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MODULAR SIMULATION OF COOLANT INTERNAL NETWORK AND ROTATING CAVITY ANALYSIS



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ABSTRACT

The increase in gas turbine performance requires very high total inlet temperatures even in heavy-duty applications. Therefore, an accurate design of both the blade cooling and the cooling network from the compressor to the blade is very important. In previous works, the authors have studied the cooling blade problems for both the stator and the rotor case. The present paper presents a simple and fast procedure to study the cooling network: a modular code has been developed for this purpose and particular attention has been focused on the study of the rotating cavities between the stator and the rotating disks.

The results we have obtained are good and the code developed is at present used in industry.

NOMENCLATURE

a	disk minimum radius value	[m]
b	disk maximum radius value	[m]
C	number of boundary equations	
C <sub>p</sub>	specific heat (constant pressure)	[J/kgK]
f	generic function	
H	enthalpy	[J/kg]
kc	conductibility	[W/(mK)]
k	coefficient (used in linear equation)	
l	number of the link equation	
m	mass flow rate	[kg/s]
M <sub>R</sub>	momentum of rotating disk	[Nm]
M <sub>S</sub>	momentum of stator disk	[Nm]
N	number of elements	
n	number of the nodes	
p	number of the properties	

p	static pressure	[Pa]
q	heat flux	[W/(m <sup>2</sup> s)]
r	radius	[m]
R	gas constant	[Nm/kgK]
S	disk surface	[m <sup>2</sup> ]
T	absolute temperature	[K]
u	number of data input and output of the element	
v <sub>φ</sub>	tangential velocity	[m/s]
v <sub>r</sub>	radial velocity	[m/s]

Greek

φ	angular coordinate	[rad]
μ	dynamic viscosity	[kg/(ms)]
ρ	density	[kg/m <sup>3</sup> ]
Ω	disk angular velocity	[rad]
ω	flux angular velocity	[rad]

Dimensionless groups

C <sub>w</sub> = m/μ <sub>cool</sub>	mass flow coefficient
Nu = hΔr/k <sub>e</sub>	Nusselt number
Pr = C <sub>p</sub> μ/k <sub>e</sub>	Prandtl number
Re <sub>φ</sub> = ρΩb <sup>2</sup> /μ	rotational Reynolds number
x = r/b	dimensionless radial coordinate
XK = ω/Ω	slip coefficient

Subscript

Cool	coolant
i	i element
il	input l

i2	input 2
o1	output 1
o2	output 2
in	input
out	output
m	average quantity

### Superscript

0	booting data
'	first iteration
''	second iteration
'''	third iteration

## INTRODUCTION

In the next twenty years the gas turbines have been become the most important equipment for energy conversion systems. The continuous performance increase has also been realised with the relevant introduction of turbine blade cooling. This cooling requires an increasingly complex design of nozzles and rotor row and the authors have been already studying the blade cooling phenomena in the past years (Carcasci et al. 1993,1994,1995,1996), but on the other hand an appropriate coolant network distribution is needed to move the compressor air bleeding to the turbine blades, cavity and other auxiliary components of gas turbine. The analysis of the cooling network is very difficult, in fact it is normally constituted by many parts arranged in complicated ramifications, and the boundary conditions are not always the same because of the insufficient experimental data.

Generally, in order to describe a cooling circuit even up to 50-60 components are required: they may be turns, fast expansion or contraction, rectilinear pipes, rotating cavity and so on. This type of problem requires a simplified approach with 0-1 dimensional approach for a fast and flexible solution. Moreover, the complexity of the circuit involves the necessity of multi-input and/or multi-output component use: a mixing or a separating component is a typical example of this situation.

However, even the more complex circuits are composed only with a series of few elementary components, each with its geometric configuration. This presupposes the use of a modular approach to solve the problem. In other words, to obtain the greater possibility of analysing whatever problem, we schematise the system in elementary modules, each one simulated by its physic-mathematics form

The authors and also in the past, many researchers have approached with different methods the development of modular codes in the energy systems simulation. We can divide these methods in two great families:

- «sequential» method
- «parallel» method

The modular sequential codes (also called «explicit») go on along the mass flux in the net circuit, solving the modules one by one, according to the precedence in the circuit: some input data

are missed in a node it is supposed and later verified in an iterative way (Cioli and Desideri, 1991; Perz, 1991).

The modular parallel codes (called «implicit» too) solve each module independently from the others and put the resulting data together with the continuity equations in an equations system that will provide the new conditions for each module (Perz, 1991, Carcasci and Facchini, 1995).

For this work, we used a «semi-parallel» method: similarly to the sequential method, it solves the modules one by one along the mass flux and it supposes the missed data, but it revises them by solving a system similarly to the parallel method.

The developed code allows then an easy analysis of any cooling network, based upon a wide module library and particular attention has been given to the rotating cavity problem. In fact, in a previous work, the authors have studied this problem developing dedicated code on it (Carcasci et al.) based on other studies too (Von Kármán, 1921; Daily and Nece, 1960; Owen and Wilson, 1995; Owen and Roger, 1989; Stewartson, 1953). For the rotating cavity module simulation we have inserted this code in the cooling network.

## MODULAR CODE

The study of any configuration of cooling systems is very important. So we have developed a code with modular characteristics:

- *Flexibility*: the user can analyse every scheme without creating or rewriting the source code.
- *Easy use*: the user has to supply to the code only the information about the system configuration.
- *PC-Based system*: at least for few a modules (40-50), the code simply requires a personal computer. Only for very complicate system it is better to use a workstation.
- *Speed*: the calculation time has to be short
- *Expandability*: a new module insertion has to be easy enough.

### Module characteristics

Globally a cooling network appears as a system of ducts where the mass flux circulates and receives physical transformation in some elements (ducts, turns, contraction etc.) Each element has some input and some output and through these «doors» it connects itself with the other elements. These doors are called «nodes»: in a node the flux is unchanged so we have the continuity of all the flow characteristics (Fig. 1).

A group of characteristic quantities sufficient to describe the flux state have to be chosen in each node. In general, to determine the flow state a quantitative parameter (as the mass flowrate), the thermodynamic state (as the pressure and the temperature) and the flow composition (chemical species concentrations) are required. Each element is characterised by the geometric dimensions that regulate the pressure drop and the heat exchanged, so, each element looks like a black box.

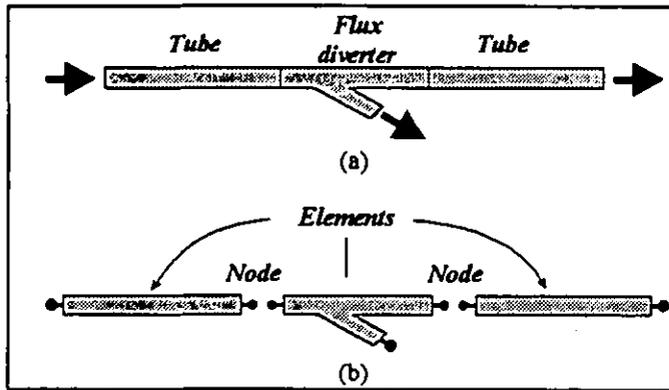


Fig. 1: Explicative example on a simple cooling network

### Mathematics formulation

When the flow characteristics in each node are known the problem is solved. Now calling  $N$  like is the number of elements in the plant for the  $i^{\text{th}}$  of them will have  $u_i$  input and output; if  $p$  is the number of the properties necessary to define the flux state, then the number of the unknown quantity of the problem is:

$$p \cdot \sum_{i=1}^N u_i$$

The kind of equations that we can write are:

- **Flow Continuity equations** : these impose the continuity of the mass, thermodynamic characteristics, fluid composition; for each node we can write  $p$  continuity equations (one for each flow property) and so if we have  $n$  nodes, we will have  $n \cdot p$  equations.
- **Unit equations**: they define the laws that control the physical quantity evolution between the inputs and the outputs of the element. Then, for the  $i^{\text{th}}$  element of the  $N$  elements, we will have  $l_i$  unit equations.
- **Boundary equations**: they define some conditions on some nodes of the plant. Let  $C$  be the number of these conditions.

The mathematical systems admits a solution only if all these equations are independent and their number is equal to the unknown quantities ( $p \cdot \sum_{i=1}^N u_i = n \cdot p + \sum_{i=1}^N l_i + C$ )

We can notice that it is possible to set the boundary conditions in whatever node .

The unit equations are generally not linear, so we linearize them with proportional coefficients (by a first order Taylor series expansion ) that will be updated in the course of the calculation (Carcasci and Facchini, 1995). With this method the mathematical system that we have to solve is always linear. This choice allows an easy and fast equation system solution.

The general approach of these methods can require a relevant calculation time even in simple cases, but for the cooling network we can introduce some simplifications:

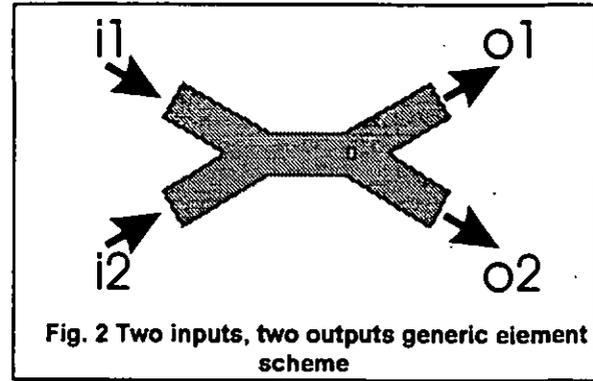


Fig. 2 Two inputs, two outputs generic element scheme

- the fluid composition is constant so the chemical compositions become known quantities.
- the fluid temperature is not considered an unknown quantity for the system resolution; in this case, it is possible to assign a fixed temperature only in the inputs of the network, so the temperature is subsequently calculated in each module: and only the continuity is imposed to the nodes.

Therefore, the user will have to supply to the code the pressure and the temperature in the inputs and the pressure in the outputs; then the code will supply to the user the mass flow rate in each part of the network solving a linear system in which pressure and mass flow rate are the unknown quantities.

### Unit equations and their linearization

The unit equations are those which characterise the module . As we said, they are linear and join the mass flow rate with the pressure variation of the element. In the most general case, for a cooling network, a module can have two inputs and two outputs and so for each module we have four unknown mass flow rates ( $m_{i1}, m_{i2}, m_{o1}, m_{o2}$ ) and four unknown pressures ( $p_{i1}, p_{i2}, p_{o1}, p_{o2}$ ) (Fig. 2).

In general, the pressure drop is a generic mass flow rate function:  $\Delta p = f(m_{i1}, m_{i2}, m_{o1}, m_{o2})$ , so if we consider its linearization with the Taylor series expansion, synthetically we have:

$$p_{i1} - p_{o1} = k_{i1}^I \cdot m_{i1} + k_{i2}^I \cdot m_{i2} + k_{o1}^I \cdot m_{o1} + k_{o2}^I \cdot m_{o2} + k_n^I \quad (1)$$

$$p_{i1} - p_{o2} = k_{i1}^{II} \cdot m_{i1} + k_{i2}^{II} \cdot m_{i2} + k_{o1}^{II} \cdot m_{o1} + k_{o2}^{II} \cdot m_{o2} + k_n^{II} \quad (2)$$

$$p_{i2} - p_{o1} = k_{i1}^{III} \cdot m_{i1} + k_{i2}^{III} \cdot m_{i2} + k_{o1}^{III} \cdot m_{o1} + k_{o2}^{III} \cdot m_{o2} + k_n^{III} \quad (3)$$

$$p_{i2} - p_{o2} = k_{i1}^{IV} \cdot m_{i1} + k_{i2}^{IV} \cdot m_{i2} + k_{o1}^{IV} \cdot m_{o1} + k_{o2}^{IV} \cdot m_{o2} + k_n^{IV} \quad (4)$$

where, for example, in the first equation:

$$k_{mn}^I = \frac{\partial(p_{i1} - p_{o1})}{\partial m_{mn}} \quad \forall m = i, o \quad \forall n = 1, 2 \quad (5)$$

and:

$$k_n^l = (p_n^0 - p_{o1}^0) - \left[ \frac{\partial(p_{n1} - p_{o1})}{\partial m_{n1}} \cdot m_{n1}^0 + \frac{\partial(p_{n1} - p_{o1})}{\partial m_{n2}} \cdot m_{n2}^0 + \frac{\partial(p_{n1} - p_{o1})}{\partial m_{o1}} \cdot m_{o1}^0 + \frac{\partial(p_{n1} - p_{o1})}{\partial m_{o2}} \cdot m_{o2}^0 \right] = (p_n^0 - p_{o1}^0) - [k_{n1}^l \cdot m_{n1}^0 + k_{n2}^l \cdot m_{n2}^0 + k_{o1}^l \cdot m_{o1}^0 + k_{o2}^l \cdot m_{o2}^0] \quad (6)$$

Moreover we can write the continuity mass equation:

$$m_{n1} + m_{n2} = m_{o1} + m_{o2} \quad (7)$$

We can demonstrate that the equations 1-4 are linearly dependent, so it is sufficient to consider only three of them together with the continuity equation (7).

Obviously we have to apply the previous method also when a reduced input or output are present. In these cases the unknown quantities are less but we consider the same system dimension introducing some pseudo-equations that simulate the missing part compared with the general case. In particular we will have:

1 input, 1 output

$p_{o2} = 0$  instead of the equation 2

$p_{i2} = 0$  instead of the equation 3

$m_{o2} = 0$

$m_{i2} = 0$

2 input, 1 output

$p_{o2} = 0$  instead of the equation 2

$m_{o2} = 0$

1 input, 2 output

$p_{i2} = 0$  instead of the equation 3

$m_{i2} = 0$

In order to determine and update the  $k$  constants we have to use their definitions (equations 5-6): we assume, for each module, a vector composed by the pressure and the mass flowrate in input ( $p_{i1}^0, p_{i2}^0, m_{i1}^0, m_{i2}^0$ ); the module then calculates the pressure and the mass flow rate in output ( $p_{o1}^0, p_{o2}^0, m_{o1}^0, m_{o2}^0$ ), after that, we repeat the same procedure another three times, but arbitrary varying the mass flow rate; finally we are able to calculate all the  $k$  constants. When the calculation is started (after the first 4 iterations) each new iteration is produced by the system resolution and each time we can recalculate the  $k$  constants using the previous three calculation results.

## MODULE CHARACTERIZATION

In general, each module modifies the thermodynamic parameters of the fluid that crosses it, introducing a pressure drop and a heat exchange.

Pressure drop and heat exchange are considered by experimental correlations available in literature. Some modules are considered punctual and consequently the heat exchange is neglected (turns, fast expansion or contraction (Idelcik, 1960).

On the contrary for the ducts and the rotating cavity considering correlations are used for both phenomena.

The model of the rotating cavity is more complex and requires a particular explanation.

## Rotating cavity module

Many studies have been done in the past for the understanding of the phenomena that we have in the rotating cavity.

By the reduction of the Navier-Stokes equations system into ordinary differential equations (Von Kármán, 1921), we can obtain the velocity profile, the non dimensional coefficients related to the resistant momentum on a stator case, and the mass flow rate pumped by the rotating disk.

In the turbulent flux case, the mass flow rate pumped by the disk has the following expression (Von Kármán, 1921; Owen and Roger, 1989):

$$C_w = 0.219 Re_p^{0.8} \quad (8)$$

About the stator-rotor cavity of the gas turbine, we refer to the case in which we have two turbulent boundary layers for the two disks. This is the so-called «IV speed region» by Daily and Nece (1960) classification. In this case a block of fluid (called «core») rotates into the cavity with a velocity,  $\omega$ , intermediate between that of the two disks and depending on the radius. The velocity of the fluid into the core has only tangential direction, while the radial equilibrium is guaranteed by the opposite effect of the centrifugal forces and the pressure gradient (Owen and Wilson, 1995). The rotor boundary layer inhales fluid by the core and pumps it in centrifugal direction. To re-establish the mass continuity some fluid is absorbed by the external fluid (hot gas) and it is brought down within the cavity along the stationary wall boundary layer. So we have the «ingestion» phenomena: the ingested gas that comes from the main hot flow seriously heats the wall cavity. To avoid this problem we generally inject fluid coming from the compressor, in the lower part of the cavity: the so-called «source region» is so generated (Stewartson 1953, Owen and Roger 1989), and the injected coolant can be inhaled by the rotor boundary layer.

The model developed is based on a one-dimensional axialsymmetric study in the radial direction, where some effects, as the case or disk momentum and thermal exchanges, have been introduced by experimental correlations (Carcasci et al., 1997).

The model is characterised by a control volumes discretization. For each of them the equations that regulate the phenomena are as follows:

1) mass continuity

$$m_{in} = m_{out} = 2\pi b \rho v_r \quad (9)$$

2) momentum for the radial direction:

$$\frac{1}{2\pi} (m_{out} v_{r,out} - m_{in} v_{r,in}) - b_m \rho v_{\theta,m}^2 \Delta r =$$

$$= -p_{out} b_{out} r_{out} - p_{in} b_{in} r_{in} + p_{in} b_m \Delta r + p_{out} r_m \Delta b \quad (10)$$

3) energy:

$$m_{out} H_{out} - m_{in} H_{in} = M_R \Omega + S_1 q_1 + S_2 q_2 \quad (11)$$

4) state equation (perfect gas):

$$p = \rho R T \quad (12)$$

5) boundary condition:

$$M_R - M_S = m_{out} r_{out}^2 \omega_{out} - m_{in} r_{in}^2 \omega_{in} \quad (13)$$

The equation 13 depends on the rotational core presence and clearly maintains that the flow rotational speed increase depends on the momentum difference between stator case and rotating disk. For the momentum calculation some experimental correlations are necessary that have the following form (Haaser et al., 1987):

$$M = c_f \rho \cdot (\omega^2 - \Omega^2) \frac{I}{2} \quad (14)$$

where:

$$c_{f,rotor} = \frac{a}{Re^b (1 - XK)^c} \quad (15)$$

$$c_{f,stator} = \frac{dXK^e}{Re^f} \quad (16)$$

$$I = \int_A r^3 dA \quad (17)$$

$$Re = \rho \Omega r_{out} \Delta r / \mu \quad (18)$$

$$\Delta r = r_{out} - r_{in} \quad (19)$$

$$r_m = \frac{1}{2} (r_{out} + r_{in}) \quad (20)$$

For the heat exchange we have used a correlation of the following type:

$$Nu = f(Re_{nb}, Pr) \quad (21)$$

where:

$$Re_{nb} = \frac{\Omega r_m (1 - XK) \rho \Delta r}{\mu} \quad (22)$$

To consider the ingestion case, we use the equation 14 with which we can evaluate the mass pumped by the rotor disk: if the cooling mass flow rate injected in the vane is less than the pumped one, then we will have ingestion and the mass ingested

is the difference. In this case, we consider that the point until we have hot gas circulation is where the mass pumped is the same of the mass injected. For each control volume, that is in the circulation zone, the recirculated mass is evaluated as the mass pumped by the rotating volume control surface (Carcasci et al., 1997).

## GRAPHIC INTERFACE

The complicated structure of the majority of the cooling networks creates the graphic interface necessity that allows to supply the correct data to the code. In fact the code requires all the connections between the elements, their geometric characteristics and the booting data of the system. So it's easy to imagine the complexity of the code-input files.

Consequently, the graphic interface has to have the following characteristics:

- *Flexibility*: the interface has to allow to obtain whatever configuration of the network with the possibility to insert or to remove any new connections or elements.

- *Expandability*: the interface has to follow the code progress and so has to allow new modules insertion.

- *Easy of use*: even an inexperienced user has to be able to speak with the code.

For the previous reasons the graphic interface has been developed with the Visual Basic 4.0\* programming language.

## RESULTS

The code developed has been applied to many problems; in order to give an example of these, in this paper we present one in which we have simulated the cooling network around the blade shank of a *Nuovo Pignone* gas turbine. The real rotating disk is shown in Fig. 3. Without entering the simulation details (for industrial secrecy reasons as well), we can notice that, even in a simple case like this, many modules are required. The resulting data of the simulation are shown in Fig 4 and in Fig 5 we show the schematization of the network as we have built by the graphic interface.

The use of this simulation provides a very important opportunity to evaluate the mass flow distribution influenced by parts fits and tolerances. As far as the rotating cavity module in Fig. 7 is concerned, we can see the pressure trend of a cavity having the geometry shown in Fig. 6. The principal parameters have been fixed to allow a comparison with the experimental data found by Owen and Rogers (1989). We can observe that the accordance between the experimental data and the calculated ones is very good but we have to say that, in this case, the cavity geometry is relatively simple. We can see a similar situation concerning the Nusselt number, considering the geometry shown in Fig 8 and referring to another experiment of Owen and Rogers, 1989 we obtain the results shown in Fig 9. The accordance between the experimental data and the calculated ones is good in this case too.

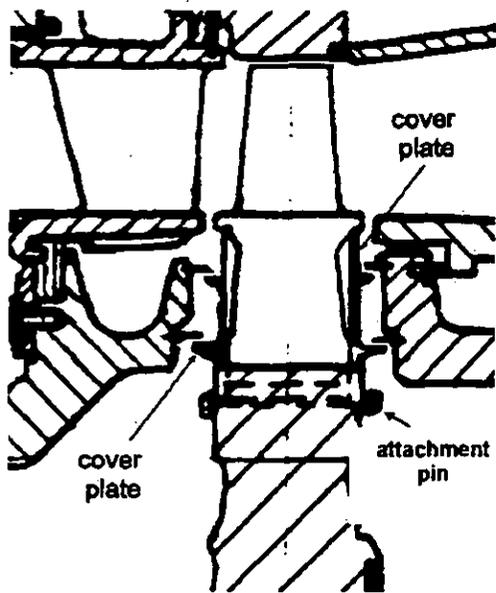


Fig. 3: gas turbine rotor row

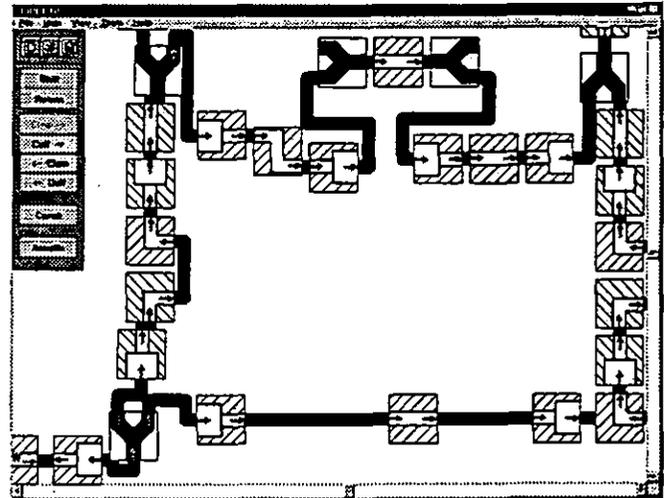


Fig. 5: module layout by graphic interface

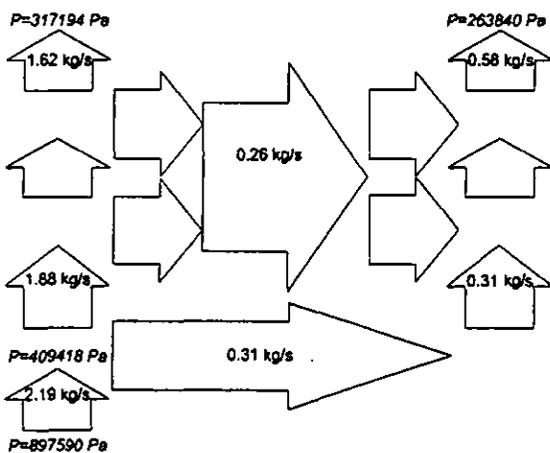


Fig. 4: simulation results: pressure and mass flow rate

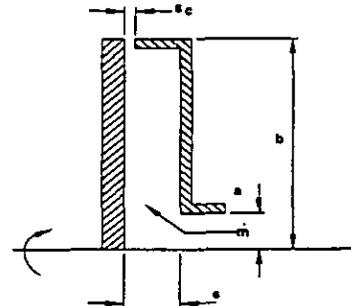


Fig. 6: test case geometry

After we tested the code in the previous simple cases (unfortunately the availability of experimental data in literature is limited) we have simulated a gas turbine real cavity which is shown in Fig 10. The obtained results are shown in figure 11, 12 and 13.

In Fig. 11 we can see the XK parameter trend: it increases with the radius due to the tangential velocity of the rotor disk, and only near the top of the cavity XK decreases due to the friction of the stator closing.

It's important to specify that the XK trend allows to understand the cavity state, in fact, if the mass flow rate decreases XK increases and vice-versa so, for an assigned angular velocity of the disk, XK provides an indication of the ingestion possibility.

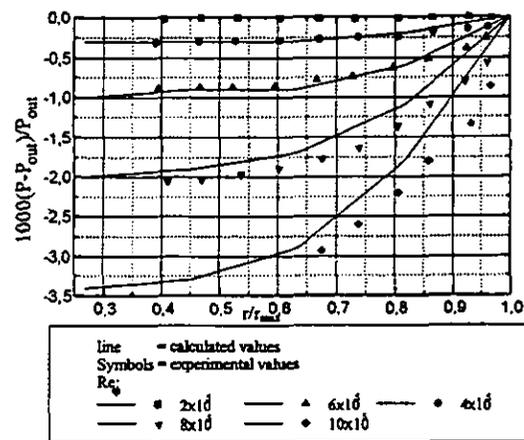


Fig. 7: pressure trend

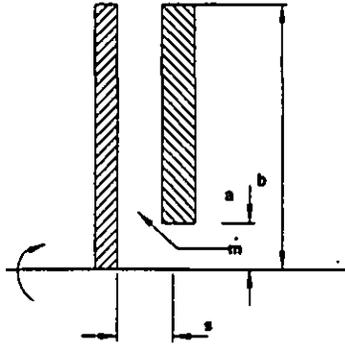


Fig 8 test case geometry

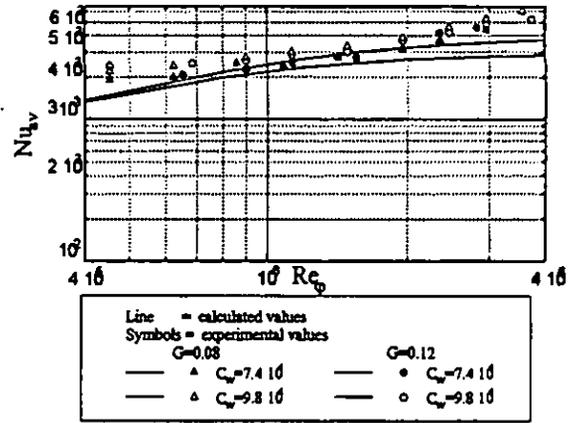


Fig 9: Nusselt number trend

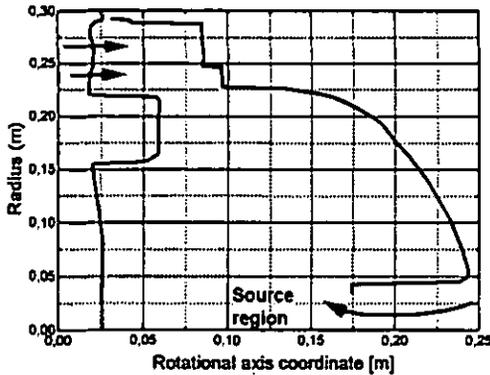


Fig 10: gas turbine cavity geometry

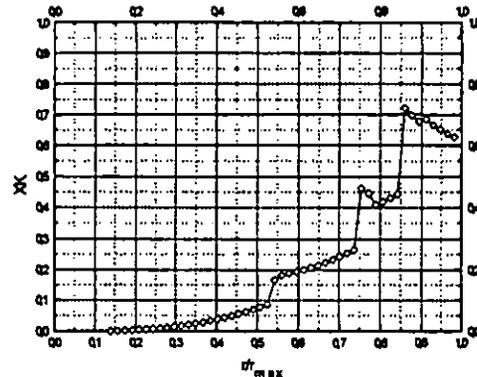


Fig 11: XK parameter trend

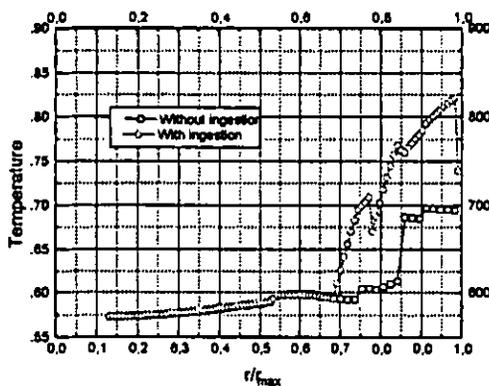


Fig 12: temperature trend for the gas turbine cavity

In fact, as we already said, if the injected mass flow rate is lower than the pumped one, we will have hot gas ingestion that causes great problems for the material resistance. This situation is shown in Fig. 12 where we can see the normal increase of the temperature due the windage effect and, in the case of ingestion, the high increase of the temperature (more than 100 K) in the high radius zone.

The evaluation of Nu number is also possible and the results are shown in fig13 where the trend of the curves is opposite for the rotating disk and stator case and the XK dependence is clearly evident; the more relevant variations are due to the real geometry tested characterized by consistent section reduction in radial direction.

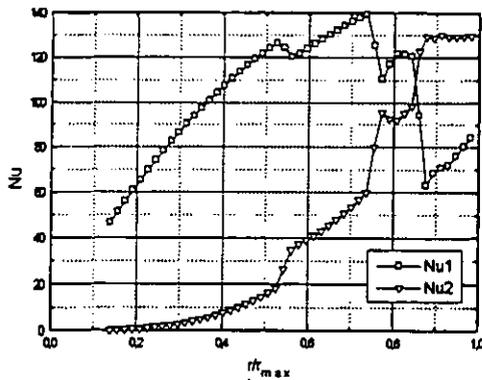


Fig 13: Nusselt number of rotating disk (#1) and stator case (#2)

## CONCLUSIONS

An easy tool for both the design and the verification of the flow network has been developed. In particular it allows the study a cooling network of the gas turbine. Moreover, we can easily analyse any configuration thanks to the modular structure of the developed code.

For a complete analysis of the cooling network, the rotating cavity module has been inserted and tested with some experimental results.

The results obtained seem good and the developed tool is applied to real gas turbine and used in industry.

## ACKNOWLEDGMENTS

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