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An Experimental Study on the Relationship Between Velocity Fluctuations and Heat Transfer in a Turbulent Air Flow

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ABSTRACT

The work presented here is the first reported study to test the general correlation for turbulent heat transfer proposed by Maciejewski and Anderson (1996). A turbulent pipe flow apparatus was built for heat transfer and fluid studies. Tests were performed for a range of Reynolds numbers from 27,000 to 90,000. The heated wall temperature, adiabatic temperature, the wall heat flux and the maximum velocity fluctuations were measured at each Reynolds number. The non-dimensional groups recommended by Maciejewski and Anderson were formed and compared to the correlation. The results verify the correlation with agreement to within ±7% (as per figure 11). This study has important implications for the study of heat transfer in a wide range of fields, including the gas turbine industry. The development of a geometry independent correlation will lead to faster turn around times and improved engine design.

NOMENCLATURE

C_p	specific heat, [J/kgK or 1/kg]
c_f	skin friction coefficient
f	friction factor
k	conductivity [W/mK or 1/ms]
l	mixing length
Nu	Nusselt number
Pr	Prandtl number
q''	Wall heat flux (W/m^2)
Re	Reynolds number
St	Stanton number
T_w	wall temperature [°C or J]
T_{ad}	adiabatic wall temperature [°C or J]
U^*	reference velocity [m/s]
$\bar{u}$	mean component of turbulent velocity [m/s]
u'	fluctuating component of turbulent velocity [m/s]
u^+	non-dimensional mean velocity = $\bar{u}/u_w$
u'^+	non dimensional turbulent velocity = u'/u_w
u'_{max}	maximum turbulent fluctuations [m/s]
u_w	wall velocity = $\sqrt{V^2 f/8}$, [m/s]
V	pipe mean velocity, [m/s]

Greek

ε_h	eddy diffusivity for heat
ε_m	eddy diffusivity for momentum
ν	kinematic viscosity [m^2/s]
μ	viscosity [Pa-s]
ρ	density [kg/m^3]
Π_0	dimensionless driving potential
Π_1	dimensionless surface heat flux
Π_2	dimensionless velocity fluctuations
Π_3	Prandtl number

INTRODUCTION

One of the most persistent and difficult problems in the gas turbine industry is the prediction of heat transfer to turbine blades and vanes. Current heat transfer theory cannot accurately predict the heat transfer to the first stage blades of a newly designed gas turbine under all engine conditions. Most turbulent heat transfer correlations are restricted to the specific geometry for which they were developed

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because they are usually based on a length-based Nusselt number and a length-based Reynolds number. Testing geometrically similar engines is the only way to generate accurate correlations for gas turbine heat transfer prediction. The hope of constructing a general correlation for turbulent heat transfer rests on the possibility that surface convective heat transfer rates in turbulent flows can be characterized in purely local terms.

Recently, Maciejewski and Anderson, (1996) developed a geometry independent correlation for turbulent flow from studies of turbulent flow in air, water and FC-77, for internal flows, external flows and flows with high levels of free stream turbulence. They proposed a new non-dimensionalization technique and found that they could predict wall temperature in terms of the adiabatic temperature, the wall heat flux, the maximum stream-wise turbulent fluctuations and the fluid properties (without reference to the flow geometry) to within ± 12% for the cases studied. To develop the correlation they made some assumptions about the adiabatic temperature rise for the external flows and about the maximum turbulent fluctuations for the internal flows. This was necessary because none of the existing data sets contained all of the necessary information. The work presented here is the first to test the correlation proposed by Maciejewski and Anderson (1996) by measuring all of the required parameters.

Review of Existing Turbulence Models

The motivation for this work is to generalize existing methods for solving turbulent heat transfer problems. Current methods for solving turbulent heat transfer include empirical models for the Nusselt number and friction factor, and semi-empirical models which solve the time-averaged momentum and energy equations using empirical inputs to model the turbulent shear stress and turbulent heat transfer terms.

Semi-Empirical Models. The time averaged momentum equation for turbulent boundary layer flow with constant density and negligible body forces is:

$$\bar{u}\frac{\partial\bar{u}}{\partial x}+\bar{v}\frac{\partial\bar{v}}{\partial y}-\frac{\partial}{\partial x}\left[\bar{v}\frac{\partial\bar{u}}{\partial y}-\overline{u'v'}\right]+\frac{1}{\rho}\frac{dP}{dx}=0 \tag{1}$$

$\overline{u'v'}$ is the turbulent shear stress and represents the "closure problem" in turbulent heat transfer. The turbulent shear stress is usually modeled by an algebraic or differential equation. One of the simplest models assumes that the turbulent shear stress is proportional to the mean velocity gradient:

$$\overline{u'v'}=-\varepsilon_m\frac{\partial\bar{u}}{\partial y} \tag{2}$$

where ε_m is the eddy diffusivity for momentum and is related to the mixing length l as follows:

$$\varepsilon_m=l^2\frac{\partial\bar{u}}{\partial y} \tag{3}$$

The Prandtl mixing length theory proposes that close to the wall, l scales on the distance from the wall, and in the outer region of the boundary layer l scales on the momentum boundary layer thickness. Using empirical models for l and further assumptions about the shape of the velocity profile one can derive an approximate but closed form relation between skin friction and Reynolds number (see Kays and Crawford, 1993):

$$\frac{C_f}{2}=0.0287\,\mathrm{Re}_x^{-0.2} \tag{4}$$

The time averaged energy equation for turbulent boundary layer flow is:

$$\bar{u}\frac{\partial\bar{t}}{\partial x}+\bar{v}\frac{\partial\bar{t}}{\partial y}=\frac{\partial}{\partial y}\left(\alpha\frac{\partial\bar{t}}{\partial y}-\overline{v't'}\right) \tag{5}$$

Where $\overline{v't'}$ is the turbulent heat flux. Similar to the momentum problem, the turbulent heat transfer problem can be related to the temperature gradient:

$$\overline{v't'}=\varepsilon_h\frac{\partial\bar{t}}{\partial y} \tag{6}$$

where ε_h is the eddy diffusivity for heat. Using the turbulent Prandtl number, $Pr_t=\varepsilon_m/\varepsilon_h \cong 0.85$ the value for ε_h can be evaluated given ε_m. This allows for an approximate solution of equation (5) which yields (see Kays and Crawford, 1993):

$$St\,\mathrm{Pr}^{0.4}=0.0287\,\mathrm{Re}_x^{-0.2} \tag{7}$$

Equations (4) and (7) agree with experimental data for constant velocity and surface temperature, but the mixing length model breaks down if the flow changes physical nature. The eddy viscosity can become zero in regions of zero velocity gradients which poses a problem because it is not physically realistic. Better models such as the κ-ε and the algebraic stress model have been developed, however these require simultaneous solution of a set of equations and don't offer a closed form solution that is easily used by a designer.

Empirical Models. Another class of models are of the experimental variety in which the data from heat transfer experiments is plotted and correlations in the form of St, Nu =f(Re, Pr) result. These correlations exist for a variety of geometries, Prandtl numbers, and Reynolds numbers and provide a simple method for calculating heat transfer.

Among the many empirical models are that of Dittus and Boelter (1930) for fully developed turbulent flow in a smooth circular pipe:

$$Nu=0.023\,\mathrm{Re}^{\frac{4}{5}}\,\mathrm{Pr}^n \tag{8}$$

where n=0.4 for heating and 0.3 for cooling. The experimental conditions are a Prandtl number range of 0.7 to 160, Reynolds number greater than 10,000, and a length to diameter ratio greater than 10.

Sieder and Tate (1936) modeled the effect of property variations and found that

$$Nu=0.027\,\mathrm{Re}^{\frac{4}{5}}\,\mathrm{Pr}^{\frac{1}{3}}\left[\frac{\mu}{\mu_s}\right]^{0.14} \tag{9}$$

μ is the viscosity evaluated at the free-stream temperature and μ_s is the viscosity evaluated at the surface temperature. The experimental conditions are a Prandtl number range of 0.7 to 16,700, Reynolds number greater than 10,000, and a pipe length to diameter ratio greater than 10.

Petukhov (1970) recommended:

$$Nu = \frac{(f/8)\,\mathrm{Re}\,\mathrm{Pr}}{(1.07 + 12.7(\frac{f}{8})^{\frac{1}{2}}(\mathrm{Pr}^{\frac{2}{3}} - 1))} \qquad (10)$$

where the friction factor, f, can be obtained from the Moody (Nikuradse) diagram. This experimental conditions are a Prandtl number range of 0.5 to 2000, Reynolds number range of 10^4 to $5*10^6$.

All of the above correlations are for fully developed turbulent flow in a smooth circular pipe and are found to agree reasonably well with experimental data. Many correlations exist for this type of flow and still others for non-circular pipes, channel flow, and external flow in a variety of configurations. For specialized applications such as flow over a turbine blade or flow over an airfoil one must develop their own correlations using similar techniques.

The empirical and semi-empirical solution methods have shortcomings. Solving the momentum and energy equations for turbulent flow requires an iterative solution to the temperature field and carries some assumptions about the shape of the velocity profile (specifically, the law of the wall). In complex geometries or adverse conditions this may not hold. In addition, the computation time can be extensive. The correlations of the form Nu=f(Re, Pr) are established by experimental data from specific configurations. If this differs from the geometry for which it is being applied, the results may not be acceptable. The lack of generality in the existing methods has created the motivation for the development of a general correlation.

DEVELOPMENT OF A GENERAL CORRELATION

Most turbulent heat transfer studies rely on a traditional non-dimensionalization method which leads to the development of such non-dimensional variables as St, Nu, Re and Pr. Recent work by Maciejewski and Anderson (1996) used an innovative approach to dimensional analysis to establish the elements of a general correlation for turbulent heat transfer in incompressible flows which is independent of the geometry of the flow and independent of the thermal boundary conditions imposed on the flow.

The development of the Maciejewski/Anderson correlation is based on three assumptions: (1) the dimensional equivalence of the units of temperature and energy; (2) the existence of a single local measure of the turbulence which incorporates the effects of geometry; and (3) the existence of a single local reference temperature which incorporates the effects of thermal boundary conditions.

Assumption (1) is contrary to the position typically adopted by heat transfer texts although texts devoted to dimensional analysis either require (Sedov, 1959) or accept it (Bridgman, 1931). Adoption of this position leads to the formulation of a set of dimensionless variables for turbulent heat transfer which do not require the use of a geometric or fluid length scale. Assumption (2) was addressed by Maciejewski and Moffat (1992a,b) who showed that the local value of the surface heat flux may be directly determined by the maximum local value of the turbulent velocity fluctuations, u'_{max}. Assumption (3) was addressed by Anderson and Moffat (1992a, b) who showed that the local driving potential for heat transfer, which accounts for the effects of upstream thermal boundary conditions, is the difference between the actual local surface temperature, T_w and the local adiabatic surface temperature, T_{ad}.

Maciejewski and Anderson (1996) assumed that a relationship of the following form exists:

$$q_w'' = f(u'_{max}, \Delta T_{w,ad}, \rho, C_p, k, \mu) \qquad (11)$$

Following Panton (1984), Callen (1985) and others, they further assumed that temperature and energy are dimensionally equivalent. (In this system, ΔT is expressed in J, C_p is expressed in kg^{-1}, and k is expressed in $m^{-1}s^{-1}$. The Boltzmann constant, $k_B = 1.38 \times 10^{-23}$ J/K, is used to convert Kelvins to Joules for the purpose of evaluating ΔT, C_p and k.) The relation between the variables can be expressed non-dimensionally as follows:

$$\Pi_0 = f(\Pi_1, \Pi_2, \Pi_3) \qquad (12)$$

where:

$$\Pi_0 \equiv \frac{q_w''}{\rho U^{*3}} \quad \Pi_1 \equiv \frac{C_p(T_w - T_{ad})}{U^{*2}} \quad \Pi_2 \equiv \frac{u'_{max}}{U^*} \quad \Pi_3 \equiv \frac{\mu C_p}{k} \qquad (13)$$

and:

$$U^* = \frac{k}{(\rho C_p)^{2/3}} \qquad (14)$$

Π_0 may be interpreted as the non-dimensional surface heat flux, Π_1 as the non-dimensional driving potential for heat transfer (temperature difference), Π_2 as the non-dimensional level of the turbulent fluctuations, and Π_3 as the non-dimensional fluid viscosity (also recognized as the Prandtl number). The variables ρ, C_p, and k serve as the basis variables for the purpose of this non-dimensionalization. The combination $U^* = k/(\rho C_p)^{2/3}$ has dimensions of velocity and serves as a reference velocity. Fluid properties are evaluated at the film temperature, $T_f = (T_w + T_{ad})/2$.

Maciejewski and Anderson (1996) used the following five data sets to develop the correlation:

1) Maciejewski and Moffat (1992a, 1992b) - a study of boundary layer heat transfer in the presence of very high levels of free stream turbulence (i.e., local free-stream turbulence levels between 20% and 60% of the local free-stream mean velocity). The study was conducted in air on a flat heat transfer surface placed in the margin of a turbulent free jet.

2) Hollingsworth and Moffat (1989) - a study of boundary layer heat transfer in the absence of free-stream turbulence which was conducted on both flat and concave heat transfer surfaces placed in a water channel.

3&4) Anderson and Moffat (1992a, 1992b: 1990) - studies in air which were conducted in rectangular channels with roughness elements on one wall. The 1992 studies employ a dense array of flat elements and the 1990 study employs a sparse array of cubical elements.

5) Garimella and Schlitz (1992) [see also Schlitz (1992)] - studies conducted in rectangular channels with roughness elements on one wall using FC77 as a coolant. They employed an array of extremely flat elements.

The resulting correlation developed by Maciejewski and Anderson is:

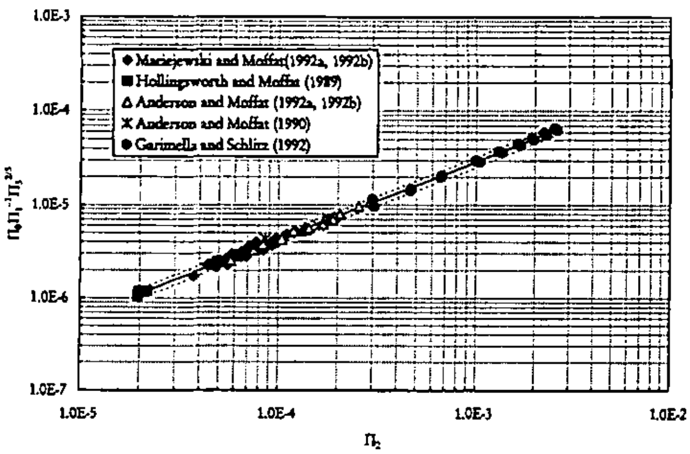

Figure 1. General Correlation.

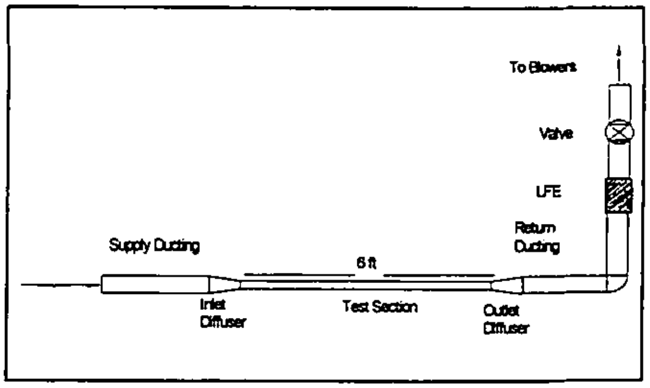

Figure 2. Schematic of Wind Tunnel.

$$\Pi_0 = 0.0092\,\Pi_1 \Pi_2^{5/6} \Pi_3^{-2/5} \tag{15}$$

The data and the correlation are shown in Figure 1. (The solid line represents the correlation and the dashed lines are ±12%). The correlation is limited to incompressible flows over the range of variables shown. The data sets used to generate the correlation represent a diverse range of experiments for which to test the hypothesis of Maciejewski and Anderson (1996). However, there were shortcomings to these data sets which prevent a clear verification of this correlation. The external flow data sets measured the maximum turbulent fluctuations, the wall heat flux, and the mean temperature of the flow but did not measure the adiabatic temperature. This was estimated using the mean and wall temperature data. The internal flow studies measured the adiabatic temperature and the wall heat flux but did not measure the maximum turbulent fluctuations. These values were inferred from the pressure drop in the test section. The study presented here includes direct measurement of the maximum turbulent fluctuations, the wall heat flux, and the adiabatic temperature.

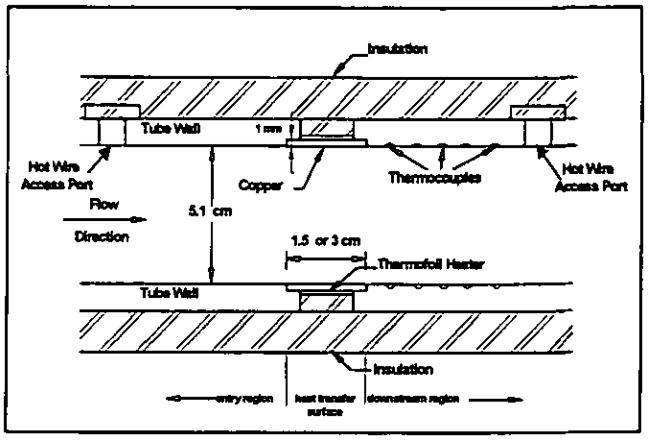

Figure 3. Schematic of Test Section.

DESCRIPTION OF EXPERIMENTAL FACILITY

A schematic of the wind tunnel is shown in Figure 2. The tunnel operates in suction mode. Air enters through a large filtered plenum into a 10.2 cm ID (4") PVC supply tube. The supply tube is connected to a 30.5 cm (12") long 10.2 to 5.1 cm (4 to 2") diffuser which decreases the inner tube diameter to the test section diameter. The test section is 1.8 m (6') long and is constructed from 5.1 cm (2") ID cast acrylic tubing. A 30.5 cm (12") long 5.1 to 10.2 cm (2 to 4") diameter diffuser joins the test sectinn to the return duct flow (10.2 cm ID PVC tube). A laminar flow element (LFE)is mounted downstream of the test section for measurement of channel flow rate and a valve is used to control the flow rate. The return duct is connected to two blowers which supply the air flow.

The test section is shown in Figure 3. It consists of an entry reginn, a heat transfer surface and a downstream region. The entry region is 88 cm long, which allows for a length to diameter ratio of 17.4 to ensure fully developed flow at the heat transfer surface. The heat transfer surface consists nf a 5.1 cm (2") ID, 1 mm (0.040") thick copper cylinder with an etched thermofoil heater glued around the outside diameter. The surface fits between the two acrylic tubes that form the entry and downstream regions. Tests were run for both a 3 and a 1.5 cm long (in the flow direction) heat transfer surface. The downstream region is 88 cm long and is instrumented with thermocouples for downstream temperature measurement. Hot wire access ports are located up and downstream of the heat transfer surface. Pressure taps are located 1 m apart for measurement of channel pressure drop.

Surface temperatures are measured on the heat transfer surface, on the entry region wall, and nn the downstream wall by calibrated 36 gauge type-K thermocouples. The inlet bulk temperature (which is also the adiabatic temperature for this study) was measured using three thermocouples placed around the center of the inlet flow approximately 1 meter upstream of the heat transfer surface.

Velocity measurements are made using a Dantec model 56C17 constant temperature hotwire anemometer with a miniature boundary layer probe (Dantec model P14) mounted on a Unislide computer controlled traversing system. The hotwire output is fed to a Keithly DAS800 A/D card mounted in a desktop PC. Data was acquired at 100 hz, with 2000 samples taken at each point. There are six access ports in the test section (three upstream and three downstream of the heat transfer surface).

Experimental Procedure

The heat transfer and velocity data were acquired separately. Velocity profiles were acquired for range of Reynolds numbers between 27,000 and 90,000. For each test the valve was set and the flow rate was measured with the laminar flow element. The test section pressure drop was measured and compared to predicted values. The hotwire calibration was checked against a pitot probe before each traverse and found to agree within 1.3% on average. At each point the computer recorded the hot-wire position, the average velocity of 2000 readings at that point, and the turbulent fluctuations based on the RMS value using 2000 readings. Each profile was then integrated over the pipe flow area to determine the flow rate and compared to the laminar flow element reading. The two agreed within 1.4% on average.

Temperatures were acquired for both the 1.5 and 3 cm heat transfer surface The flow rate was set to that of a corresponding velocity profile and the power to the heater was set to achieve a 20°C temperature rise at the copper surface. Temperatures were monitored until steady state was reached and then recorded for all thermocouple locations.

Data Reduction

The maximum turbulent fluctuations were found by direct examination of the velocity profiles. All profiles presented here were measured at the upstream port location. Centerline turbulence levels ranged from 1.2% at Re = 90,000 to 2.5% at Re = 27,000.The adiabatic temperature for this configuration is the measured inlet temperature so it's measurement does not require any additional tests (see Moffat and Anderson, 1990 for further explanation). The wall temperature was measured by the embedded thermocouples in the acrylic and copper sections. Fluid properties were evaluated at the film temperature. The value for the heat flux was calculated as the input power minus the conductive losses due to lead-wire conduction, and axial and radial conduction through the tube.

Lead-wire losses were calculated using a simple 1D model and were typically less than 0.1% of the input power. The tube conduction losses were calculated using a finite element model of the test section. The model inputs were the experimentally determined steady state temperature of the copper inner and outer surface, the downstream wall temperatures the outer tube temperatures and the input power minus the lead-wire losses. The tube conduction heat losses were found to be 6-10% of the input power for the 3 cm heater and 10-15% of the input power for the 1.5 cm heater.

Uncertainty Analysis

An Nth order uncertainty analysis was performed to determine the uncertainty on the two main calculated quantities, the wall heat flux and the maximum fluctuating velocity. The standard single sample uncertainty analysis as recommended by Kline and McClintock(1953) and extended by Moffat (1988) was used. All uncertainties are quoted at 20:1 odds. The uncertainty in adiabatic temperature rise was estimated as ±0.15 °C (0.7%). The uncertainty in the fluctuating velocity was based on the Chi-square variable for 2000 readings and was found to be approximately ±3%. The largest uncertainty was in the estimation of the wall heat flux due to the uncertainty in the tube conduction losses. This value was determined by perturbing the finite element program inputs and is estimated at ±50% which yields an uncertainty in the wall heat flux of 3 to 8%.

EXPERIMENTAL RESULTS

Aerodynamic Studies

Wall friction factors, f, were measured for Reynolds numbers from 20,000 to 90,000. The results are plotted in Figure 4 and show excellent agreement (for Re > 30,000) with a correlation for turbulent pipe flow. Uncertainty in the friction factor ranged from 1 to 5% and error bars are shown on the plot. The measured friction factor values for Re < 30,000 are lower than the predicted value which may indicate that the flow is not yet fully turbulent.

The structure of the flow was verified by making velocity traverses across the test section over the range of Reynolds numbers. Figure 5 plots the mean turbulent velocity versus distance from the wall in wall coordinates where:

$$u^+ = \frac{\bar{u}}{u_w} \text{ and } u_w = \sqrt{\frac{f}{8}V^2} \tag{16}$$

$$y^+ = \frac{y u_w}{\nu} \tag{17}$$

u_W is the friction velocity, ν is the kinematic viscosity, y is the distance from the wall and V is the mean velocity. The profiles collapse well in the log region and confirm that the flow is turbulent.

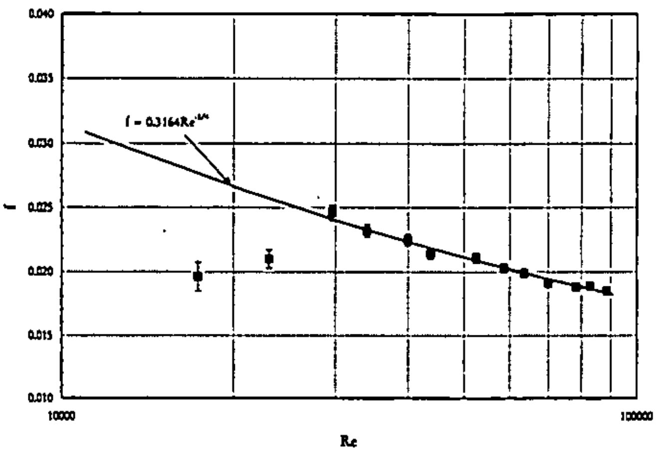

Figure 4. Friction Factor versus Reynolds Number.

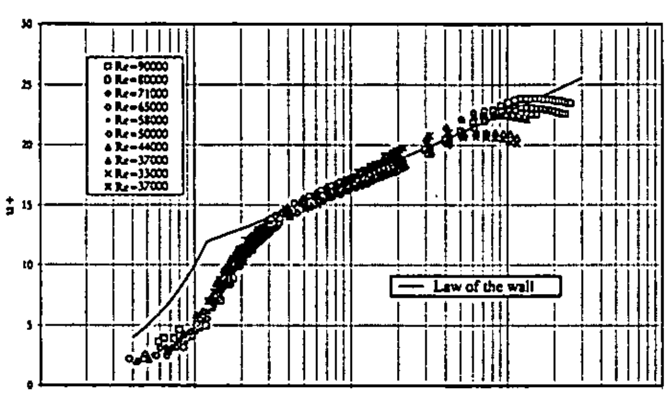

Figure 5. Mean velocity profile plotted in wall coordinates.

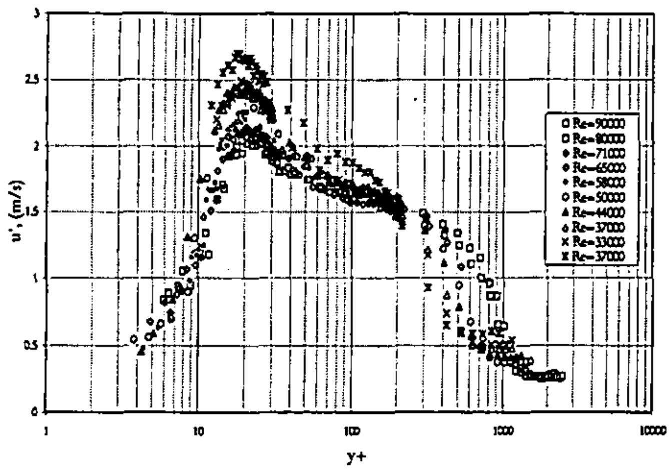

Figure 6. Fluctuating Velocities in wall coordinates.

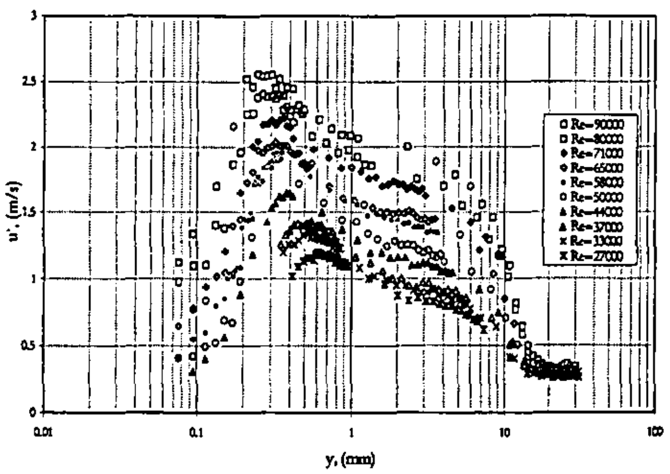

Figure 7. Fluctuating Velocity Profile, u' versus y.

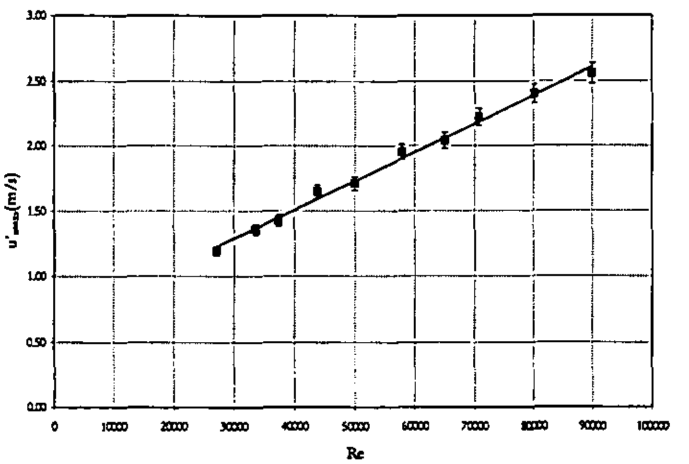

Figure 8. Maximum velocity fluctuations versus Re.

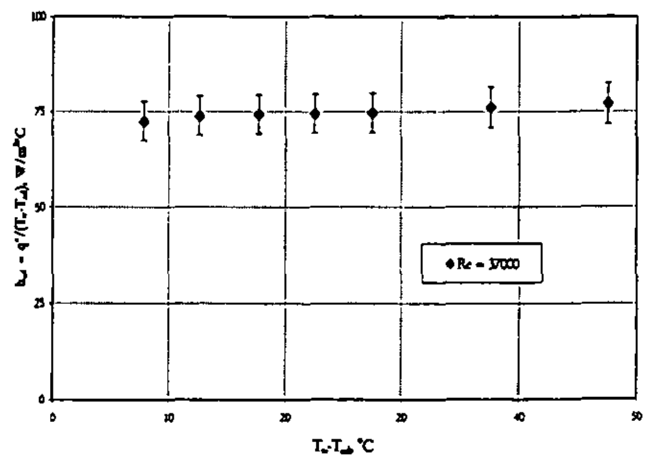

Figure 9. Heat transfer coefficient vs temperature rise.

Figure 6 plots the non-dimensional velocity fluctuations in wall coordinates where:

$$u'^{+} = \frac{u'}{u_w} \tag{18}$$

u'^{+} peaks between y^{+} = 10 and 20 which agrees well with turbulent boundary layer data (Hinze, 1975). The maximum value decreases from about 2.7 at a Reynolds number of 27,000 to 2.0 at a Reynolds number of 90,000.

Figure 7 plots u' versus y for all cases tested. In this case, the maximum value of u' increases from about 1.2 m/s at Re = 27,000 to 2.6 m/s at Re = 90,000. The peak location moves closer to the wall as the Reynolds number increases. Figure 8 plots u'_{max}, as determined by observation of the profiles, versus Reynolds number. For the range of flows studied the relationship is fairly linear as shown in the figure. All data values are listed in Table 1 (1.5 cm heater) and Table 2 (3 cm heater).

Heat Transfer Data

Figure 9 plots the average heat transfer coefficient (at the heat transfer surface) versus the wall temperature rise for a temperature rise from 5 to 50 °C. The plot verifies the data reduction scheme showing that the values are independent of temperature rise. The uncertainty in the adiabatic heat transfer coefficient is about 7% for this Reynolds number as shown.

Figure 10 plots the heat transfer coefficient versus Reynolds number for the 1.5 and 3 cm heaters. For the range tested h increases fairly linearly with Re. The data is also compared with values predicted by equation (7) for fully developed turbulent pipe flow. As expected the experimental data (which is in the thermal entry region of the pipe) is higher than the correlation for the fully developed region. Data values are listed in Tables 1 and 2.

Correlation Predictions

The non-dimensional variables recommended by Maciejewski and Anderson (1996) were formulated as shown in Tables 1 and 2. Figure 11 plots the new data as Π_2 versus $\Pi_0\, \Pi_1^{-1}\, \Pi_3^{2/5}$. The solid line is

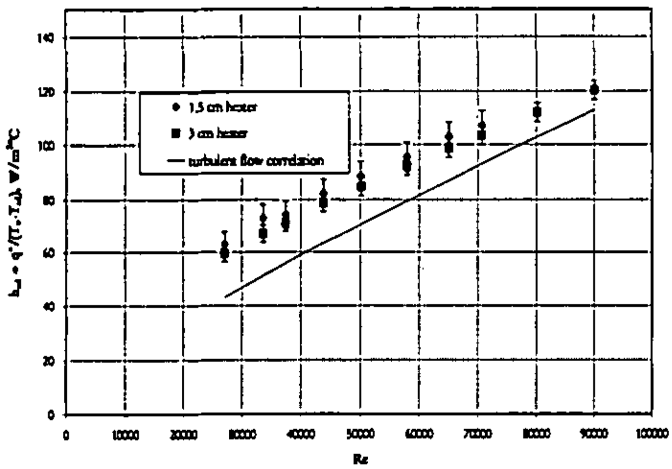

Figure 10. Heat transfer coefficient versus Re.

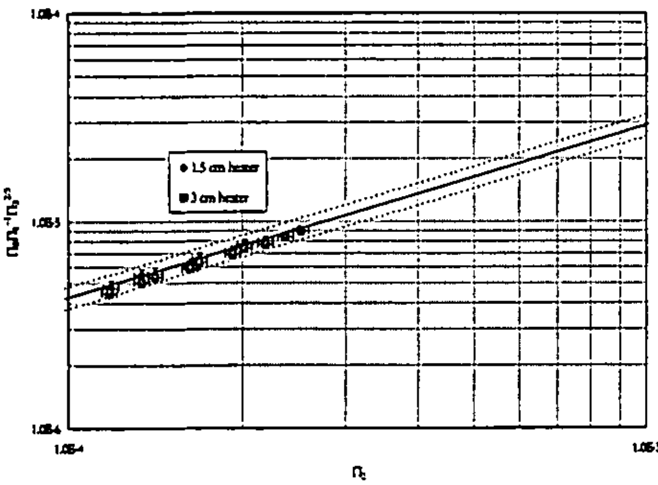

Figure 11. Data in Maciejewski/Anderson variables.

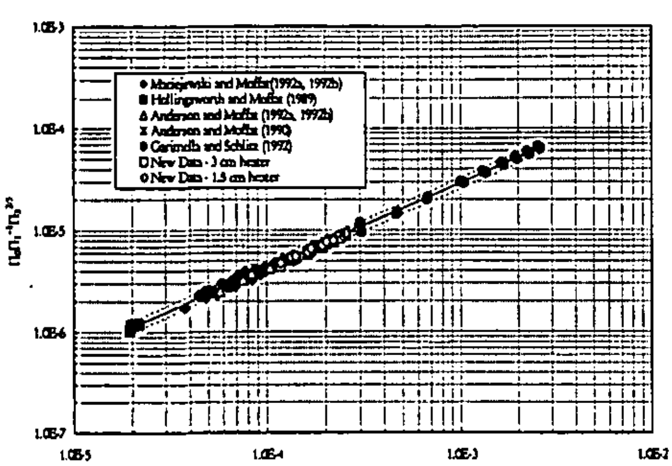

Figure 12. Maciejewski and Anderson Correlation.

the Maciejewski/Anderson correlation and the dashed lines are ±12%. The 1.5 cm heater data appear to lie on top of the correlation, while the 3 cm data is slightly below. Comparing each to the correlation, the 1.5 cm data agrees within a standard error of 3% and the 3 cm data agrees within a standard error of 7%. Figure 12 plots the same data along with all the other data examined by Maciejewski and Anderson (1996).

DISCUSSION

There are four elements of a general method for predicting turbulent heat transfer: (1) a method for predicting the local surface heat flux, q_w'', in terms of the local variables u'_{max}, T_w-T_{ad}, ρ, C_p, μ, and k, (2) a method for estimating u'_{max}, (3) a method for estimating T_w-T_{ad}, and (4) a method for estimating ρ, C_p, μ, and k.

The foundation for the first of these elements was proposed by the work of Maciejewski and Anderson (1996) and confirmed by this work.

This study has not addressed element 2. Estimation of u'_{max} is non-trivial and it is essential to the proposed method for evaluating turbulent heat transfer. However this falls within the domain of turbulence simulation and turbulence modeling. (Existing k-ε models could be used to estimate u'_{max}.) Or it can follow the work of Anderson and Moffat (1992a,b) who developed a method for estimating u'_{max} based on the channel pressure drop.

The third element is a method for estimating the local value of T_{ad}. (Once T_{ad} is determined, ρ, C_p, μ, and k can be evaluated via thermodynamic equations of state.). The estimation of T_{ad} can be conducted within the framework of linear mathematics, i.e., superposition. In flows which exhibit a clear marching direction, the superposition method can be applied to calculate the adiabatic temperature. In a recirculating flow the value of the adiabatic temperature will approach the value of the bulk-mean temperature due to strong thermal mixing. An estimate of the bulk mean temperature may suffice for use in the correlation.

If these elements of a general method for predicting turbulent heat transfer can be established, then a general method for predicting turbulent heat transfer might consist of the following sequence of steps:

1. Use an existing technique in turbulence simulation or in turbulence modeling to estimate u'_{max} throughout the domain of the analysis in question.
2. Use the superposition method to evaluate T_{ad} at the first "point" along the boundary of the domain of the analysis.
3. Use the general correlation for turbulent heat transfer to evaluate q_w'' (given T_w) or T_w (given q_w'') at the first "point" along the boundary of the domain of the analysis.
4. "March" along the boundary of the domain of the analysis, applying step (ii) and step (iii) at each "point" along the boundary.

CONCLUSION

This correlation verifies that turbulent heat flux can be predicted by purely local variables, independent of the global geometry of the flow and of the thermal boundary conditions. The level of the turbulent velocity fluctuations characterizes the flow boundary conditions, while the adiabatic temperature characterizes the thermal boundary conditions.

This work is also part of the development of a general method for solving turbulent flow heat transfer. The motivation for this work was to test the Maciejewski/Anderson correlation using a facility which would directly measure the wall temperature, the adiabatic temperature, the wall heat flux and the maximum turbulent velocity fluctuations. The results presented here confirm the Maciejewski/Anderson correlation for an internal turbulent pipe flow of air. Although only a small range of the correlation has been tested the fact that it lies directly on the correlation validates the original range of flows tested. Further work should look at extending the range tested under a variety of turbulent flow conditions.

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Re	u'_{max} (m/s)	T_{ad} (°C)	T_w (°C)	q''_{wall} (W/m^2)	Π_0 x 10^9	Π_1 x 10^4	Π_2 x 10^4	Π_3
27104	1.19	22.6	45.4	1442.6	1.22	2.25	1.18	0.71
33618	1.36	24.5	45.1	1510.1	1.27	2.02	1.34	0.71
37441	1.43	24.6	45.2	1532.1	1.29	2.02	1.41	0.71
43820	1.65	22.6	45.2	1859.6	1.58	2.24	1.63	0.71
50000	1.71	24.7	45.1	1809.1	1.52	2.00	1.69	0.71
57897	1.95	24.6	45.1	1960.1	1.65	2.01	1.93	0.71
64984	2.04	22.9	45.4	2320.2	1.97	2.22	2.02	0.71
70687	2.22	24.8	45.1	2178.9	1.83	1.99	2.19	0.71

Table 1. 1.5 cm Heater Data

Re	u'_{max} (m/s)	T_{ad} (°C)	T_w (°C)	q''_{wall} (W/m^2)	Π_0 x 10^9	Π_1 x 10^4	Π_2 x 10^4	Π_3
27104	1.19	24.8	45.1	1211.8	1.02	1.99	1.18	0.71
33618	1.36	24.8	45.1	1363.6	1.15	1.99	1.34	0.71
37441	1.43	24.8	44.8	1428.3	1.20	1.96	1.41	0.71
43820	1.65	24.9	45.2	1599.8	1.34	1.99	1.63	0.71
50000	1.71	24.7	45.3	1745.3	1.46	2.02	1.69	0.71
57897	1.95	24.8	45	1867.2	1.57	1.98	1.93	0.71
64984	2.04	24.8	45	1995	1.68	1.98	2.01	0.71
70687	2.22	24.8	45.2	2110.1	1.77	2.00E	2.19E	0.71
80137	2.40	24.8	45	2264.9	1.90	1.98E	2.37E	0.71
90060	2.56	25	45.3	2442.5	2.05	1.99E	2.52E	0.71

Table 2. 3 cm Heater Data

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