In these, a solution technique for the case of a dry friction damper is developed by means of the Harmonic Balance Method (HBM). The solution is given initially for a one degree of freedom oscillator, and has been later extended to structures with many degrees of freedom. The main contribution consists in the approximation of the force transmitted by the damper, that makes use of different expressions for the cases of sticking or slipping. More recently [11], the case of a whole blade array has been faced by means of the cyclic symmetry theory [14].

Srinivasan and Cutts [2] built an experimental system to measure the vibrations damping of a single blade due to slipping at the interface with the disk.

Muszynska and Jones [3] studied the reciprocal interaction of the blades in the array, adopting a simplified approach to model the blades and the forces generated in the dampers. A similar approach was adopted by Dominic to study the influence of some characteristic parameters of the array.

Sinha [9] considered a single blade, modeled as simple oscillator, in order to optimize the friction force for damping of random vibrations.

Wang and Chen [10] refined the Harmonic Balance Method adding terms up to third order in the Fourier series that approximates the force transmitted by the damper.

Ewins et al. [12] adopted a different version of the Harmonic Balance Method for the case of bidimensional slipping, adapting the model to the case of microslip. Recently [13], they studied the case of a single blade with several dampers by means of a three-dimensional finite element model.

In all of the above mentioned researches rotor blades are considered. The present work deals with stator blades of a recent engine for jetliners. In fact, in modern engines also the length of the stationary blades has reached values such that the vibrational phenomena are relevant (in this case the length to mean chord ratio is about 5).

The considered blade array is part of a low pressure stage. The latter is formed by segments manufactured by casting, each of them contains six blades connected by
Detailed finite element modeling of the segment (17,000 element HEXA8 - 40,000 nodes)

Modal reduction

Non linearity

Numerical integration of the motion equations  Approximate analysis with harmonic solution

Fig. 1. Procedure adopted in the study.

the "upper platform" at the external radius and by the "lower platform" at the internal radius. The upper platform of each segment is fixed to the engine casing, the lower platform is in contact (on surfaces normal to the turbine axis) with the lower platforms of the two neighboring segments. The mating surfaces are mounted with interference in axial direction, thus a normal force is exerted and the segments are elastically pre-deformed (interlocking). During operation, the relative slipping of the surfaces generates the friction that is used to reduce vibrations.

Modeling the complete system (twenty eight segments and contact surfaces) would lead to excessive computational effort. The procedure adopted in this work, with the aim of reducing the problem size without over-simplifying it, includes the following steps (Fig. 1):
1) detailed modeling of the single segment, to accurately describe both the blade shape (twisted and tapered) and the geometrical details of the platforms;
2) modal reduction of the segment model, to reduce the problem dimensions saving the relevant vibrational properties;
3) analysis of a single segment or of a pair (with the possibility to take advantage of the cyclic symmetry), introducing only in this step the non linear aspects of the phenomenon (contact with friction).

Regarding the latter step, two approaches are suitable:
3a) numerical integration of the equations of motion; 3b) approximate analysis based on harmonic solution.

The first choice is potentially closer to reality, since it does not require any strong simplification. The second is based on some limiting assumptions, but reduces the computation time.

Both choices have been adopted in this work. The presented results correspond to the case of a pair of segments undergoing relative slipping. This does not claim to reproduce the behavior of an actual array, but represents a comparative benchmark for the two methods.

Tab. I. Mode shapes included in the condensation.

<table>
<thead>
<tr>
<th>n°</th>
<th>Hz</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>141</td>
<td>segment circumferential translation</td>
</tr>
<tr>
<td>2</td>
<td>276</td>
<td>segment axial translation</td>
</tr>
<tr>
<td>3</td>
<td>341</td>
<td>segment radial rotation</td>
</tr>
<tr>
<td>4</td>
<td>459</td>
<td>blade bending</td>
</tr>
<tr>
<td>5</td>
<td>464</td>
<td>blade bending</td>
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<tr>
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<td>467</td>
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<td>7</td>
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<td>8</td>
<td>474</td>
<td>blade bending</td>
</tr>
<tr>
<td>9</td>
<td>527</td>
<td>blade bending + platform translation</td>
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<tr>
<td>10</td>
<td>686</td>
<td>blade torsion</td>
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<td>blade torsion</td>
</tr>
<tr>
<td>15</td>
<td>764</td>
<td>blade torsion + platform translation</td>
</tr>
</tbody>
</table>

FINITE ELEMENT MODEL

Figure 2 shows the finite element model (NASTRAN) of the segment, including about 17,000 solid elements HEXA8 and 40,000 nodes (120,000 degrees of freedom). Table I lists the values of the eigenfrequencies and the features of the mode shapes: some of them concern the whole segment, others mainly the blades with almost still platforms. Only the modes not exceeding 1000 Hz are considered, since the excitation amplitude above this threshold is negligible.

For the subsequent calculations the model has been condensed using the "generalized dynamic reduction" [15] which differs from the usual "component mode synthesis"
because in the evaluation of eigenvalues and eigenvectors the not condensed degrees of freedom may be either constrained or unconstrained. For each contact surface, the degrees of freedom of one node have not been condensed and they have been assumed unconstrained in the calculation of the eigenvectors. Thus, the number of degrees of freedom used in the analysis for each segment is 21 (15 modal and 6 physical). However the stiffness and inertia properties of the segment are retained, with the usual approximation typical of the reduction technique.

NUMERICAL INTEGRATION

Contact modeling

For the solution of the non-linear problem an ad hoc code has been developed in the MATLAB environment. The contact is simulated by a gap element connecting two nodes, in which the contact and slipping forces are concentrated (Fig. 3a). This model is based on the penalty formulation schematically shown in Fig. 3b,c. It involves the following definitions:

- a local system x y z, where x is the gap closing and opening direction, y and z are lying in the tangential plane where slipping occurs;
- the value \( i \) (Fig. 3b) of the relative displacement of the nodes in x direction to simulate an initial interference or an initial clearance; in the present analysis an initial interference is imposed to account for the interlocking;
- the penalty stiffnesses: \( k_a \) in x closing direction that inhibits relative penetration of the mating surfaces, \( k_b \) which allows gap opening, \( k_a \) connecting the nodes during the sticking condition. The values of \( k_a \) and \( k_b \) are equal and approximately three orders of magnitude greater than the largest term of the stiffness matrix of the structure.

The solution is obtained by numerical integration with the Newmark method. During the time the contact condition can change from stick to slip. At each step the sticking/slipping condition is detected and the non-linearity is solved iteratively.

In the following it is briefly summarized how the program detects the change of contact conditions during the time integration.

- Change from stick to slip condition. At each time step it is checked whether the resultant of the tangential forces on the gap nodes becomes greater than the limit force \( \mu N \) (\( \mu \) is the static friction coefficient, \( N \) is the normal force), the exact time of this transition is detected since the equations of motion during the time step are known from the Newmark rule. After this time on the two nodes a tangential friction force equal to \( \mu N \) (\( \mu \) is the slipping friction coefficient), and directed opposite to their relative velocity, is imposed. More iterations for each time step are usually necessary in order to detect the direction of the friction force when slip starts.
- Change from slip to stick condition. At each time step it is checked whether the relative tangential velocity has the same direction of the force \( \mu N \) imposed to the node; if this occurs the stick condition has been achieved. The time of this transition is detected by recalculation adopting a reduced the time step.

The transition from opening to closing and from closing to opening are not described since, as it will be explained in the following, the gap is always closed.

Time integration results

The program uses as input the condensed matrices of the detailed model of the segment. Two segments clamped at the upper platforms are connected by a gap...
element interposed between two nodes of the slip surfaces of the lower platforms.

The local x direction of the gap (closing direction) is the global axial direction, the local y and z directions (slipping directions) are respectively the circumferential and radial directions.

At the present stage of the research, further information being still unavailable, the following assumptions have been adopted:

- the loading condition is a pressure distribution formed by a static component (estimated from experimental data) and an alternate component, the latter equal to 5% of the former; the frequency is close to the resonance with the lowest segment mode (141 Hz) and the phase changes according to the angular position of the segments;
- the damping matrix is proportional to mass and stiffness and the modal damping ratios are increasing with the mode order [16]; due to the lack of experimental data, we adopted values ranging from 0.01 for to first mode up to 0.1 for the fifteenth.

These assumptions have to be intended as testing conditions to check the method, and to evaluate the influence of the characteristic parameters. The actual values of the data, matching with the experimental results, will be identified in the further development of the research.

During operation, the force between segments in contact caused by the interference is increased by the gas pressure. In order to evaluate the role of the force, different analyses have been carried out, assuming for the mean normal contact force values up to about 400 N (including the pressure effect). Above this threshold the phenomenon is no longer interesting because slipping is excessively reduced. The values assumed for static and kinematic friction coefficients are 0.25 and 0.2.

As example, Fig. 4 shows the time history (after the end of the transient) of the relative displacement components of the two nodes in contact (that is, connected by the gap element), for the case of mean contact force equal to 260 N.

It can be noticed that the axial component is constant (and equal to the interference), since the nodes remain in contact, whilst the other diagrams are with good approximation sinusoidal.

Furthermore the motion amplitude in radial direction is much smaller than in circumferential direction.

In Fig. 5 are plotted, for several values of the mean normal contact force, the radial and circumferential components of relative displacement of the nodes in contact. The trajectory of the relative motion is an orbit strongly elongated in circumferential direction, except for
high mean normal contact force. No stop and motion reversal occurs, thus the system is always in slip condition and the friction force assumes the value given by the horizontal line of the diagram in Fig. 3c.

In Fig. 6 are plotted, for the same cases of Fig. 5, the radial and circumferential components of the friction force as a function of the homologous components of relative displacement between the surfaces in contact. Therefore, the area of every cycle is proportional to the energy dissipated by friction along that direction: the major importance of the amount related to the circumferential direction is evident. Furthermore, the circumferential force component remains practically constant during slip (parts approximately horizontal in the cycles).

These results are summarized in Fig. 7, where the energy dissipated by friction (expressed as a percentage of the maximum elastic energy in the cycle) is plotted as a function of the normal force. The total dissipated energy reaches a maximum when the contact force in the range 250-300 N.

In Fig. 8 the elastic energy is plotted, again as a function of the normal force: in this case the curve reaches a minimum under the same force range.

The maximum of the former curve and the minimum of the latter represent different aspects of a sort of optimum condition in which the motion is minimized by friction.

The results lead to the following observation about the behavior of the system, valid for interlocking values close to the optimum condition:

- despite the non-linearity, the time response is substantially harmonic;
- slipping concerns mainly the circumferential direction (both for the displacement and for the dissipated energy), except for the case of high mean normal contact force (i.e. high interlocking), in which the trajectory of the relative motion becomes almost circular;
- along the circumferential direction the friction force is almost constant over the cycle.

These properties justify the assumptions which are the grounds of the approximated analysis based on harmonic solution described in the next section.

**HARMONIC SOLUTION**

**External friction damper**

Here are reported the relevant aspects of the approach based on the harmonic balance method [6] that will be used in the following analysis. The oscillating system of Fig 9a, from which the case with many degrees of freedom is derived, includes an external damper connected to the ground, reacting with a constant force equal to \( \mu N \) during slipping and that is connected to the mass by a spring with stiffness \( k_p \), that can be regarded as the tangential penalty mentioned in the previous section.

The displacement \( u(t) \) is written as superposition of two terms: \( u_E(t) \), due to the external and known excitation \( F_E \), and \( u_N(t) \), due to the non-linear force \( -F_N \) transmitted by the damper. It is assumed that the system response is harmonic, in the form:

\[
  u(t) = u_E(t) + u_N(t) = B \cos \theta
\]

where \( \theta = \lambda t - \phi \). The force \( F_N \) has, over the time, the cyclic behavior shown in Fig. 9b. The cycle is formed by sticking periods, during which \( F_N \) is linear with respect to displacement, and slipping periods, during which \( F_N \) is constant and equal to \( \mu N \).

The cyclic behavior of \( F_N \) is approximated by a
Fourier series truncated to the first order:

\[ F_N = F^c \cos(\theta) + F^s \sin(\theta) + \ldots \]  \hspace{1cm} (2)

In slipping condition the coefficients take the values:

\[ F^c = \frac{k_p B}{\pi} \left( \theta^* - \frac{\sin 2\theta^*}{2} \right) \]  \hspace{1cm} (3a)
\[ F^s = -\frac{4\mu N}{\pi} \left( 1 - \frac{\mu N}{k_p B} \right) \]  \hspace{1cm} (3b)

where \(\theta^* = \arccos \left(1 - \frac{2\mu N}{k_p B}\right)\). In sticking condition \((\theta^* = \pi)\):

\[ F^c = k_p B \quad F^s = 0 \]  \hspace{1cm} (4a)

\[ F^s = \frac{4\mu N}{\pi} \left( 1 - \frac{\mu N}{k_p B} \right) \]  \hspace{1cm} (4b)

The amplitude \(B\) and the phase \(\phi\) are obtained by equating the amplitude of the cosine and sine term (harmonic balance) in eq. (1).

It is important to notice that, unlike the case numerically solved, in which there is continuous motion along a closed trajectory (Fig. 5), in this case the monodimensional motion reversal requires a quick transition slipping/sticking/slipping instead of the "cornering" observed at each end of the bidimensional trajectory. Therefore, neglecting the difference between the forces transmitted during slipping and sticking results to be correct, since the sticking condition is never met in the simulation that accounts for bidimensional relative motion.

**Internal friction damper**

In the present work the method mentioned in the previous section has been extended to the case of friction damper inserted between degrees of freedom \(j\) and \(i\) belonging to segments \(J\) and \(I\) of the array, as sketched in Fig. 10.

Assuming that the displacements of the degrees of freedom connected by the damper are harmonic in time, the cycle of the transmitted force is again that in Fig. 9b, provided that \(u_j - u_i\) is assumed as abscissa instead of \(u\):

\[ u_j - u_i = U_j \cos(\lambda t - \varphi_j) - U_i \cos(\lambda t - \varphi_i) \]  \hspace{1cm} (5)

where \(D = \sqrt{U_j^2 + U_i^2 - 2U_j U_i \cos(\varphi_j - \varphi_i)}\), \(\theta = \lambda t - \varphi\), \(\tan \varphi = \frac{U_j \sin \varphi_j - U_i \sin \varphi_i}{U_j \cos \varphi_j - U_i \cos \varphi_i}\).

The coefficients of the Fourier series of \(F_N\) are again given by eqs. (3a,b) and (4a,b), in which \(B\) is replaced by \(D\).

Similarly to the case of the one-degree-of-freedom system, the response of the j-th degree of freedom due to the external loads and to the force \(-F_N\) transmitted by the damper is:

\[ u_j(t) = u_{JE}(t) + u_{jn}(t) \]  \hspace{1cm} (6)

The displacement due to the external loads is:

\[ u_{JE} = u_{JE}^c \cos \lambda t + u_{JE}^s \sin \lambda t \]  \hspace{1cm} (7)

Fig. 9. Oscillating system with damper connected to ground: a) scheme; b) cyclic damper force.
The components in sine and cosine can be expressed by modal superposition:

\[ u_{jE}^c = \sum_{n=1}^{N_m} \Phi_{jn} k_n \left( f_{cn}^c H_n^c - f_{cn}^s H_n^s \right) \]
\[ u_{jE}^s = \sum_{n=1}^{N_m} \Phi_{jn} k_n \left( f_{cn}^s H_n^c + f_{cn}^c H_n^s \right) \]  \hspace{1cm} (8a,b)

where \( N_m \) is the number of modes included in the analysis and, for the \( n \)-th mode, \( \Phi_{jn} \) is the \( j \)-th component of the eigenvector, \( k_n \) is the modal stiffness, \( f_{cn}^c \) and \( f_{cn}^s \) are sine and cosine components of the modal external forces, \( H_n^c \) and \( H_n^s \) are the direct and quadrature components of the modal receptance.

The displacement caused by the force \(-F_N\) can be written as:

\[ ujn = \left( F^c r_{jj}^c - F^s r_{jj}^s \right) \cos \theta - \left( F^s r_{jj}^c + F^c r_{jj}^s \right) \sin \theta \]
\[ + \left( F^c r_{jj}^c - F^s r_{jj}^s \right) \cos \theta - \left( F^s r_{jj}^c + F^c r_{jj}^s \right) \sin \theta \]  \hspace{1cm} (9)

where \( r_{jj}^c \) and \( r_{jj}^s \) are the direct and quadrature components of the receptance for the \( j \)-th degree of freedom, that can be expressed by modal superposition:

\[ r_{jj}^c = \sum_{n=1}^{N_m} \Phi_{jn}^2 k_n H_n^c \]
\[ r_{jj}^s = \sum_{n=1}^{N_m} \Phi_{jn}^2 k_n H_n^s \]  \hspace{1cm} (10a,b)

Keeping in mind eqs. (7), (8), (9), (10), after some algebraic manipulation eq. (6) becomes:

\[ u_j = \]
\[ = \left( u_{jE}^c \cos \varphi + u_{jE}^s \sin \varphi + F^s r_{jj}^s - F^c r_{jj}^c \right) \cos \theta + \left( u_{jE}^s \cos \varphi - u_{jE}^c \sin \varphi - F^c r_{jj}^c - F^s r_{jj}^s \right) \sin \theta \]
\[ + \left( u_{jE}^c \cos \varphi + u_{jE}^s \sin \varphi + F^s r_{jj}^s - F^c r_{jj}^c \right) \cos \theta - \left( u_{jE}^s \cos \varphi - u_{jE}^c \sin \varphi - F^c r_{jj}^c - F^s r_{jj}^s \right) \sin \theta \]  \hspace{1cm} (11)

The response of the \( i \)-th degree of freedom to the external load and to the force \(+F_N\) transmitted by the damper can be obtained with the same procedure leading from eq. (6) to eq. (11).

The relative displacement in eq. (5) becomes:

\[ \left( u_{jE}^c \cos \varphi + u_{jE}^s \sin \varphi + F^s r_{jj}^s - F^c r_{jj}^c \right) \cos \theta + \]
\[ + \left( u_{jE}^s \cos \varphi - u_{jE}^c \sin \varphi - F^c r_{jj}^c - F^s r_{jj}^s \right) \sin \theta = \]
\[ = D \cos \theta \]

Also in this case, by means of the harmonic balance the amplitude \( D \) and the phase \( \varphi \) are obtained. From these, the response to the external loads and to the damper force can be evaluated for all degrees of freedom belonging to segments \( J \) and \( I \).

**Results**

The curves in Fig. 11 give the amplitude of relative displacement in circumferential direction of the damper ends (i.e. slipping of the two nodes), in the neighborhood of the first eigenfrequency, under the same values of mean normal contact force previously considered. The isolated points are control results given by numerical integration: the two types of solution are in excellent match, except for the case of mean contact force equal to 407 N. Such a discrepancy can be explained by the following argument. The hypothesis of monodimensional slipping motion assumed for the harmonic balance solution implies also that the friction force \( \mu N \) is totally applied along the circumferential direction. In case of high normal contact force the radial components of the motion and of the friction force are not negligible, as can be observed in Fig. 6.

The monodimensional assumption is no longer fulfilled, and this leads to underestimating the circumferential motion amplitude. However, it can also be remarked that such case lies far from the optimum condition previously identified and is potentially less interesting.

As a conclusion it can be stated that, when the hypotheses are respected, the harmonic balance method give results substantially equal to those obtained by numerical integration, with obvious advantages in terms of computation time. In the considered cases, the typical CPU time on a Pentium PC is 20 minutes for the integration of a single time history instead of a few seconds for a frequency response curve.

**CONCLUSIONS**

In the study carried out up to now the large non-linear initial problem has been condensed keeping only the number of parameters required to reproduce the system dynamics. The simplified problem has been solved both
by numerical integration and by approximate harmonic solution, the formulation of the latter has been extended to this case.

The comparison of the two methods shows that the results of the harmonic solution coincide with those given by the numerical integration. The findings, even if referred to a simplified case with two segments only, point out the existence of an optimal value of interlocking, confirmed by the experience.

ACKNOWLEDGMENTS

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REFERENCES

SYMBOLS

Variables:

\( B, U \) : displacement amplitude
\( N_m \) : number of modes
\( c \) : damping
\( r \) : receptance
\( D \) : differential displacement amplitude
\( t \) : time
\( F \) : force
\( u \) : displacement
\( f_0 \) : excitation force amplitude
\( x, y, z \) : gap local coordinates
\( F_1 \) : tangential gap force
\( \Delta u_t \) : relative tangential displacement
\( H \) : modal receptance
\( \Delta u_c \) : relative circumferential displacement
\( I, J \) : segment index
\( \Delta u_r \) : relative radial displacement
\( k \) : stiffness; \( \lambda \) : frequency
\( k_a \) : closing penalty stiffness of the gap
\( \theta \) : sine or cosine argument
\( k_b \) : opening penalty stiffness of the gap
\( \mu \) : friction coefficient
\( k_p \) : tangential penalty stiffness of the gap
\( \mu_s \) : static friction coefficient
\( m \) : mass
\( \phi \) : phase angle
\( N \) : normal contact force
\( \Phi \) : eigenvector

General sub- and superscripts:

\( c \) : cosine or direct component
\( N \) : non linear
\( E \) : external
\( n \) : mode index
\( i, j \) : degree of freedom index
\( s \) : sine or quadrature component