CHARACTERISTICS OF DISCHARGE COEFFICIENT IN A ROTATING DISK SYSTEM

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ABSTRACT

The discharge coefficient of a long orifice in a rotating system is measured to examine the rotational effect on discharge behavior. The rotating system is comprised of a rotating disk and two stators on both sides of the rotating disk. Test rig is constructed to simulate the real turbine operating conditions. Pressure ratios between upstream and downstream cavities of the orifice range from 1.05 to 1.8, and rotational speed of the rotor disk is varied up to 10,000 rpm. The orifice hole bored through the rotor disk has length-to-diameter ratio of 10. For a better interpretation of discharge behavior, three-dimensional velocity field in the downstream and upstream cavities of the rotor is measured using a Laser Doppler Velocimetry. A new definition of the rotational discharge coefficient is introduced to consider the momentum transfer from the rotor to the orifice flow. Additional loss in the discharge coefficient due to pressure loss in the orifice hole at the inlet and exit regions is quantitatively presented in terms of the Rotation number and the compressibility factor. The effect of corner radiusing at the orifice inlet is also investigated at various rotational conditions.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>orifice cross-sectional area</td>
</tr>
<tr>
<td>C_d</td>
<td>discharge coefficient</td>
</tr>
<tr>
<td>C_prev</td>
<td>rotational discharge coefficient</td>
</tr>
<tr>
<td>C_loss</td>
<td>additional loss coefficient</td>
</tr>
<tr>
<td>c_p</td>
<td>specific heat at constant pressure</td>
</tr>
<tr>
<td>d</td>
<td>orifice hole diameter</td>
</tr>
<tr>
<td>f</td>
<td>compressibility factor (= (p_s - p)/\frac{1}{2} pU^2)</td>
</tr>
<tr>
<td>h</td>
<td>enthalpy</td>
</tr>
<tr>
<td>l</td>
<td>orifice hole length</td>
</tr>
<tr>
<td>Ma</td>
<td>Mach number of discharged jet (= U/\sqrt{RT_o})</td>
</tr>
<tr>
<td>m</td>
<td>measured mass flow rate through orifice</td>
</tr>
<tr>
<td>m_a</td>
<td>ideal mass flow rate through orifice</td>
</tr>
<tr>
<td>P_s</td>
<td>static pressure at orifice inlet</td>
</tr>
<tr>
<td>P_t</td>
<td>stagnation pressure</td>
</tr>
<tr>
<td>P_t</td>
<td>rotational total pressure</td>
</tr>
<tr>
<td>R</td>
<td>gas constant</td>
</tr>
<tr>
<td>R_o</td>
<td>radius of rotor disk</td>
</tr>
<tr>
<td>R_h</td>
<td>pitch radius of orifice hole</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number of discharged jet (= Ud/v)</td>
</tr>
<tr>
<td>Ro</td>
<td>Rotation number (= \omega R_h / U)</td>
</tr>
<tr>
<td>r</td>
<td>radial coordinate</td>
</tr>
<tr>
<td>r_c</td>
<td>corner radius of orifice inlet</td>
</tr>
<tr>
<td>s</td>
<td>gap distance between rotor and stator disks</td>
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<tr>
<td>T_o</td>
<td>temperature at orifice inlet</td>
</tr>
<tr>
<td>T_w</td>
<td>temperature at orifice outlet</td>
</tr>
<tr>
<td>T_r</td>
<td>rotational temperature (= R_h^2 \omega^2 / 2c_p)</td>
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<tr>
<td>T_t</td>
<td>torque</td>
</tr>
<tr>
<td>T_o</td>
<td>rotational total temperature (= T_o + T_r)</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>U</td>
<td>ideal exit velocity of discharged jet</td>
</tr>
<tr>
<td>V_w</td>
<td>exit velocity of discharge considering rotational work transfer</td>
</tr>
<tr>
<td>\bar{V}_r</td>
<td>averaged circumferential velocity component</td>
</tr>
<tr>
<td>W_p</td>
<td>shaft work</td>
</tr>
<tr>
<td>x</td>
<td>axial location from orifice outlet in the orifice flow direction</td>
</tr>
<tr>
<td>xx</td>
<td>axial location from orifice inlet in the direction opposite to the orifice flow</td>
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Greek Symbols

<table>
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<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tr>
<td>\gamma</td>
<td>specific heat ratio</td>
</tr>
<tr>
<td>\nu</td>
<td>kinematic viscosity</td>
</tr>
<tr>
<td>\Pi</td>
<td>pressure ratio between upstream and downstream cavities (= p_s/p_w)</td>
</tr>
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The orifice jet and crossflow. Hay and Spencer studied the effect of crossflow and outlet, and described it with the momentum ratio between main flow rate from the compressor. This large amount of airflow is supplied to aircraft engines but the flow does not directly contribute to the turbine power but performs several important functions such as blade and disk cooling, control of bearing axial loads, and control of turbine blade tip clearances (The Jet Engine, 1986).

The internal airflow rate reaches up to more than 20% of the main flow rate from the compressor. This large amount of airflow supplied by additional compressor work directly affects the overall turbine efficiency. Consequently, it is demanded to supply an appropriate amount of airflow to each component of engines through an accurate estimation of the performance of various parts.

Since airflow passages are subject to numerous operating conditions, and the shape of orifice is far from the standardized one such as ASME orifice, numerous studies have been conducted to investigate the orifice discharge characteristics under various operating conditions.

Bragg (1960) considered the effect of compressibility on the discharge coefficient. He made a simplified assumption on the flow pattern at the upstream wall of an orifice, and compared his theoretical predictions with experimental results. Lichtarowicz et al. (1965) observed discharge behaviors in a wide range of length-to-diameter ratios, and obtained the critical discharge coefficient at large Reynolds numbers. Rohde et al. (1969) determined the effect of approaching angle of inlet flow relative to orifice axis. Hay et al. (1983) investigated the crossflow effect on the discharge coefficient. Their results show that the influence of crossflow is strong and complex, particularly on the inlet side. Hay et al. (1987) defined the additive loss due to the crossflow of orifice inlet and outlet, and described it with the momentum ratio between orifice jet and crossflow. Hay and Spencer (1992) studied the effect of radiusing and chamfering of orifice edge. They acquired about 10–30% increase in the discharge coefficient, and reported that the chamfered orifice shows more desirable performance. Recently, Hay and Lampard (1996) reviewed works on the discharge behavior of turbine cooling holes.

However, all of the studies mentioned above are limited to the stationary conditions, and thus, there is a lack of information on a rotating system. Meyfarth and Shine (1965) focused on the discharge behavior of rotating orifices, but this study is confined to the orifice with very short length-to-diameter ratio, which is far from the mainly used orifice in the internal air system of gas turbine.

The internal airflow should pass inevitably through rotating components because a turbine system is composed of stationary and rotating parts. Recent development of design technology makes it possible to reach high rotational speeds. In this case, discharge behavior is easily expected to considerably deviate from stationary one. The effect of rotation thus should be estimated for a balanced supply of airflow to each component, and consequently for the reduction of overall amount of airflow.

In this study, a turbine disk system was constructed to simulate real operating conditions. The test rig is composed of a rotating disk with orifices and two stationary disks. Characteristics of the discharge behavior in the rotating disk system is scrutinized through measurements of the discharge coefficient under various rotating conditions in a wide range of pressure ratio between upstream and downstream of the orifice. The effect of corner radiusing of the orifice inlet is also investigated.

**EXPERIMENTAL APPARATUS AND PROCEDURE**

**Experimental Facility**

Schematic diagram of the test facility is depicted in Fig. 1. This test rig is constructed at the Institut für Thermische Strömungsmaschinen, Universität Karlsruhe, Germany. Compressor of 85.3 kW power supplies airflow at a maximum mass flow rate of 500 g/s. Supply pressure level can be controlled up to 4 bars by a bypass valve. Mass flow rate is precisely determined in an orifice-metering system, which consists of three orifices with different measurable ranges. Thermocouples and pressure taps are installed to monitor temperature and pressure around orifices and inside of pipes.

The test rig is shown schematically in Fig. 2. A rotor disk where eight orifices are installed and two stator disks on each side of the rotor compose the turbine disk system. Turbulence mesh is positioned to remove large scale fluctuations from the settling chamber. Airflow comes into the upstream cavity through the inside annular slot, and then discharged into the downstream cavity through the rotating orifices. Dimensions of the rotating disk system are summarized in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>Diameter</td>
<td>100 mm</td>
</tr>
<tr>
<td>Length</td>
<td>50 mm</td>
</tr>
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</table>

Due to the requirement of high rotational speed, high precision bearings are used, and labyrinth seal is adopted for sealing the gap between the rotor and housing. Gap distance is minimized to about 0.2 mm. The maximum allowable rotational speed is 10,000 rpm, which is equivalent to a circumferential velocity of 150 m/s at the outer edge of the rotor disk. An encoder which generates 4096 pulses per one revolution is attached on the shaft, and its signal is sent to a motor controller.

High flexibility considering geometrical variations is obtained by modular design of the rig. The orifices are bored into cylindrical shells. These are inserted into the disk to obtain easy modification of the geometry by replacing the shells. Furthermore, the distance of the stationary disks to the rotor disk can be continuously adjusted. Glass windows are installed on both front and rim sides of the test rig to acquire optical access for LDV measurements.
Two dimensional velocity field in the upstream cavity, and three-dimensional velocity field in the downstream cavity is measured using an LDV system because the flow in the downstream cavity could be highly three-dimensional. The differential doppler technique for optical heterodyning is used in the back scatter mode of receiving optics.

A 10W Argon-Ion laser (Coherent INNOVA 90) is used in multi-mode operation as a light source. The multi-wavelength beam passes the transmitter box (Dantec), in which it is first split into two beams, one of which passes a bragg cell to be shifted with a frequency of 40 MHz for the detection of the flow direction. Subsequently both of them are separated into the particular colors for two or three components (476.5 nm, 488 nm and 514.5 nm) and linked into two fiber probes (Dantec, 60 x 65; 60 x 67). Three channels of the Burst Spectrum Analyzer (Dantec, 57N20, 57N35) are used to process simultaneously the multiplied signals. A commercial aerosol generator (Pallas AGF5.0) is used to seed the flow with atomized particles of DEHS solution. The mean diameter of the generated particles ranges from 0.21 to 0.27 μm. Detailed description of LDV system and signal processing procedures are well described in Jakoby et al. (1997). Statistical uncertainties in the measured velocity field are estimated using the procedures described by Snyder et al. (1984). The maximum uncertainty levels for each component of the axial and the circumferential velocities are estimated to be 2.8 % and 2.5 %, respectively, over the whole measured area.

Measurement of Discharge Coefficient

Precise measurement of reference temperature and pressure, and mass flow rate through orifices in the rotor is essential to determine the discharge coefficient under various operating conditions. NiCr-Ni thermocouples and pressure taps are installed inside upstream and downstream cavities to identify inlet and exit conditions of the orifice. Static pressure taps of 1 mm diameter are drilled at right angle on disk surfaces, and thermocouple junctions are protruded by 2 mm from the disk surface to measure the fluid temperature. During the entire measurements, the orifice exit pressure is kept constant at 2 bars, and inlet pressure is changed according to the required pressure ratio.

In spite of the labyrinth seal, there is leakage flow through the gap between the rotor and stator. The amount of leakage mass flux should be considered for the exact evaluation of the discharge coefficient. The leakage mass flux, however, also changes with variation of experimental conditions such as rotational speed of the rotor, pressure ratio and geometric parameters. Thus, additional experiment for compensating the leakage flow is conducted. Waschka et al. (1992) found that the leakage flux of labyrinth seal can be determined by the inlet and exit conditions of the leakage and rotational speed. In each experiment, the leakage flux is measured with orifice shells being replaced by those with no orifice hole, keeping all other experimental conditions including rotational speed of rotor, pressure ratio and geometric parameters the same as those in corresponding discharge coefficient measurement. The precise seal leakage rate as a function of pressure ratios and the rotor speeds is well described in Maeng (1998).

The discharge coefficient is defined as

$$ C_d = \frac{\dot{m}}{\dot{m}_{\text{ideal}}} \tag{1} $$

where $\dot{m}_{\text{ideal}}$ is the mass flux through the orifice in an ideal process.

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Table 1 Dimensions of rotating disk system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orifice hole diameter $d$ (mm)</td>
<td>4</td>
</tr>
<tr>
<td>Orifice hole length $l$ (mm)</td>
<td>40</td>
</tr>
<tr>
<td>Corner radius of round inlet $r_c$ (mm)</td>
<td>2</td>
</tr>
<tr>
<td>Pitch circle radius of orifice $R_p$ (mm)</td>
<td>105</td>
</tr>
<tr>
<td>Rotor disk radius $R_D$ (mm)</td>
<td>160</td>
</tr>
<tr>
<td>Gap distance between rotor and stator $s$ (mm)</td>
<td>25</td>
</tr>
</tbody>
</table>
In general, the following relation can be deduced for an ideal discharge system from energy conservation, entropy and ideal gas relations.

\[ T_e = T_o + \frac{U^2}{2c_p} \]  

\[ m_{\text{ideal}} \] can be described in terms of inlet conditions using Eq. (2), which is given by

\[ m_{\text{ideal}} = \frac{p_o A}{\sqrt{T_o}} \left[ \frac{2\gamma}{(\gamma-1)R} \left( \frac{1}{\Pi^2} - \frac{1}{\Pi^2} \right) \right] \]  

Thus, the discharge coefficient can be defined as follows:

\[ C_d = \frac{\sqrt{T_o}}{p_o A} \left[ \frac{2\gamma}{(\gamma-1)R} \left( \frac{1}{\Pi^2} - \frac{1}{\Pi^2} \right) \right] \]  

Uncertainty in the measurement of the discharge coefficient is estimated less than 3.8% by Kline and McClintock’s (1953) method for a single-sample experiment.

RESULTS AND DISCUSSION

Flow Field in Upstream and Downstream Cavities

The coordinate system and disk configuration is depicted in Fig. 3. \( x, r \) and \( \phi \) are axial, radial and circumferential coordinates, respectively, which are fixed at the rotor disk rotating in the clockwise direction viewed from the downstream face. The origin of \( x \) and \( xx \) coordinates is located on the disk surfaces facing downstream and upstream cavities, respectively. Three-dimensional velocity components are measured at a rotational speed of 6,000 rpm and a pressure ratio of 1.05.

Figure 4 shows the velocity vectors in the \( x-\phi \) plane from \( \phi = -10 \) to 20 degrees at a fixed radial location, \( r = R_o \). Flow measurement near the rotor surface in upstream cavity cannot be made because of difficulty in probing. As shown in Fig. 4(b), flow in the region \( xx/s > 0.5 \) is relatively unidirectional which does not have the axial component. It seems that the flow only in the vicinity of the orifice is influenced by the orifice flow. On the other hand, the flow in the downstream cavity shows strikingly different structure from that in the upstream cavity, and shows strong three-dimensional behavior because jet-like orifice flow interacts with the downstream stator which is located at \( xx/s = 1 \). The backward flow shown near the downstream stator is due to redirection of the flow after colliding with the stator. A rapid deceleration of the flow near the stator surface might occur because the velocity vector preserves its magnitude far down to the stator surface. Flow structure in the downstream cavity is described in detail by Jakoby et al. (1997).

Variation of circumferential velocity component along the radial direction is illustrated in Fig. 5. The circumferential velocity plotted in this figure is the averaged values over 360 degrees in the circumferential direction. Figure 5(a) shows the variation of the circumferential velocity in the downstream cavity at the axial locations of \( x/s = 0.40 \) and 0.76. Note that the orifice center is located at \( r/R_o = 0.656 \), and its linear velocity of rotation is 66 m/s. If there is no orifice, the circumferential velocity should increase linearly proportional to the radial position. As shown in Fig. 5(a), however, the circumferential velocity variation in both radial and axial directions appears insignificant. This is because the downstream cavity flow is not directly affected by the shear caused by the rotation of the disk but strong rotational motion of the orifice flow preserves its angular momentum further into the axial direction. Thus, it also has its magnitude close to the orifice rotational velocity, 66 m/s.

Figure 5(b) shows variation of the circumferential velocity at \( xx/s = 0.50, 0.66 \) and 0.82. Comparing with the downstream cavity
Fig. 4 Velocity vectors along the circumferential direction at a fixed radial location, \(r = R_h\), when \(\omega = 6,000\) rpm, \(\Pi = 1.05\)

flow, the flow shows nearly inviscid behavior except near the rotor surface. The magnitude is quite reduced from the rotor speed and increases in the radial direction.

Rotational Work Transfer from Rotor to Orifice Flow

The discharge coefficient \(C_d\) as a function of pressure ratio \(\Pi\) is plotted for different rotational speeds in Fig. 6. Figure 6(a) and Fig. 6(b) correspond to different orifice inlet shapes: square edged inlet and corner radiused inlet, respectively. In general, the discharge coefficient is higher at high rotational speed of the rotor, and this trend is more distinctive when the pressure ratio is low.

In the case of the corner radiused inlet, the discharge coefficient value exceeds unity at \(\omega = 7,500\) and \(10,000\) rpm, which respectively correspond to \(V_e = 82.5\) and \(110\) m/s at the orifice hole center. This is because the rotational momentum of the rotor is transferred to the orifice flow when it passes through the long orifice. The present orifice has the length-to-diameter ratio of 10. Thus, the definition of the discharge coefficient should be modified by reevaluating ideal mass flux considering this rotational momentum transfer. When energy balance is applied to a control volume including the orifice and its surroundings, energy equation is reduced as follows for a one-dimensional discharge system with adiabatic assumption.

\[
\frac{dW}{dt} = \dot{m}(h_e + \frac{1}{2}V_e^2) - \dot{m}(h_s + \frac{1}{2}V_s^2)
\]  

From the flow measurement data, the following assumption can be deduced.

\[
\frac{V_e}{V_s} \approx \frac{V_{e,0}}{\sqrt{V_{e,0}^2 + (\omega R_e)^2}} < 1
\]  

The shaft work transferred from the rotor can be determined by applying the angular-momentum theory. It follows that
From Eq. (5) - (7), the energy relation can be rearranged as follows:

\[ T_s + \frac{R^2 \omega^2}{2c_p} = T_e + \frac{V^2}{2c_p} \]  

(8)

When compared with Eq. (2), the second term on the left hand side is newly added because of the rotational energy transfer. This term can be defined as the rotational temperature \( T_{rot} \), that is

\[ T_{rot} = \frac{R^2 \omega^2}{2c_p} \]  

(9)

With definition of \( T_{rot} \), the rotational discharge coefficient considering the rotational work transfer, \( C_{d,rot} \), can be written as follows:

\[ C_{d,rot} = \frac{\dot{m}}{\sqrt{T_s}} \frac{1}{p_i A} \left[ \frac{2y}{(y-1)R} \left( \frac{1}{\Pi} \right)^{\frac{y}{2}} \left( 1 + \frac{T_{rot}}{T_0} \right) - \left( \frac{1}{\Pi} \right)^{\frac{y}{2}} \right] \]  

(10)

Meyfarth et al. (1965) described discharge behavior of a rotating system in terms of the velocity ratio of rotating orifice to discharged jet to take into account the rotational effect. In the present study, this velocity ratio is defined as the Rotation number, \( Ro \), which is given by

\[ Ro = \frac{\omega R}{U} \]  

(11)

where \( U \) is the ideal exit velocity of discharge defined by Eq. (2) and Eq. (3) as follows:

\[ U = \left[ \frac{2yRT_s}{(y-1)} \left[ 1 - \left( \frac{1}{\Pi} \right)^{\frac{y}{2}} \right] \right]^{\frac{1}{y-1}} \]  

(12)

Figure 7 shows the variation of the rotational discharge coefficient \( C_{d,rot} \) for the square edged inlet with the Rotation number. Plotted are also the present data and Meyfarth et al. (1965)'s evaluated from the conventional definition of the discharge coefficient \( C_d \). In the study of Meyfarth et al. (1965), their orifice is so short that the rotational work transfer effect is negligibly small. As can be seen in this figure, the rotational discharge coefficient now reasonably represents the discharge behavior which shows the same trend as Meyfarth et al. (1965)'s. The rotational discharge coefficient, \( C_{d,rot} \), decreases with increasing Rotation number. The difference in the discharge coefficient values between present \( C_{d,rot} \) and Meyfarth et al. (1965)'s is owing to location of vena contracta. In the present study, vena contracta is located inside the long orifice hole, and thus the flow is reattached as it expands. Visualized Flow (1988) shows this for a stationary set-up. The enlarged vena contracta leads to higher discharge coefficient than that of Meyfarth et al. (1965), because their orifice is very short.

Equation (10) can be rearranged into a similar form as the conventional definition of the discharge coefficient given by Eq. (4). The rotational total pressure in the isentropic process can be defined as follows:

\[ \frac{P_\text{rot}}{P_0} = \frac{T_{\text{rot}}}{T_0} \left( 1 + \frac{T_{rot}}{T_0} \right)^{-\frac{y}{2}} \]  

(13)

The ideal mass flux considering rotational work transfer can be rewritten in terms of \( p_i \) and \( T_i \) as follows:

\[ \dot{m}_{\text{ideal,rot}} = \frac{P_i A}{\sqrt{T_i}} \frac{2y}{(y-1)R} \left[ \left( \frac{1}{\Pi} \right)^{\frac{y}{2}} \left( 1 + \frac{T_{rot}}{T_0} \right) - \left( \frac{1}{\Pi} \right)^{\frac{y}{2}} \right] \]  

(14)

Finally, \( C_{d,rot} \) can be expressed in terms of the rotational pressure ratio, \( \Pi_{rot} \), as
where the rotational pressure ratio is defined as

$$\Pi_{rot} = \frac{\Pi}{\Pi_0}$$

Equation (15) has exactly the same formula as Eq. (4) if $T_o, P_o$ and $\Pi$ are correspondingly replaced by $T_i, P_i$ and $\Pi_{rot}$. It has been shown that the rotational work transfer in a rotating orifice system can be considered by introducing the rotational discharge coefficient $C_{d,rot}$ which can be easily evaluated with the orifice exit and modified inlet conditions.

Figure 8 is a reproduction of Fig. 6 as a function of the rotational pressure ratio. The magnitude of the discharge coefficient can be presented in the reversed order, that is, it decreases with increasing rotational speed. It should also be noticed that in the case of the corner radiused inlet (Fig. 8(b)), the rotational discharge coefficient values are now meaningfully evaluated to be less than unity.

Additional Loss Due to Orifice Rotation

In addition to the rotational work transfer effect on the discharge behavior, an additional loss may be generated due to the relative motion of the orifice to upstream and downstream cavity flows. As shown in Fig. 7, results of both Meyfarth et al. (1965) and present $C_{d,rot}$ diminish as the Rotation number increases. The additional loss can be generated at both inlet and outlet of the orifice. In the inlet region, flow separation bubble is getting larger as the Rotation number increases because the crossflow at the orifice inlet, which flows at right angle with respect to the orifice flow, is getting stronger, and consequently produces pressure loss. The pressure loss is also produced in the orifice exit region due to the interaction of the jet-like orifice flow and the stator, which is developed into a complex three-dimensional flow as shown in Fig. 4(a). This pressure loss is expected to result in the deficit of discharge coefficient.

In this section, it is attempted to systematically isolate the effect of the above pressure loss on the discharge behavior. To do this, the additional loss coefficient, $C_{loss}$, is defined as follows:

$$C_{loss}(\Pi_{rot}, \omega) = \frac{C_{d,rot}(\Pi_{rot}, \omega = 0) - C_{d,rot}(\Pi_{rot}, \omega)}{C_{d,rot}(\Pi_{rot}, \omega = 0)}$$

where $C_{d,rot}(\Pi_{rot}, \omega = 0)$ and $C_{d,rot}(\Pi_{rot}, \omega)$ represent the curves at different rotational speeds shown in Fig. 8. The loss coefficient $C_{loss}$ is determined in such a way that at a given rotational pressure ratio the difference between $C_{d,rot}(\Pi_{rot}, \omega = 0)$ and $C_{d,rot}(\Pi_{rot}, \omega)$ values are calculated from the data shown in Fig. 8 using an interpolation scheme proposed by Akima (1970), and then it is normalized by $C_{d,rot}(\Pi_{rot}, \omega = 0)$ value.

The loss coefficient as a function of the Rotation number is shown in Fig. 9. As expected the loss production increases with the Rotation number. This is because the higher Rotation number implies higher orifice rotational velocity compared relatively to the discharged orifice flow velocity at a given rotational pressure ratio. This fact agrees well with Hay et al. (1983) who observed the
diminishing discharge coefficient of a long orifice, as the inlet or exit crossflow velocity increases.

However, the Rotation number alone is not sufficient to properly describe the loss characteristics because of the compressibility effect. Mach number of the present orifice jet, Ma, ranges from 0.26 to 0.88. As flow turns into the orifice hole, the streamline, which tends to enlarge the area of vena contracta. Such a change in the streamline curvature can be the primary cause of an increase in the discharge coefficient at high pressure ratio for a stationary orifice, as indicated by Bragg (1960). This effect is more pronounced in the presence of relative motion of surrounding flow. Furthermore, the exit velocity of dischargement increases in the presence of the wall friction, explained as Fanno process. This leads to the further flow expansion at a given mass flux.

In order to take into account the compressibility effect, another parameter should be introduced such as the dynamic pressure compressibility factor in addition to the Rotation number. The dynamic pressure compressibility factor, $f$, is defined as the ratio of the difference between stagnation pressure $p_s$ and static pressure $p_o$ to the dynamic pressure as follows:

$$ f = \frac{p_s - p}{\frac{1}{2} p U^2} \quad (18) $$

The behavior of the dynamic pressure compressibility factor is related closely to the Mach number as described in Saad (1993). The loss coefficient is plotted in Fig. 10 including the compressibility effect. It looks very well correlated with $Ro^{13}/f^a$ regardless of the rotational speed $\omega$. The exponents were determined by the least-error-square method.

**Effect of Inlet Corner Radius**

As shown in Fig. 6, the orifice with corner radiused inlet has higher discharge coefficient distribution than that of the square edged one in the entire experimental range. This is due solely to geometric difference of these two because the amount of the rotational work transfer for each inlet shape should be identical at the same experimental conditions. Thus, the difference in the rotational discharge coefficient results solely from flow interaction with surrounding near the orifice hole.

As previously explained, additional loss is related closely to the streamline curvature at the orifice inlet. Enlargement of vena contracta by inlet radiusing is well illustrated for the stationary orifice in Visualized Flow (1988). In a rotating system, vena contracta is reduced because the flow approaches asymmetrically with respect to the orifice axis. Thus, the discharge behavior can deviate from the stationary one, and can be more susceptible to corner radiusing.

Figure 8(b) depicts the rotational discharge coefficient distribution of the orifice with corner radiused inlet. It shows more uniform distribution compared with that of the square edged orifice (Fig. 8(a)).

The increment in the discharge coefficient due to inlet radiusing is shown in Fig. 11, where $C_4$ is the discharge coefficient of the squared inlet and $\Delta C_4$ is the difference between the discharge coefficients of two different inlet shapes. The effect of inlet radiusing is well presented by a single parameter $Ro$, regardless of pressure ratio and rotor speed. The increment amounts to more than 20% at high Rotation numbers.
SUMMARY AND CONCLUSIONS

Discharge characteristics of a long orifice with length-to-diameter ratio of 10 is investigated at various rotational conditions. The pressure ratio is in the range between 1.05 and 1.8, and rotational speed of the rotor up to 10,000 rpm. The Reynolds number based on the ideal exit velocity ranges from 45,650 to 152,500. Some important observations are noted and summarized below.

(1) The discharge coefficient in a rotating system increases compared to the stationary one because of the rotational momentum transfer to the orifice flow from the rotor. The rotational discharge coefficient is introduced which can reasonably represent the discharge behavior of the rotating system.

(2) The additional loss due to relative motion of the rotating orifice and surrounding flow is qualitatively presented by two parameters: the compressibility factor and the Rotation number.

(3) The orifice with corner radiused inlet shows higher distribution of the discharge coefficient than that of the orifice with square inlet, and the difference increases with the Rotation number. The increase in the discharge coefficient can be predicted when the Rotation number is given.

REFERENCES


