ABSTRACT

A two-equation turbulence model with additional terms for Coriolis and rotational buoyancy has been used for prediction of heat transfer from the leading and trailing sides of rotating square and rectangular channels with radially outward flow. Test cases with different Reynolds and rotation numbers are considered. The coolant air used is pressurized and operating conditions are selected to closely match real turbine operating parameters. Results show that previous experimental data can be extrapolated to predict the heat transfer characteristics of coolant passages of actual turbine blades. The internal secondary flow vortex structures for some of the aspect ratio channels are found to be different from the expected vortex structures based on earlier lower rotation speed, lower temperature, and lower pressure operating conditions.

NOMENCLATURE

- Local air temperature, wall temperature and inlet air temperature
- Velocities in x, y, and z directions
- Velocity vector, distance measured from start of heating along z
- Average axial velocity through the channel

Greek Symbols

- Volume expansion coefficient, \( \beta = 1/T \)
- Turbulent and laminar viscosity
- Rotational speed, density of cooling air

INTRODUCTION

Rotor blades of advanced high temperature turbines have both internal and external cooling arrangements. The internal cooling is achieved by passing air through built-in serpentine coolant passages in each blade. The flow inside these coolant channels is turbulent and is affected by rotational forces, i.e., Coriolis and rotational buoyancy. The extremely hot thermal operating conditions coupled with high rotation speed of turbines make it prohibitive to design and run experiments in actual conditions. Here we present for the first time a detailed study of heat transfer in rotor blade coolant passages of different practically important aspect ratios. The difficulty of collecting such data in the laboratory has led us to perform numerical experiments. Five different aspect ratios of the coolant passage are selected to cover different regions of a turbine blade. The leading edge of a blade can accommodate a low aspect ratio coolant channel and the narrow trailing edge can have a high aspect ratio coolant passage.

Majumdar et al. (1977) showed that parabolic predictions with the standard k-\( \varepsilon \) model are not satisfactory for moderate to high rotation numbers and suggested the need for modifications in the model to account for the rotational effects. Later, Howard et
al. (1980) used Coriolis modified turbulence models and improved the predictions of fully developed rotating flow. Launder et al. (1987) developed a second-moment closure turbulence model and satisfactorily predicted rotating fully developed flow without heat transfer. Iacovides and Launder (1990) and Younis (1993) studied flow patterns in rectangular rotating channels without rotational buoyancy effects. Recent work on heated channels by Prakash and Zerkle (1992) and Tekriwal (1994a) included thermal buoyancy effects in the momentum and predicted heat transfer results with a high-Reynolds number k-e model obtaining reasonable qualitative agreement with experimental profiles of local Nusselt numbers. However, trailing wall Nusselt number was significantly underpredicted. Low-Re k-e model predictions by Tekriwal (1994b) showed satisfactory results for Nusselt numbers at the leading wall with lower Reynolds number flows (Re ≤ 5000) but the predictions were again not satisfactory at the trailing wall.

Improvements in the prediction of internal cooling in rotating channels has evolved through various stages. Dutta et al. (1994) improved on the heat transfer predictions of Prakash and Zerkle (1992) by including a Coriolis turbulence production term from Howard et al. (1980). Encouraged by the results, the model was extended to predict other features of heated rotational flows in Dutta et al. (1995a, 1995b). Figure 2 shows predictions of Dutta et al. (1995b) and illustrates satisfactory comparison of model predictions with established experimental data. In this paper heat transfer and flow results in actual operating conditions (no experimental data available in the public domain) are numerically predicted with a turbulence model of Dutta et al. (1995a, 1995b), which has been tested and verified in less stringent experimental conditions. In previous studies, the rotation speed was of the order of 1000 RPM, inlet temperature was of the order of 300 K, and maximum operating pressure was 10 atmospheres. Moreover, primarily the square channel with aspect ratio of 1:1 was considered in detail. In this work five different aspect ratios ranging from AR=1:4 to AR=4:1 are studied. The rotation number is varied from Ro=0.3 to Ro=0.7 to simulate the conditions in ground based power generating turbine units. The inlet and wall temperatures are 900 K and 1200 K respectively. The rotational speed is maintained at 10,000 RPM. The coolant air is at a pressure of 30 atmospheres. The hydraulic diameter of different aspect ratio channels is 6 mm.

**MATHEMATICAL MODEL**

Figure 1 shows the coordinate and rotation geometry. The following governing equations are based on a coordinate system rotating with the channel. A constant density for the advection terms is used since a density increase associated with an increase in bulk hydrostatic head due to rotation is balanced by an increase in the temperature. However, for the second pass (not considered here) a constant density may not be appropriate, since the coolant density will decrease from tip-to-root due to both a decrease in pressure and an increase in temperature. Moreover, analysis of the second pass presents other difficulties such as the turn region, that awaits future numerical research. The continuity equation is:

\[ \mathbf{\nabla} \cdot (\rho \mathbf{V}) = 0 \quad (1) \]

and the solved momentum equations are

\[ \mathbf{\nabla} \cdot (\rho \mathbf{u} - \mu_{\text{eff}} \mathbf{V}) = - \left( \frac{1}{ho} \frac{\partial p}{\partial x} \right) - 2\rho \Omega x \cdot \rho \Omega y (T - T_{\text{in}}) \quad (2) \]

\[ \mathbf{\nabla} \cdot (\rho \mathbf{v} - \mu_{\text{eff}} \mathbf{V}) = - \frac{2\Omega}{\rho} \quad (3) \]

\[ \mathbf{\nabla} \cdot (\rho \mathbf{w} - \mu_{\text{eff}} \mathbf{V}) = - \left( \frac{1}{ho} \frac{\partial p}{\partial z} \right) + 2\rho \Omega z - \rho \Omega x (T - T_{\text{in}}) \quad (4) \]

where \( \mathbf{V} \) is the velocity vector (u, v, w).

The second and third terms on the right hand side in the x and z momentum equations, (2) and (4), represent Coriolis and rotational buoyancy forces, with rotation radii \( r_x \) and \( r_z \) in x and z directions (see Figure 1). The temperature variation in the fluid is about 25 percent and Boussinesq approximation of the buoyancy term may be questionable. As a correction, \( \beta \) has been calculated based on local temperature. This calculation procedure showed satisfactory results with a 22 percent variation in the temperature (Dutta et al., 1995b). The effective viscosity, \( \mu_{\text{eff}} \), includes laminar,
Figure 3. Effect of aspect ratio on the Nusselt number ratio

\[
\mu = \mu + \mu_t; \quad \mu_t = \rho C_v \frac{k^1}{\rho}
\]

The enthalpy, \( i \), transport equation is

\[
\nabla \cdot (\rho_i \nabla - \Gamma_i \nabla) = 0; \quad \Gamma_i = \frac{\mu}{\rho} + \frac{\mu_t}{\rho_t}
\]

Since Mach number is less than 0.1, compressible effects in the energy equation are neglected. The \( k \) and \( \varepsilon \) transport equations are

\[
\nabla \cdot (\rho_i \nabla - \mu \nabla) = P - \rho_e + P_e + P_s
\]

\[
\nabla \cdot (\rho_i \nabla - \mu \nabla) = (C_1 P + P_e + C_2 P_e \frac{\varepsilon^2}{k} - C_1 P_e \frac{\varepsilon^3}{k})
\]

where \( P \) is the usual Reynolds stress turbulence production term given as

\[
P = \mu \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 \left( \frac{\partial v}{\partial y} \right)^2 \left( \frac{\partial w}{\partial z} \right)^2 + \left( \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} \frac{\partial w}{\partial z} \right)^2 \right]
\]

The buoyancy and Coriolis generated turbulence production terms, \( P_b \) and \( P_o \), are taken as

\[
P_b = \frac{\mu}{\rho} \nabla \cdot \Gamma_{b}; \quad P_o = \rho \Omega \nabla \cdot \frac{\partial \Gamma_{b}}{\partial t}
\]

Figure 4. Effect of rotation number on Nusselt number ratio at selected axial locations

The buoyancy production term, \( P_b \), arises from a Boussinesq approximation of the velocity-temperature cross-correlation (Hossain and Roh, 1982) and \( C_2 = 0.9 \). The Coriolis modified term, \( P_o \), is included from Howard et al. (1980). In general, \( P_b \) is positive near the trailing wall and negative near the leading wall. A positive \( P_b \) increases turbulence and a negative \( P_b \) suppresses turbulence. The other model constants have the following values (Launder and Spalding, 1974): \( \alpha_1 = 1.0 \), \( \alpha_2 = 1.314 \), \( C_1 = 0.09 \), \( C_2 = 1.44 \), and \( C_3 = 1.92 \). To obviate the need for a fine grid at the wall the model uses a non-equilibrium wall function approach of Launder and Spalding (1974). Figure 2 shows that using this modified \( k-\varepsilon \) model predictions are in good agreement with the corresponding experimental data of Wagner et al. (1991).

OPERATING CONDITIONS

The channel hydraulic diameter, \( D \), is 6 mm for all aspect ratio channels. The aspect ratios are AR=1:4, 1:2, 1:1 (square), 2:1, and 4:1 as shown in Figure 1. The mean rotating radius, \( R \), is 50 D. The coolant is air at 30 atmospheric pressure. The inlet coolant temperature is 900 K and the surrounding heated surfaces are at 1200 K in the heated test section. The density ratio obtained is DR=0.25. The channel is rotated at 10,000 RPM. The Reynolds number (based on the channel hydraulic diameter and inlet conditions) is varied from \( Re=40,000 \) to \( Re=15,000 \) to give rotation numbers of \( Ro=0.3 \), 0.4, 0.5, 0.6, and 0.7.
ratio channels and therefore less space for the fluid to move in the predictions.

The 4:1 aspect ratio channel, which was modelled with a 8 x 10 x 32 grid, had a higher aspect ratio than the square channel results are bounded by the Nusselt number ratios of square rotating channels, and Guidez (1989) used a 1:2 aspect ratio rectangular channel. Soong et al. (1991) published average Nusselt numbers in rotating rectangular channels. Since they did not provide the local Nusselt numbers, their data are not used here for comparison. The axial locations of Wagner et al. (1991) are z_0/D=4.7 and 8.5, and for Han et al. (1994) the locations are z_0/D=5 and 9. Whereas the axial location of Guidez's (1989) data is z_0/D=7.4. Results show that the trend in the existing experimental data may be used to estimate the coolant channel performance in the real operating condition. The trailing wall of 1:2 aspect ratio channel shows the highest Nusselt number ratios. However, the variations of this surface Nusselt number ratio is not smooth with rotation number. The trailing surface Nusselt number ratio is in general higher than the leading surface Nusselt number ratio. Coriolis force with rotation increases the axial flow velocity near the trailing wall compared to that near the leading wall. Moreover, the secondary flow redistributes the turbulence favoring heat transfer enhancement from the trailing wall.

Figure 5 shows the effect of rotation number on Nusselt number ratio at selected axial locations. Data of Wagner et al. (1991), Han et al. (1994), and Guidez (1989) are included for comparison. Wagner et al. (1991) and Han et al. (1994) used square rotating channels, and Guidez (1989) used a 1:2 aspect ratio rectangular channel. Soong et al. (1991) published average Nusselt numbers in rotating rectangular channels. Since they did not provide the local Nusselt numbers, their data are not used here for comparison. The axial locations of Wagner et al. (1991) are z_0/D=4.7 and 8.5, and for Han et al. (1994) the locations are z_0/D=5 and 9. Whereas the axial location of Guidez's (1989) data is z_0/D=7.4. Results show that the trend in the existing experimental data may be used to estimate the coolant channel performance in the real operating condition. The trailing wall of 1:2 aspect ratio channel shows the highest Nusselt number ratios. However, the variations of this surface Nusselt number ratio is not smooth with rotation number. The trailing surface Nusselt number ratio is in general higher than the leading surface Nusselt number ratio. Coriolis force with rotation increases the axial flow velocity near the trailing wall compared to that near the leading wall. Moreover, the secondary flow redistributes the turbulence favoring heat transfer enhancement from the trailing wall.

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hydraulic diameters by changing the bulk flow velocity and the rotation speed. This part of the work deviates from the rest of the analysis in that, the hydraulic diameters are different from 6 mm and the rotation speed is different from 10,000 RPM. Results show some variation at the developing section of the trailing side. But in general the Nusselt number distribution patterns are comparable.

Figures 6, 7, and 8 show the flow parameters at different axial locations for Re=25,000 and Ro=0.5. Figure 6 shows the axial velocity distribution. The flow separates on the leading wall upstream of the start of heating and therefore the plot shows negative axial velocity near the leading surface. The buoyancy effects are significantly stronger in the present analysis compared to earlier published results. It is interesting to note that fluid near the trailing surface decelerates for aspect ratios of 1:1, 2:1, and 4:1. Whereas, the fluid accelerates near the trailing surface for lower aspect ratios. This is mainly driven by buoyancy as is clear from Figure 7.

Figure 7 shows, the higher aspect ratios have a smoother gradient of temperature from the trailing region to the core flow region. Whereas, the lower aspect ratio channels have a steeper gradient in temperature and the trailing side temperature is lower than the core flow. Therefore, the denser fluid near the trailing wall gets accelerated by the centrifugal force in the low aspect ratio channels. Steeper temperature gradients near the trailing surfaces of lower aspect ratio channels (AR = 1:2 and 1:4) are reflected in the higher heat transfer coefficients from these surfaces (see Figure 3).

Figure 8 shows the predicted turbulent viscosities indicating the turbulence mixing strengths in different aspect ratio channels. In general, the lower aspect ratio channels show lower turbulence mixing than the higher aspect ratios. Unlike predictions of Dutta et al. (1995b), the turbulence in the core flow of square channel is significantly high. This is because the separated flow region is significantly big in the present analysis (covering almost 40 percent of the channel) as is shown in Figure 6. This larger separation bubble enhances turbulence in the core flow. The square channel of AR = 1:1 shows maximum turbulent eddy viscosity at the core flow. It can be seen that walls dampen the turbulent eddy viscosity. The core flow region of a square channel is farthest from the surrounding walls compared to other aspect ratio channels. This facilitates a higher eddy viscosity in the square channel.

Figure 9 compares the secondary flow vectors of the Dutta et al. (1995b) to the present analysis. Dutta et al. (1995b) predicted the flow conditions of Wagner et al. (1991). The prediction of Dutta et al. (1995b) was based on coolant air at 10 atmosphere, at an inlet temperature of 300 K, and the rotation speed was 575 RPM. The present analysis is for simulated engine conditions with a higher rotation speed of 10,000 RPM and the coolant is at 30 atmospheres...
and the inlet temperature is 900 K. The contours in the plots are for the axial velocity. A steeper gradient in the axial velocity distribution is observed in the present analysis.

Figures 10 and 11 show the vector plots of different aspect ratios at two different axial locations. The changes in the secondary flow vectors are primarily due to the effects of buoyancy. The plots show that the effect of buoyancy is less significant on the low aspect ratio channels (AR = 1:2 and 1:4) compared to that on higher aspect ratio channels (AR = 1:1 and 2:1). The aspect ratios 1:1 and 2:1 show that secondary flows near the middle of the channels are moving from trailing to the leading surfaces at z/D=10 (Figure 10). From previous predictions and reasoning based on the Coriolis force, the secondary flow in a rotating channel is expected to move from the leading to the trailing surface. It can be argued that the higher buoyancy forces active in the present rotating flow are responsible for this discrepancy between prior and present predictions. However, at a downstream location (z/D=17, Figure 11), where thermal boundary layers are thicker, the buoyancy effects are less and the secondary flows at the core move from leading to the trailing surfaces.

CONCLUSIONS

This numerical analysis predicts the turbulent flow and heat transfer patterns in different aspect ratio rotating channels.

Figure 8. Turbulent viscosity distribution at different axial locations. Re=25000, Ro=0.5

Figure 9. Comparison of secondary flow vectors and axial flow contours (w/wg) in rotating square channels (AR=1:1)

This work is different from earlier publications in a sense that it includes a detailed discussion of the channel aspect ratio effects on heat transfer and flow distribution at simulated turbine operating conditions. The aspect ratio is varied from 1.4 to 4.1. The coolant is air at 30 atmospheres with an inlet temperature of 900 K. The following conclusions are drawn from this numerical work:

The actual performance of coolant channels in rotating turbine blades may be estimated from the existing experimental data. The trailing side Nusselt number ratio for AR=1:2 is highest among the different aspect ratio channels considered. The low aspect ratio channels are more affected by rotation than the high aspect ratio channels. The difference between the Nusselt number ratios from the trailing and the leading sides is more for 1:2 and 1:4 aspect ratio channels. Though the spanwise averaged heat transfer results (Nu/Nu,) of the present analysis are comparable to those for lower temperature and lower rotation speed conditions, the internal secondary flow structures are significantly different from commonly observed Coriolis force driven patterns for aspect ratios 1:1 and 2:1 due to stronger buoyancy effects.

Predictions indicate that the leading side prediction requires more attention and may be improved by a combined high and low Re k-ε model. The predicted heat transfer coefficient in the leading side is significantly low thus indicating a possible high uncertainty in future experiments. Results from different hydraulic diameter channels are comparable, therefore the laboratory results from larger channel cross sections may be used for smaller cross section rotor blade coolant passages.
Figure 10. Secondary flow vectors and axial flow contours (ww_in) in different aspect ratio channels. Re=25000, Ro=0.5, z_0/D=10

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