INVESTIGATION OF A TURBULENCE MODEL FOR WALL COOLING OF COMBUSTION CHAMBERS

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ABSTRACT
The trend in gas turbine engine combustor design is to use forced convection cooling with enhanced surfaces instead of film cooling. Therefore there is a need to develop reliable design tools for the combustor cooling channels. This work investigates the ability to predict heat transfer and pressure drop in a wide channel with ribs by using the Low Re k-ε model of Launder and Sharma with Yap correction. Different rib sizes and Reynolds numbers are tested and the results are compared to experiments. The results show a large variation in predictive ability for different rib sizes. Some care is recommended when using this turbulence model in engineering applications.

INTRODUCTION
Numerical simulations have been used to predict flow and heat transfer in so-called enhanced cooling channels for a rather long time. Unfortunately, the gas turbine engineers are still left without a reliable tool to apply for the cooling problems of gas turbine blades and combustion chambers.

This work is based on the problems of the combustor wall cooling. The demands of efficiency and emissions on future gas turbines make forced convection cooling instead of traditional film cooling very attractive. Forced convection cooling is implemented by pumping air through channels on the outside of the hot combustor walls. The high wall temperatures make enhanced heat transfer geometries necessary. By using this cooling concept, the air that is used for cooling can be used for combustion afterwards, which reduces losses. But this also means that the pressure drop of the cooling fluid is not allowed to be too high. Therefore there is a need to optimize the heat transfer of the cooling channels for a given pressure drop.

The complex flow and heat transfer associated with enhanced geometries are indeed very difficult to handle, as there is separated flow and turbulence present. To the engineers comfort, computer resources are increasing with time, and these give them new hopes for the future. The limitations in computer power ten years ago forced the engineers to start out with simple eddy viscosity (SEV) turbulence models and wall functions. This approach soon proved to be too crude to account for the complex flow phenomena of the separated flows present (see for example Kobayashi et al (1985) or Ciofalo and Collins (1992)). Therefore different kinds of adjustments and corrections to the SEV model were implemented. An example of these is Lee et al (1988) who used a streamline curvature correction.

Another way to improve the predictions, at least in theory, is to consider the turbulent stresses as anisotropic as they must be in the separated regions of the flow. This can be done with the turbulence model of Speziale and was performed by Acharya et al (1993). Both streamline curvature correction and an anisotropic turbulence model was used at the same time by Liou et al (1993).

All the investigations above used wall functions that force the velocity profiles next to the walls to follow the law of the wall. This is not a very good description of a separated flow, as has been indicated in many studies (for example Arman and Rabas (1994)). A way of avoiding the wall functions is to use low Reynolds number models (Low Re) or two layer models (TL). The main drawback of these approaches is that they will require a much more powerful computer than the wall function method. Still, there has been a number of tests performed with the Low Re- and TL-models. Investigators using the Low Re models are: Chang and Mills (1993), Arman and Rabas (1994), Fusegi (1995) and Jacovides and Raisee (1997). The TL-model approach has been adopted in studies by Arman and Rabas (1994) and Jacovides and Raisee (1997).

A third way of simulating an enhanced geometry is to consider the enhanced surface as a rough one. In this way the detailed modelling of the enhancement promoters, for example ribs, is avoided. The ribs are thought of something that just modifies the law of the wall instead of something that perturbs and separates the flow. This modification can be implemented through the use of wall functions of a rough wall, which are based on experimental correlations. The temperature field of course also needs to be adjusted, and this is done by using Reynolds analogy. Simulations that use wall
functions of a rough wall have been presented by Cunha (1992) and Youn et al (1994).

Unfortunately, many of the numerical simulations leave too many questions unanswered concerning the performance of the models from an engineer's point of view. The key parameters to keep track on to an engineer when designing a cooling channel are the mean heat transfer coefficient and friction factor of the enhanced channel. These are the properties that will be the backbone of the results in this investigation. There is also a need to consider different geometries with different flow situations and stresses to make conclusions about the performance of the model. Some of the tests above have for example only considered one size of the ribs of the enhanced channels. A model may perform well for small rib sizes as the stresses in the fluid are only moderate. A larger rib size may induce higher levels of stresses and much greater anisotropy effects and totally deteriorate the predictive ability. The performance can of course also be affected by the Reynolds number of the flow and therefore different numbers should be considered.

This work hopefully fulfills the demands outlined above. It is a more thorough test of a Low Re model, already considered by Chang and Mills (1993), which in their study showed promising results. The model used is the Launder and Sharma with Yap correction and it has also been investigated by Iacovides and Raisee (1997) for a gas turbine blade cooling application. The main objective of this work is to decide whether or not the model is applicable for calculations in enhanced cooling channels for gas turbine engine combustors.

**PROBLEM FORMULATION**

Prediction of heat transfer and pressure drop in cooling channels with application to gas turbine engine combustion chambers is considered in this work. The channels are of large aspect ratio, rectangular, and with attached ribs which serve as turbulence and heat transfer promoters. For engineering purposes the friction factors and mean Nusselt numbers of the channel will be calculated for different rib sizes and Reynolds numbers. Comparisons to experiments by Han et al. (1978,1979) will be made and this will be the basis on which conclusions are drawn.

The investigation by Han et al. (1978,1979) was carried out on a rectangular channel (AR=12) with symmetrically attached ribs on the two broader walls. In the experiments Reynolds numbers between 5000-20000 were used and a constant heat flux was supplied to the enhanced walls. The ribs were manufactured in a highly conductive material in different sizes and of different shapes. Also the pitch between the ribs were varied in the study, and the pitch to rib height ratio p/e ranged from 5 to 20. Air was used as the fluid and it was heated approximately 30 °C from room temperature through the channel. The temperature difference between the wall and the fluid was maintained around 20 °C in the experiments. Han used thermocouples and also an overall heat balance to evaluate the heat transfer.

To simulate these conditions (i.e. Han et al. (1978,1979)) the fluid is considered as incompressible air and the viscous dissipation and buoyancy effects are neglected. The flow and temperature fields are regarded as steady and periodically fully developed (no entrance effects present). The fluid properties are assumed constant which might be a reasonable assumption compared to the corresponding experiments. However, for the intended industrial application the termophysical properties may vary since these depend on temperature and pressure. Nevertheless, if accurate predictions are not possible for constant physical properties, it is not likely that predictions for varying properties are of any value at all. Thus the chosen test case is justified. In the simulations the computational domain is taken as two-dimensional (see Fig. 1) since due to the large aspect ratio the three dimensional effects will be small except close to the sidewalls. These effects are assumed to have negligible influence on the friction factor and overall heat transfer coefficient. The thermal boundary conditions is a uniform wall heat flux on the walls (including the rib surface area) which corresponds to that in the experiments.

**GOVERNING EQUATIONS**

The governing equations to be solved are the momentum, continuity and energy equations and the transport equations for the turbulent kinetic energy and dissipation added through the turbulence model. If the pressure and temperature are splited up in one periodic part and one linearly increasing part, periodic boundary conditions may be applied for all the variables:

\[ P(x,y) = P^*(x,y) - \beta \cdot x \]  \hspace{1cm} (1)

\[ T(x,y) = T^*(x,y) + \sigma \cdot x \]  \hspace{1cm} (2)

In the expressions above \( \beta \) represents the pressure gradient in the main flow direction (x-axis) and \( \sigma \) represents the temperature gradient in the same direction. Both of these are assumed to be constant over every...
geometry period (i.e. pitch) in the channel. The temperature gradient is defined as:

$$\sigma = \frac{\partial T_b}{\partial x} = \frac{q_w \cdot A}{m \cdot c_p \cdot \rho}$$  \hspace{1cm} (3)

If the expressions above are inserted in the governing equations one obtains:

$$\frac{\partial U_i}{\partial x_i} = 0$$  \hspace{1cm} (4)

$$\frac{\partial (U_j \cdot U_i)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial}{\partial x_i} \left( \frac{p^* + \frac{2}{3} \rho \cdot k}{\sigma} \right) + \frac{\beta}{\rho} +$$

$$+ \frac{\partial}{\partial x_i} \left[ \frac{v \cdot \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - u_i \cdot u_j}{\sigma} \right] \hspace{1cm} (5)

$$\frac{\partial (U_j \cdot T^* + U_i \cdot u_j)}{\partial x_j} = \frac{1}{\rho} \frac{\partial}{\partial x_i} \left( \frac{v + u_i}{\sigma} \right) \cdot T^* -$$

$$\left( U_i \cdot \sigma - \frac{\partial}{\partial x_i} \left[ \frac{v + u_i}{\sigma} \right] \cdot \sigma \right) \delta_{ij} \hspace{1cm} (6)

where

$$-u_i \cdot u_j = v_t \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} k \cdot \delta_{ij}$$  \hspace{1cm} (7)

$$v_t = f' \cdot C' \cdot \frac{k^2}{c} \hspace{1cm} (8)

f' = \exp \left( \frac{-3.4}{\left( 1 + \frac{R_i}{50} \right)^2} \right) \hspace{1cm} (9)

R_i = \frac{k^2}{T} \hspace{1cm} (10)

The transport equations for turbulent kinetic energy and dissipation are given by:

$$\frac{\partial (U_j \cdot k)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \frac{v + u_i}{\sigma_k} \cdot \frac{\partial k}{\partial x_j} \right] -$$

$$\frac{\partial u_i}{\partial x_j} \frac{\partial U_j}{\partial x_i} - \epsilon$$  \hspace{1cm} (11)

$$\frac{\partial (U_j \cdot \varepsilon)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \frac{v + u_i}{\sigma_e} \cdot \frac{\partial \varepsilon}{\partial x_j} \right] -$$

$$C_1 \cdot u_i \cdot u_j \frac{\partial U_j}{\partial x_i} \frac{\varepsilon}{k} -$$

$$-f_2 \cdot C_2 \cdot \frac{\varepsilon^2}{k} + E$$  \hspace{1cm} (12)

where

$$\varepsilon = \varepsilon \cdot 2 \cdot v \cdot \left( \frac{\partial k}{\partial x_k} \right)$$  \hspace{1cm} (13)

$$f_2 = 1 - 0.3 \cdot \exp (-R_i^2)$$  \hspace{1cm} (14)

$$E = 2 \cdot v \cdot u_j \left( \frac{\partial^2 U_i}{\partial x_j^2} \right) + S_e$$  \hspace{1cm} (15)

The equations above are from the Launder and Sharma (1974) model with a correction added by Yap (1987). In the equations above the constants are set to:

$$\sigma_k = 1.0, \sigma_\varepsilon = 1.217, \sigma = 0.9, C_1 = 0.09, C_2 = 1.44, C_3 = 1.92.$$

The correction term is added to the transport equation of turbulent dissipation and is given as:

$$S_e = 0.83 \frac{k^3}{4} \left( \frac{k^3}{4} \right) \left( \frac{k^3}{4} \right)$$  \hspace{1cm} (16)

As a non-equilibrium relation between turbulent production and dissipation exists in the separated flow field, this term enhances the dissipation near the walls. In practice this means that the overprediction of Nusselt number encountered with the Launder-Sharma model is strongly reduced near the reattachment point (as reported by Heyerichs and Pollard (1996) and Chang and Mills (1993)). The constant $c_L$ is set to 2.5 in the correction term above.

**DEFINITION OF VARIABLES**

The hydraulic diameter of the channel is defined as:

$$D_h = 2 \cdot H$$  \hspace{1cm} (17)

The Reynolds number is then given as:

$$Re = \frac{U \cdot D_h}{v}$$  \hspace{1cm} (18)

where the mean velocity is calculated through:

$$\bar{U} = \frac{\int U \cdot dA}{\int dA}$$  \hspace{1cm} (19)

for a plane situated in between the ribs.

The friction factor is calculated from:

$$f = \frac{\beta \cdot D_h}{\left( \frac{\rho \cdot U^2}{2} \right)}$$  \hspace{1cm} (20)

The average Nusselt number is defined as:

$$Nu = \frac{q_w \cdot D_h}{\lambda \cdot (T_w - T_b)}$$  \hspace{1cm} (21)

where
\[
\frac{T_w - T_b}{dA} = \frac{\int (T_w - T_b) \cdot dA}{\int dA} \quad (22)
\]

\[
T_b = \frac{\int T \cdot |U| \cdot dA}{\int |U| \cdot dA} \quad (23)
\]

The bulk-temperatures used for the front and rear walls of the ribs are the ones calculated in the slabs directly before and after the ribs, respectively.

**NUMERICAL SOLUTION METHOD**

A finite volume technique with a non-staggered body fitted grid is employed for solving the governing equations. The hybrid-scheme (Versteeg and Malalasekera (1995)) is used for discretization of the convection terms and the pressure-velocity coupling is handled by the SIMPLEC algorithm (Versteeg and Malalasekera (1995)). For modelling the turbulence field, the low Reynolds number \( k-\epsilon \) model of Launder and Sharma (1974) is chosen and the so called Yap-correction (Yap (1987)) is implemented. The equations are solved in an iterative fashion and the code CFX 4.1b (AEA Industrial Technology (1992)) is used.

For each of the geometries investigated here, non-uniform grids are used and tests have been performed with the aim to demonstrate grid influence. The procedure for each geometry is then to begin with a coarse grid (about 4000 cells) and then increase the number of cells with a factor about 2 and 1.5 in the \( x \)- and \( y \)-directions, respectively, for each new grid. An example of a fine grid is demonstrated in Fig. 2.

![Figure 2. Example of fine grid near the rib front and top surface (\( e/D_h = 0.056 \)).](image)

Limitation in computer power and the aim to have reasonable computing times set the maximum number of grid refinements possible. Table 1 shows the results for the three geometries under consideration. The grids used for the results to be presented later in this paper will be the finest ones in Table 1. For these grids the near-wall nodes are placed well within the laminar sublayer, i.e. \( y^+ \approx 0.2 \).

**Table 1. Grid refinement tests for various geometries.**

<table>
<thead>
<tr>
<th>( e/D_h )</th>
<th>NX</th>
<th>NY</th>
<th>( f \times 10^2 )</th>
<th>Nu</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.032</td>
<td>90</td>
<td>48</td>
<td>5.20</td>
<td>84.7</td>
</tr>
<tr>
<td>0.032</td>
<td>180</td>
<td>84</td>
<td>5.07</td>
<td>77.2</td>
</tr>
<tr>
<td>0.032</td>
<td>360</td>
<td>150</td>
<td>4.95</td>
<td>72.9</td>
</tr>
<tr>
<td>0.056</td>
<td>90</td>
<td>47</td>
<td>9.90</td>
<td>107.7</td>
</tr>
<tr>
<td>0.056</td>
<td>185</td>
<td>75</td>
<td>9.84</td>
<td>97.3</td>
</tr>
<tr>
<td>0.056</td>
<td>338</td>
<td>120</td>
<td>9.73</td>
<td>93.4</td>
</tr>
<tr>
<td>0.102</td>
<td>85</td>
<td>38</td>
<td>27.92</td>
<td>159.2</td>
</tr>
<tr>
<td>0.102</td>
<td>140</td>
<td>58</td>
<td>27.69</td>
<td>147.9</td>
</tr>
<tr>
<td>0.102</td>
<td>370</td>
<td>119</td>
<td>27.60</td>
<td>142.3</td>
</tr>
</tbody>
</table>

From Table 1 it is obvious that totally grid independent solutions were not achieved. By Richardson extrapolations one may find that the results with the finest grids are still some per cent off.

The convergence criteria to be fulfilled in the simulations are a scaled massflow residual less than \( 4 \times 10^{-5} \) and that the change in friction factor is less than \( 10^{-6} \) for 200 successive iterations. Underrelaxation is used and the URF's in the simulations are set to 0.7 for momentum- and 0.5 for the \( k- \) and \( \epsilon- \)equations. In addition to these, the source terms of the \( \epsilon- \)equation are underrelaxed by a URF of 0.5 in every iteration to increase convergence. The energy equation is solved after a converged solution of the flow field has been obtained to save computational time and no underrelaxation of this equation is necessary.

The constant pressure gradient \( \beta \) is specified to create a flow field. Through the rate of mass flow, the Reynolds number is coupled to \( \beta \). Thus various values of \( \beta \) correspond to various Reynolds numbers.

**RESULTS AND DISCUSSION**

The results are compared to the data of Han (1978, 1979) in Figs. 3 to 6 and Figs. 8 to 9.

![Figure 3. Friction factor for \( e/D_h = 0.032 \). Full line is Han (1978,79) and dotted line is present simulations.](image)
For the small ribs ($e/D_h=0.032$) the prediction of friction factor is good (between -13% and +2%) but the heat transfer is underpredicted quite a lot (30%).

For the medium size ribs ($e/D_h=0.056$) the prediction of friction factor is moderate, and the friction factor is overpredicted by 13-18% as the Nusselt number is underpredicted by 20-34%. These results can be compared to the ones obtained by Chang and Mills (1993). In their study, the same geometry ($e/D_h=0.056$ and $p/e=10$) and Reynolds number range was used. Their results showed a heat transfer underprediction of 15-20% but the friction factor was in better agreement with the data of Han. This may be explained by several differences in the simulations: used grid, boundary conditions, convergence criteria, definition of mean velocity.

Chang and Mills used a rather coarse grid ($NX=84$, $NY=74$) and did not specify the distance to the wall of the near wall nodes, which may affect the results. This is illustrated in Fig. 7 where the local Nusselt number (at $Re=20100$) is presented along the heated bottom wall and the rib surface for a coarse ($NX=90$, $NY=47$) and a fine ($NX=338$, $NY=120$) grid. By using the coarse grid, a separation bubble which is present on top of the rib is poorly predicted (the Nusselt numbers are high along the whole top surface). The fine grid predicts effects of the bubble much stronger with a corresponding dip in the Nusselt number profile. As can be expected, the coarse grid profile is much better in agreement with the Chang and Mills results than the fine one.

A different boundary condition was used by Chang and Mills with specified profiles of velocity, temperature and turbulence properties at the inlet of the computational domain, but the outlet boundary condition was not specified. This may not necessarily influence the results to a large extent but is still a difference.

The convergence criteria for continuity used in their work were weaker than the ones used here, which may be of greater importance. In addition to this, Chang and Mills did not specify whether the largest or the narrowest cross-section of the channel was used when calculating the mean velocity.

For the large ribs ($e/D_h=0.102$) the prediction of friction factor is poor (+70%) but the Nusselt number is in reasonable agreement with Han's data (from -13% to +4%).

To conclude, the friction factor is predicted well for the small ribs but is severely overpredicted as the size of the ribs increase. The heat transfer is generally underpredicted but the predictions become better the bigger the ribs are.
Figure 9. Nusselt number for e/D<sub>e</sub>=0.102. Full line is Han (1978,79) and dotted line is present simulations.

As shown by the figures above, the Launder and Sharma model with Yap correction cannot handle the flow phenomena encountered with large ribs satisfactorily. On the other hand, it seems to work quite well for small rib sizes, if the deviation of the Nusselt number (around -30%) is considered as acceptable. This may be due to the lower stresses induced, and lower level of anisotropy present in the flow, as the rib size decreases. Also, a more complex flow field can be observed for the largest rib size compared to the smallest one (see Figs. 10-12) in this investigation. The recirculating zones upstream and downstream the rib are more compressed and compact for the large ribs than for the small ones, which makes the gradients in the flow stronger. In addition to this, the flow is separated on top of the big ribs but stays attached on top of the smallest ones. The distribution of the local Nusselt number, shown in Fig. 7, is also a reflection of the flow pattern found in Figs. 10-12. The Nusselt number is low at the lower rear and front sides of a rib due to the slow fluid motion there. In the reattachment region between two ribs the Nusselt number becomes high. On the top surface of a rib strong variation in the Nusselt number occurs due to flow separation and reattachment.

CONCLUSIONS
The model of Launder and Sharma with Yap correction has been tested in a two dimensional symmetrically ribbed channel for three different rib sizes. The results of the simulations show that the model in general overpredicts friction factor and underpredicts heat transfer for the geometries tested. As the size of the ribs is increased from small to large, the predictions of the flow field deteriorate but the heat transfer results are improved. The predictive ability of the model varies from -13% to +70% deviation for the friction factor and from -34% to +4% for the heat transfer. Studies of the local flow and Nusselt number improved the understanding of the mechanisms of momentum and heat transport. Some care is recommended as turbulence models are used in engineering applications.

ACKNOWLEDGEMENTS
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NOMENCLATURE

\begin{itemize}
  \item \(\text{A}\) total heat transfer area \([\text{m}^2]\)
  \item \(\text{AR}\) aspect ratio of channel = \(W/H\)
  \item \(c_1, c_2\) turbulence model constants
  \item \(C_1, C_2\) turbulence model constants
  \item \(e\) specific heat \([\text{J kg}^{-1} \text{K}^{-1}]\)
  \item \(D_h\) hydraulic diameter = \(2H\)
  \item \(e, f\) rib height and width \([\text{m}]\)
  \item \(E\) turbulence model term \([\text{m}^2 \text{s}^{-4}]\)
  \item \(f\) Fanning friction factor
  \item \(f_1, f_2\) turbulence model functions
  \item \(H\) channel height \([\text{m}]\)
  \item \(f\) turbulent kinetic energy \([\text{m}^2 \text{s}^{-2}]\)
  \item \(NX\) number of grid cells in x-direction
  \item \(NY\) number of grid cells in y-direction
  \item \(Nu\) average or local Nusselt number
  \item \(p\) rib pitch \([\text{m}]\)
  \item \(P\) pressure \([\text{Pa}]\)
  \item \(P'\) periodic pressure \([\text{Pa}]\)
  \item \(q\) heat flux \([\text{W m}^{-2}]\)
  \item \(Re\) turbulent Reynolds number
  \item \(Re\) Reynolds number
  \item \(S_c, S_y\) Yap correction term \([\text{m}^2 \text{s}^{-1}]\)
  \item \(T\) temperature \([\text{K}]\)
  \item \(T'\) periodic temperature \([\text{K}]\)
  \item \(U\) velocity \([\text{m s}^{-1}]\)
  \item \(W\) channel width \([\text{m}]\)
  \item \(x, y\) coordinates \([\text{m}]\)
  \item \(y_p\) distance from point to to wall \([\text{m}]\)
  \item \(y^*\) dimensionless distance
  \item \(\alpha\) pressure gradient \([\text{Pa m}^{-1}]\)
  \item \(\epsilon\) turbulent dissipation rate \([\text{m}^2 \text{s}^{-3}]\)
  \item \(\lambda\) thermal conductivity \([\text{W m}^{-1} \text{K}^{-1}]\)
  \item \(\nu\) molecular kinematic viscosity \([\text{m}^2 \text{s}^{-1}]\)
  \item \(\nu_t\) turbulent kinematic viscosity \([\text{m}^2 \text{s}^{-1}]\)
  \item \(\rho\) density \([\text{kg m}^{-3}]\)
  \item \(\sigma\) bulk temperature gradient \([\text{K m}^{-1}]\)
  \item \(\sigma_c, \sigma_k\) turbulence model constants
  \item \(\sigma_t\) turbulent Prandtl number
\end{itemize}

Subscripts

\begin{itemize}
  \item \(i, j, k\) coordinate indices
  \item \(b\) bulk
  \item \(w\) wall
\end{itemize}

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Figure 10. Flow field for $e/D_h=0.032$ near the bottom wall and ribs at Re=20353.

Figure 11. Flow field for $e/D_h=0.056$ near the bottom wall and ribs at Re=20187.

Figure 12. Flow field for $e/D_h=0.102$ near the bottom wall and ribs at Re=20222.