THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
345 E. 47th St., New York, N.Y. 10017

97-GT-241

DEVELOPING CENTRIFUGAL COMPRESSOR TRAIN OPTIMIZATION MODELS FOR PERFORMANCE EVALUATION

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ABSTRACT

Recent advances in numerical optimization software now allow problems arising in the evaluation of complex centrifugal compressor train performance to be readily solved by machinery users. Analyses of compressor operations often requires consideration of multiple non-linear constraints involving pressure, temperature, flow rate, composition and power. Based on these constraints, the operational boundaries of the machinery must be evaluated. Frequently operating points of greatest interest exist where one of the operational variables, such as flow rate, is maximized subject to operational constraints. A methodology is proposed which can be used to apply commercially available optimization software to problems of this type. A review of gas equation of state relationships needed to determine thermodynamic properties is provided, as well as methods for calculating compressor discharge pressure based on polytropic head and efficiency. Recycle control is considered and a model is proposed.

INTRODUCTION

The task of evaluating how a centrifugal compressor will operate in a given process frequently reduces to evaluating machine performance at operating points which are defined by process constraints. These constraints usually involve pressure or temperature and may be flow dependent. Compressor performance, on the other hand, is characterized by polytropic head which is thermodynamically related to pressure and temperature and is also flow dependent. In situations where compressor flow is known the associated system pressures and temperatures can easily be calculated. Unfortunately, evaluating compressor performance in the context of plant operations often involves situations where the flow is unknown and must be determined based on pressure or temperature constraints which are also flow dependent. In this case, a lengthy iteration is required to find the operating point where compressor performance characteristics and system constraints are simultaneously satisfied. This situation may be further complicated when gas turbines are used as machinery drivers and the effect of compressor recycle control is considered. In fact, the problem may become sufficiently complex that interactions between the compressors, turbines and recycle controllers may not be clearly understood at all.

This provides the motivation for the development of a numerical methodology which can be used to find machinery operating points while simultaneously satisfying operational constraints. In the usual case, where many viable solutions may exist, this methodology should also be able to optimize the solution with respect to some defined objective function.

If we consider the necessary features of such a methodology two aspects of the problem are apparent. The first of these is the need for a system model which can be used with a numerical optimization routine and the second is the optimization routine itself.

It is the intent of this paper to describe in detail one such model which was developed for use with a commercially available optimization program. It is not the goal of this paper to describe the techniques used to perform numerical optimization; however, a brief discussion of optimization methodology is presented to familiarize the reader with the topic.

The Optimization Problem

Describing compressor performance as part of an operating plant usually requires that several simultaneous constraints be satisfied while some other operational variable is maximized. A likely situation might involve a plant which wishes to maximize gas processing rate subject to constraints involving compressor discharge pressure. To achieve this objective, suction pressure, inlet flow rate and compressor speed might be manipulated. Mathematically this problem can be formulated as a constrained optimization, where the objective function, which is to be maximized, is the compressor flow rate and the constraints involve power and delivery pressure.

Presented at the International Gas Turbine & Aeroengine Congress & Exhibition
Orlando, Florida — June 2–June 5, 1997
This paper has been accepted for publication in the Transactions of the ASME
Discussion of it will be accepted at ASME Headquarters until September 30, 1997

NOMENCLATURE

$ACFM$ = actual cubic ft per minute

C_p = specific heat, $N\,m/kg\,K$ ($ft\,lbf\,/\,lbm\,R$)

C_v = specific heat, $N\,m/kg\,K$ ($ft\,lbf\,/\,lbm\,R$)

$C\,K\,F_a Y$ = flow metering constants, Kg/sec (lbm/min)

H_p = polytropic head, $N\,m/kg$ ($ft\,lbf\,/\,lbm$)

K = control parameter usually 0.5

$\dot{M}$ = mass flow, kg/sec (lbm/min)

MW = molecular weight, $kg/kg\,mol$ ($lbm/lb\,mol$)

P_{gas} = compressor power, W (hp)

$P_{turbine}$ = turbine power, W (hp)

Q = flow, standard m^3/day (ft^3/day)

Q_a = flow, actual m^3/min (ft^3/min)

Q_{min} = flaw minimum, standard m^3/day (ft^3/day)

$Q_{recycle}$ = flow recycle, standard m^3/day (ft^3/day)

R = gas constant, $8314/MW\ N\,m/kg\,K$ ($1545/MW\,ft\,lbf/lb\,R$)

R_u = gas constant, $8314\ N\,m/kg\,mol\,K$ ($1545\,ft\,lbf/lb\,mol\,R$)

S = speed, fraction of rated speed

$SCFD$ = standard cubic ft per day

T = temperature, K (R)

T_{amb} = ambient temperature, C (°F)

Z = compressibility

a = acoustic velocity, $\sqrt{kg_c Z T_1 R_u / MW}$

b = surge control safety margin

d = orifice diameter, cm (in)

$f_{objective}$ = optimization objective function

f_1 = functional relation based surge data

f_2, f_3 = constraint functions

$g_{constraint}$ = constraint evaluation vector

g_c = dimensional constant, 1 SI units ($32.2\,lbm\,ft/lbf\,sec^2$)

h = enthalpy, $N\,m/kg$ ($ft\,lbf\,/\,lbm$)

h_w = orifice pressure drop, cm (in H_2O)

k = isentropic exponent

n = polytropic exponent

p = pressure, N/m^2 (lbf/ft^2)

s = entropy, $N\,m/kg\,K$ ($ft\,lbf\,/\,lbm\,R$)

s_s = proximity to surge

s_c = proximity to surge control

u = interal energy, $N\,m/kg$ ($ft\,lbf\,/\,lbm$)

v = specific volume, m^3/kg (ft^3/lbm)

y = mole fraction

σ = polytropic group, $n/n-1$

η_p = polytropic efficiency

ΔP_o = flow variable, fraction of transmitter range

ρ = density, gm/cm^3 (lbm/ft^3)

Subscripts / Superscripts

$_a$ = actual, m^3/min (ft^3/min)

$_{ave}$ = average af inlet and outlet

$_i$ = component or increment

$_{ref}$ = reference state at low pressure

$_{std}$ = standard conditions

$_1$ = inlet or stage one

$_2$ = outlet or stage two

$_3$ = stage three

$_A$ = machinery train A

$_B$ = machinery train B

$_v$ = evaluated at constant volume

$_p$ = evaluated at constant pressure

^{o} = evaluated at low pressure state

$'$ = invariant form based on inlet conditions

Assuming that a suitable model for the system can be established, a numerical optimization program can be used to maximize the objective function subject to the constraints. This optimization program, or optimizer, will have to be capable of dealing with compressor characteristics and operating constraints which are non-linear. Until recently, development costs and computing requirements limited the application of such routines; however, today these programs are increasingly available as features included in popular numerical computation software packages (Grace et al. 1993). Because of their ready availability and the powerful set of subroutines offered with these packages options exist for users to develop optimization models for their own compressor installations.

The specific compressor operating scenario to be considered is from an actual Arctic oil production facility. At this facility two parallel three stage gas compression trains are driven by gas turbines and are used to process natural gas as shown in the flow diagram of Fig. 1. All compressors have associated with them recycle valves and controllers. Each of the three stages, which constitute a machinery train, are connected by a common drive shaft. From a process perspective, the two trains are tied together between stages such that inter-stage pressures are equalized. The process includes a natural gas liquids extraction plant which is fed from the discharge of the first stage. In this process heavy hydrocarbon components are removed from the gas stream prior to entering the second stage thereby reducing the molecular weight of the gas in the later compression stages. Flow rate dependent delivery pressure constraints exist for both the second and third stages.

Clearly a large number of operating parameters influence overall system performance. Some of these are, ambient temperature, molecular weight, inter-stage pressure drop, recycle control settings and performance characteristics of individual compressors. Because so many parameters affect system performance, efforts which were undertaken to experimentally determine relationships between operating variables and gas processing rate were only partially successful. Testing difficulties arose because of process noise and ever-changing process conditions. Modeling provided a means of overcoming these difficulties and allowed the effects of operating variables to be quantified. In several circumstances these effects were surprising and proved to be counter intuitive to operational personnel.

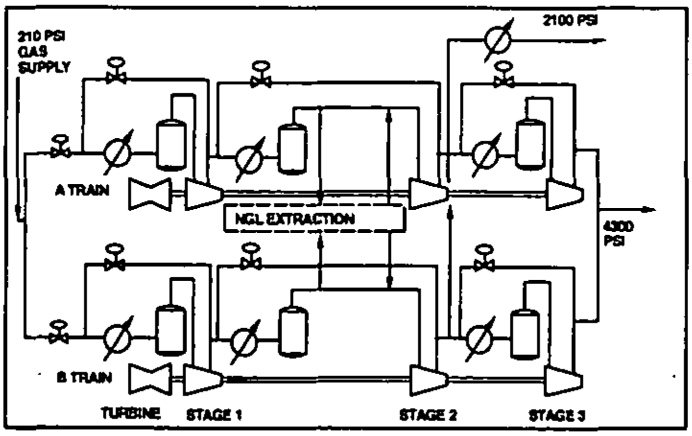

Figure 1. Three stage parallel train compression process with natural gas liquids extraction after first stage. Intermediate gas extraction after second and third stage.

Thermodynamic Relationships

To model a system such as this it is necessary to introduce several thermodynamic relationships. As a first step we must select a suitable equation of state which in this analysis will be the Redlich-Kwong equation, Eq. (1). This two parameter cubic equation of state provides a relatively simple means of obtaining thermodynamic properties. With little additional work other similar, but slightly more complex, three parameter cubic equations of state including the Soave form of the Redlich-Kwong equation or Peng-Robertson equation could be used (Reid et al. 1987). All of these equations can be manipulated to form cubic polynomials similar to Eq. (2). From this equation the compressibility is found as a root of the polynomial. In the case of a superheated vapor the polynomial has one real root and two complex roots whereas for a saturated mixture three real roots will be found where the largest root is associated with the gas phase and the smallest root is associated with the liquid phase (Henley and Seader 1981). Since the model being developed will deal only with superheated gases it will not be necessary to distinguish between real roots.

$$p = \frac{R \cdot T}{v-b} - \frac{a}{v^2 + bv} \tag{1}$$

With $Z = \frac{p \cdot v}{R \cdot T}$ a cubic equation may be defined.

$$Z^3 - Z^2 + \left(A^* - B^* - B^{*2}\right) \cdot Z - A^* \cdot B^* = 0 \tag{2}$$

$$A^* = \frac{a \cdot p}{R^2 \cdot T^2} \tag{3}$$

$$B^* = \frac{b \cdot p}{R \cdot T} \tag{4}$$

$$a = \frac{0.42748 \cdot R^2 \cdot T_c^{2.5}}{p_c \cdot T^{1/2}} \tag{5}$$

$$b = \frac{0.08664 \cdot R \cdot T_c}{p_c} \tag{6}$$

When gas mixtures are being considered mixing rules must be applied to determine a_m and b_m. The simplest mixing rules apply to the Redlich-Kwong equation and are given in Eq. (7) and (8) where for hydrocarbon pairs the parameter k_{ij} is usually zero.

$$a_m = \sum_i \sum_j y_i \cdot y_j \cdot \left(a_i \cdot a_j\right)^{1/2} \cdot \left(1 - k_{ij}\right) \tag{7}$$

$$b_m = \sum_i \left(y_i \cdot a_i^{1/2}\right)^2 \tag{8}$$

Using either mixture or pure substance values for a and b, departure functions can be calculated for the desired thermodynamic properties of enthalpy, internal energy and entropy based on the equation of state resulting in Eq. (9) (10) and (11). These departure functions allow accurate calculation of properties at pressures where the assumption of ideal gas behavior is no longer valid. The departure functions are combined with low pressure ideal gas relations to give Eq. (12) to (15) where the thermodynamic quantity calculated is relative to a low pressure reference state.

$$h - h^* = p \cdot v - R \cdot T - \frac{3 \cdot a}{2 \cdot b} \cdot \ln\left(\frac{v+b}{v}\right) \tag{9}$$

$$u - u^* = -\frac{3 \cdot a}{2 \cdot b} \cdot \ln\left(\frac{v+b}{v}\right) \tag{10}$$

$$s - s^* = R \cdot \ln\left(\frac{v-b}{v}\right) - \frac{a}{2 \cdot b \cdot T} \cdot \ln\left(\frac{v+b}{v}\right) + R \cdot \ln\left(\frac{v}{v^*}\right) \tag{11}$$

$$\Delta h = \left(h^* - h_{p1}\right)_{T_1} + \int_{T_1}^{T_2} C_p^\circ \cdot dT - \left(h^* - h_{p1}\right)_{T_2} \tag{12}$$

$$\Delta u = \left(u^* - u_{p1}\right)_{T_1} + \int_{T_1}^{T_2} C_v^\circ \cdot dT - \left(u^* - u_{p1}\right)_{T_2} \tag{13}$$

$$\text{where } C_v^\circ = C_p^\circ - R \tag{14}$$

$$\Delta s = \left(s^* - s_{p1}\right)_{T_1} + \int_{T_1}^{T_2} \frac{C_p^\circ}{T} \cdot dT - \left(s^* - s_{p1}\right)_{T_2} \tag{15}$$

Data for C_p° is readily available for a wide variety of gases in the form of third order polynomials, Eq.(16) (Reid, 1987).

$$C_p^\circ = c_0 + c_1 \cdot T + c_2 \cdot T^2 + c_3 \cdot T^3 \tag{16}$$

where for gas mixtures,

$$C_{pm}^\circ = \sum_i y_i \cdot C_p^\circ \tag{17}$$

With the forgoing equations in hand the task of numerically evaluating enthalpy, internal energy and entropy at any pressure or temperature is a straight forward task.

Polytropic Head

Polytropic head is the quantity usually provided by equipment manufacturers to describe compressor performance. Data is typically in the form of a family of curves showing head as a function of flow, Q_a and compressor speed, S. Where flow is usually expressed in units of volume at actual inlet flowing conditons.

For the purposes of developing a system model these curves can be conveniently reduced to Eq. (18) which is a polynomial in Q_a, S and Q_a / S. This polynomial has eight constants which are found by performing a multivariable linear regression on manufacturer data.

$$H_p = c_1 Q_a^4 + c_2\left(\frac{Q_a}{S}\right)^2 + c_3\left(\frac{Q_a}{S}\right) + c_4 Q_a^2 + c_5 Q_a + c_6 S^2 + c_7 S + c_8 \tag{18}$$

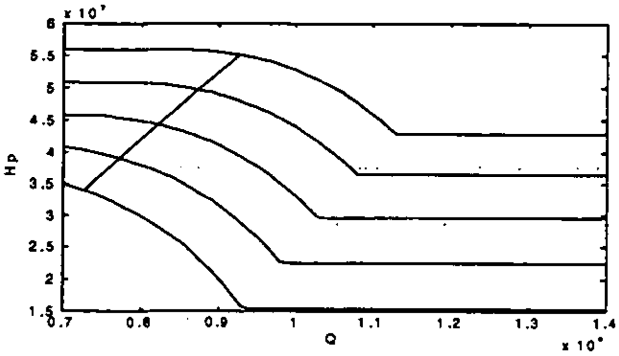

Figure 2. Polytropic Head (ft.lbf/lbm) vs. Flow (ACFM) for speeds 0.85, 0.90, 0.95, 1.0, 1.05

A plot of this polynomial is shown in Fig. 2. In Fig. 2 constant speed lines describing polytropic head as a function of flow have been extended from minimum and maximum flow values as horizontal lines using relations given in Eq. (19) and (20). Although this is not representative of actual compressor performance these equations allow polytropic head to be fully defined for all speeds and flows. This avoids computational difficulties that may arise if polytropic head is calculated for an unusually large or small flow rate which is outside of the compressor's normal operating range. Of course, precautions need to be taken to ensure that data is not erroneously used from these regions in any final solution obtained.

$$\text{for } Q \le Q_{min},\ H_p = H_p\big|_{Q_{min}} \text{ where } Q_{min} = 9000 \cdot S^{1.5} \tag{19}$$

$$\text{for } Q \ge Q_{max},\ H_p = H_p\big|_{Q_{max}} \text{ where } Q_{max} = 13000 \cdot S^{1.5} \tag{20}$$

Polytropic Efficiency

Polytropic efficiency curves may be dealt with in a completely analogous manner to the polytropic head curves just described. Using the same polynomial form Eq. (21) is obtained.

$$\eta_p = c_1 Q_a^4 + c_2\left(\frac{Q_a}{S}\right)^2 + c_3\left(\frac{Q_a}{S}\right) + c_4 Q_a^2 + c_5 Q_a + c_6 S^2 + c_7 S + c_8 \tag{21}$$

As before, the constant speed curves are extended as horizontal lines outside the normal operating range of the compressor, Fig. 3.

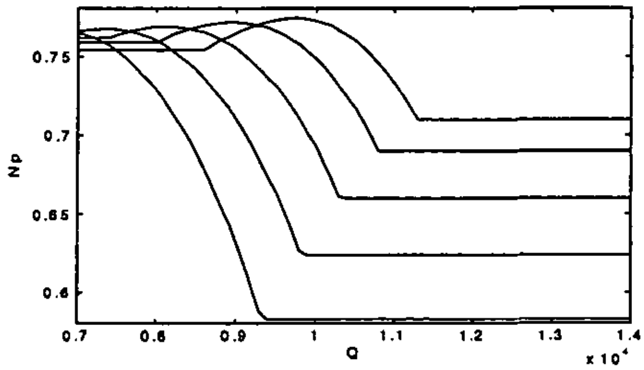

Figure 3. Polytropic Efficiency vs. Flow (ACFM) for speeds 0.85,0.90.0.95.1.0,1.05

The polytropic head and efficiency curves just introduced are based on manufacturer data which, strictly speaking, is only valid for one set of specified inlet conditions. Since it is our intent to use these curves under inlet conditions that will differ from those originally specified it is worthwhile commenting on the errors which will be introduced. Four types of errors will arise because of differences in Mach number, Reynolds number, capacity speed ratio and, most importantly for multistage compressors, volume ratio effects (ASME, 1992). Should we decide to deal with these errors rigorously a model of the compression process which considers each impeller of the compressor would be needed; therefore, a family of curves similar to those just presented would be required (Sapiro, 1982) (Gresh, 1991). For the present analysis it will be assumed that the deviation from originally specified conditions will be small thereby allowing a single curve to be used.

It is worth noting that when significant variations in inlet conditions are expected, Eq. (18) and (21) can be made invariant to inlet Mach number with relative ease. To accomplish this, independent and dependent variables are converted to their invariant forms using design inlet conditions and regressed using Eq. (18) and (21). In this case, invariant forms of the variables become, $Q_a' = Q_a / a$, $S' = S / a$, $\eta_p' = \eta_p$ and $H_p' = H_p / a^2$ (Batson, 1996). For simplicity, these invariant forms have omitted terms involving the impeller diameter. When this characteristic length is included the invariant forms become dimensionless.

Calculating Discharge Pressure

Accurately relating polytropic head and efficiency to inlet and outlet pressures and temperatures is one of the principle difficulties associated with developing a compressor model. In the situation being considered, we are given polytropic head and efficiency from manufacturer curves and an unknown pressure and temperature must be found. Let us consider first the more common situation which is that of finding an unknown polytropic head and efficiency given inlet and outlet conditions. If we assume the compression path follows that of a polytropic process, $pv^n = constant$, then the familiar polytropic head expression given in Eq. (22) is found. Here it is assumed that the polytropic exponent and thus the isentropic exponent k can be adequately described by averages taken at the inlet and the outlet (Lapina, 1982).

In our case since the head and efficiency are known, Eq. (22) can be easily rearranged to solve for pressure, where Eq. (23) is used to find the polytropic exponent.

$$H_P = Z_{ave} \cdot \frac{R_u}{MW} \cdot T_1 \cdot \frac{(n_{ave}-1)}{n_{ave}} \cdot [(p_2/p_1)^{\frac{n_{ave}}{n_{ave}-1}} - 1] = \eta_p \cdot \Delta h \quad (22)$$

$$\frac{n_{ave}}{n_{ave}-1} = \frac{k_{ave}}{k_{ave}-1} \cdot \eta_p \quad \text{where } k = \frac{C_p/C_v}{1 - \frac{Z}{p}\frac{\partial Z}{\partial p}\Big|_T} \approx \frac{C_p}{C_v} \quad (23)$$

It is noteworthy that average compressibility and polytropic exponent values have been used. The rationale behind this is that the compressibility decreases continuously as the gas pressure increases. Average compressibility values tend to account for this behavior. The same reasoning applies to average values for the polytropic exponent although, in this case, the exponent of the pressure ratio in Eq. (22) offsets changes in the polytropic term (Lapina, 1982).

In Eq. (23), the isentropic exponent k may be taken as the low pressure ratio of specific heats without too great a loss in accuracy. It should, however, be pointed out that the correct form of k is as given in Eq. (23) (*Fluid Meters*, 1971). At high pressures, approaching the critical point, real gas k values become quite large. Because of this and the averaging approach used, the more accurate form of the isentropic exponent does not always yield better overall results when compared to the simpler ratio of specific heats given in Eq. (22). This point is at least partly support by Edminster (1961).

In the case where Eq. (22) is rearranged to solve for an unknown discharge pressure and temperature, the average compressibility and ratio of specific heats cannot be found until the unknown pressure and temperature are found. The necessary iterative solution is easily accomplished and converges rapidly. Equations (22) and (23) can give surprisingly good results, usually within 2-3%, of values obtained using more rigorous methods.

For greater accuracy the unknown pressure and temperature may be calculated directly using Eq. (25) (Sorenson, 1983). Where the polytropic path is defined such that, "the ratio of reversible work input to enthalpy rise is a constant" (*Power Test Code*, 1992) or as a differential equation, Eq. (24). Using the thermodynamic relations given earlier an unknown pressure and temperature can be calculated that satisfy Eq. (25). To do this, we start from a known pressure temperature point and calculate the isentropic enthalpy change $\Delta h_{\Delta s=0}$ for an incremental pressure change. This amounts to finding an unknown temperature such that $\Delta s = 0$ for the pressure change. Next we iterate to find a second temperature point at the same pressure that results in a Δh such that $\Delta h = \Delta h_{\Delta s=0}/\eta_p$ is satisfied. This process is repeated using pressure-temperature points associated with the previous Δh as the starting point for the next increment of pressure along the compression path. The process is repeated for n increments until the entire path has been defined and the required polytropic head rise across the compressor is achieved.

$$\eta_p = \frac{v \cdot dp}{dh} \quad (24)$$

$$\eta_p \approx \frac{\sum_{i=1}^{n} \Delta h_{\Delta s=0}}{h_n - h_1} = \frac{\sum_{i=1}^{n} (h_{(i+1)ds=0} - h_{(i)})}{h_n - h_1} = \frac{H_p}{\Delta h} \quad (25)$$

Recycle Control

In modeling the overall compression system a key component is the recycle control scheme. In the system being analyzed, recycle control is shown in Fig. 2 as the positively sloped line intersecting the constant speed curves. This line defines the minimum flow through the compressor. Rates below this minimum are unstable and will result in flow reversals or surge.

The recycle controllers being modeled utilize Eq. (26) to (29). In these equations the parameter s_s is defined such that its value is equal to one when the compressor is operating on the surge line. Equation (26) can, therefore, be regarded as a ratio of polytropic head to flow parameter ΔP_o where f_1 is an almost linear function obtained from compressor testing or from manufacturer surge estimates.

$$s_s = \frac{K \cdot f_1(H_{p_{reduced}}) \cdot p_1}{\Delta P_o} \quad (26)$$

$$H_{p_{reduced}} = \frac{(p_2/p_1)^{\sigma} - 1}{\sigma} \quad (27)$$

$$\sigma = \frac{\ln(T_2/T_1)}{\ln(p_2/p_1)} \quad (28)$$

The control variable is then calculated, Eq. (29).

$$s_c = s_s + b \quad (29)$$

The surge control algorithm manipulates the recycle valve of the compressor such that the proximity to surge, s_c, is maintained at a value less than or equal to one. Since s_s also uses this criterion we recognize that the variable b creates an offset between the control variable s_c and the actual surge point defined by s_s, Eq. (29).

Because of the relative complexity of this control scheme and frequent changes made to K, b and f_1 variables, the antisurge control algorithm was simulated exactly. This allowed model parameters to be modified in the same way that the field control settings were entered in the controllers. To do this it was necessary to scale all variables in the control equations by their respective operating ranges, just as field instrument transmitters would function. As an example, let us consider the differential pressure flow signal ΔP_o. For simulation purposes this parameter is related to compressor inlet mass flow by Eq. (30). Using Eq. (30) and (31), ΔP_o may be calculated. Likewise, Eq.

(26) to (29) must also be evaluated using pressures and temperatures expressed as fractions of transmitter ranges.

$$\dot{M} = C K F_a Y d^2 \sqrt{h_w \cdot \rho_1}\ , \quad C = 0.03475\ (5.983) \qquad (30)$$

$$h_w = \Delta P_o \times [transmitter\ range, cmH_2O\ (inches H_2O)] \qquad (31)$$

Turbine and Compressor Power

As was the case with polytropic head and efficiency maximum available turbine power can also be characterized using a polynomial, which in this case, is a function of speed and ambient temperature, Eq. (32), Fig. 4.

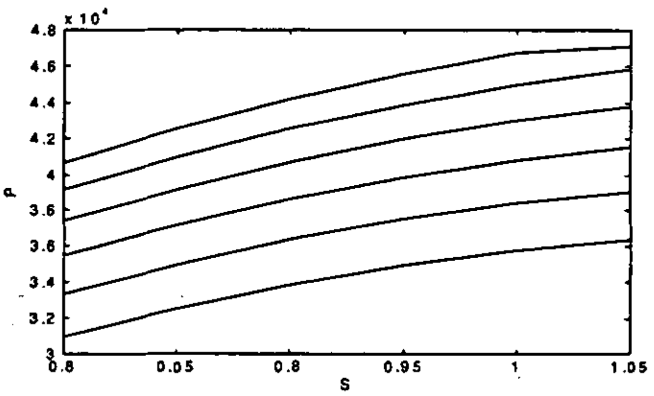

Figure 4. Maximum Turbine Power (HP) vs. Speed (1=100%) for Ambient Temperatures (-40,-20, 0 20 ,40 ,60 °F)

$$P_{turbine} = c_1 S T_{amb} + c_2 (T_{amb})^2 + c_3 (T_{amb}) + c_4 S^2 + c_5 S + c_6 \qquad (32)$$

It is worth noting that the polynomial only represents maximum power available from the turbine. As before, constants are found by performing a multivariable linear regression on data from manufacturer curves.

Ultimately it is our intention to compare turbine power to the total power utilized by the compression train. Equation (33) serves this purpose and is used to calculate the thermodynamic power absorbed by individual compressors. Of course, to calculate total compressor power mechanical losses also need to be included.

$$P_{gas} = \frac{\dot{M} \cdot H_p}{\eta_p} \quad \left(P_{gas} = \frac{\dot{M} \cdot H_p}{33000 \cdot \eta_p} \right) \qquad (33)$$

Building a Model

Having introduced the necessary equations required to develop a basic process simulator let us pause to review how various system attributes might be considered in our calculations. Since our objective is to formulate a computational scheme which can eventually be turned into a computer program our functions will henceforth be described as subroutines with their respective independent input and output variables identified. For convenience these subroutines have been given names and further organized into a three level hierarchical structure. We shall consider first the level one thermodynamic subroutines.

Level 1: Thermodynamic Subroutines Since the conceptual basis for these subroutines has already been discussed in detail no further attention will be given to them here. These relationships are summarized in Table 1.

Table 1. Level 1 Subroutines

Name	Subroutine function	Input Variables	Output Variables
z_find	calculates compressibility	p, T, C_p a_m, b_m, MW	Z
h_find	calculates enthalpy	p, T, C_p a_m, b_m, MW	$h - h_{ref}$
u_find	calculates internal energy	p, T, C_p a_m, b_m, MW	$u - u_{ref}$
s_find	calculates entropy	p, T, C_p a_m, b_m, MW	$s - s_{ref}$
k_find	calculates ratio of specific heats	p, T, C_p a_m, b_m, MW	k
p_find	calculates unknown pressure and temperature	p_1, T_1, H_p, η_p a_m, b_m, C_p, MW	p_2, T_2

Level 2: Machine Level Subroutines We are now prepared to introduce the level 2 subroutines which deal with the behavior of the individual compressors and which are listed in Table 2. The first of these subroutines "look_up" and "turbine_power", contain little more than the coefficients of the respective polynomials that describe head and efficiency for the various compressors or, in the case of the gas turbine driver, the output power. The latter routines "surge_speed", "surge_flow" and "stage_perform", need to be explained in greater detail.

Considering first the subroutine "surge_speed", we note that the objective of this routine is to calculate the minimum flow through a given gas compressor. The compressor will have associated with it a set of recycle control variables K, b and f_1 which define a surge control line based on the criterion $s_c = 1$ as described earlier. Given a set of input variables, the subroutine is to calculate a value of s_c and compare this value to one. In the case where $s_c < 1$ the machine has a flow rate which is greater than the minimum flow and the recycle controller is inactive.

In the case where $s_c > 1$, the flow rate being provided to the machine is below the minimum required. In this case, recycle flow is present at some unknown rate. To find the recycle flow, the inlet flow corresponding to $s_c = 1$ must first be found at the specified operating speed. Once this flow is known the recycle rate can be determined as the difference between the minimum required rate defined by the condition $s_c = 1$ and the rate being fed to the compressor from the upstream source. The only difficulty arising is that criterion s_c is not a function of speed but rather p_1, T_1, p_2, T_2 and ΔP_o; therefore, we cannot directly solve for the quantity ΔP_o.

Table 2. Level 2 Subroutines

Name	Subroutine function	Input Variables	Output Variables
look_up	calculates polytropic head and efficiency	Q_a, S	H_p, η_p
turbine_power	calculates available turbine driver power	$S, T_{ambient}$	P
surge_flow	calculates minimum flow as a function of inlet and outlet pressures and temperatures.	p_1, T_1, p_2, T_2	Q_{min}
surge_speed	calculates minimum flow for compressor as a function of compressor speed and inlet conditions	p_1, T_1, Q, S	$Q_{min}\|_S$
stage_perform	calculates discharge pressure, temperature and recycle flow for a given compression stage	p_1, T_1, Q, S	$H_p, \eta_p, p_2, T_2, Q_{recycle}$

```
Function [Hp,ηp,p2,T2,Qrecycle] = stage_perform (p1,T1,Q,S)
[Hp,ηp] = look_up (Q,S)
[p2,T2] = p_find (p1,T1,Hp,ηp)
[Qmin] = surge_flow (p1,T1,p2,T2)
if Qmin > Q
   [Qmin|S] = surge_speed (p1,T1,S)
   [Hp,ηp] = look_up (Qmin|S,S)
   [p2,T2] = p_find (p1,T1,Hp,ηp)
   Qrecycle = Qmin|S − Q
else
   Qrecycle = 0
end
```

Table 3. Pseudo Code Subroutine "Stage_perform"

In the author's work, the task of finding the flow where the condition $s_c = 1$ was satisfied at a given speed was accomplished using a second subroutine "surge_speed", in combination with "surge_flow". In this case, "surge_speed" performed an interation, at a constant speed, using "surge_flow."

In the iteration just described it is noteworthy that the reduced polytropic head specified hy Eq. (27) is not the same polytropic head found using the polynomial in "look_up." Rather the form of Eq. (27) and (28) requires that compressor outlet pressures be calculated so that the reduced polytropic head can be found.

The last of the level 2 subroutines is "stage_perform". This subroutine is used to simulate the performance of an actual compressor. Not unexpectedly, it makes use of all of the subroutines introduced thus far. The set of input variables to this subroutine includes inlet pressure, temperature, compressor speed, and flow rate. The basic function of the subroutine is described in the pseudo code fragment in Table 3. Where "()" define subroutine input variables and "[ ]" define returned variables.

Level 3: System Level Subroutines At this point we have developed a series of subroutines which are able to describe the thermodynamic performance of a single compressor given a set of input variables, p_1, T_1, Q and S. We have not, however, come up with a strategy to connect multiple compressors in some cohesive scheme. This is the function of the last two system level subroutines which we shall call the level 3 subroutines, Table 4. The first of the these is "opt_form". The function of this subroutine is to sequentially execute "stage_perform" for each of the six compressors in the process.

The sequence of progression is important and is from low to high pressure stages. As this is done, unknown discharge pressures and temperatures are calculated and populated in a state table which looks much the same as Table 5. The subroutine "opt_form" also performs mass continuity calculations so that flow variables are reduced to the minimum independent set which in this case is Q_{1A}, Q_{1B}, Q_{2A} and Q_{3A}. Here we recognize that mass passing through lower compression stages must equal the total passing through the higher stages less any extracted flows which are known constants.

Upon executing the subroutine "opt_form", using a set of manipulated variables, including $Q_{1A}, Q_{1B}, Q_{2A}, Q_{3A}, S_A, S_B, p_{11A}, p_{11B}$ and T_{amb}, the state table can be completely populated. At this point, the table will accurately represent the operating characteristics of individual compressors; however, it would not necessarily satisfy system operating constraints such as equilibrium discharge pressure between trains, maximum absorbed horsepower, or minimum delivery pressure requirements. In addition, the objective function $Q_{3A} + Q_{3B}$ would not yet be maximized. Restating this another way, we would have a physical model of the machinery where operational characteristics of the compressors at a machine level were met; however, characteristics of the process system or process constraint would not be met. In the latter case the degree to which these constraints were violated would be easily determined using values already available from the state table.

We also observe that by manipulating the input variables used in "opt_form" we can eventually satisfy the system level constraints while simultaneously maximizing the chosen objective function.

At this point our problem has become one of applied mathematics and can be restated as a system of objective and constraint equations as given by Eq. (34) to (37). Constraints which were built into the model at a the machine level have also been noted, Eq. (37). In these equations it is important to realize that the volumes and mass flows, Q and $\dot{M}$ respectively, are forward moving flows in the process and as such do not include recycle gas which may also be present. The gas flows Q are calculated at standard conditions and in the absence of molecular weight changes behave in the same manner as mass flows.

Objective Function:

$$f_{objective} = Q_{3A} + Q_{3B} \quad (34)$$

Manipulated Variables:

$$Q_{1A}, Q_{1B}, Q_{2A}, Q_{3A}, p_{11A}, p_{11B}, S_{1A}, S_{1B} \quad (35)$$

Level 3 System Level Constraints:

$$\begin{aligned}
g_{constraint}(1) &= p_{21B} - p_{21A} = 0 \\
g_{constraint}(2) &= p_{22B} - p_{22A} = 0 \\
g_{constraint}(3) &= p_{23B} - p_{23A} = 0 \\
g_{constraint}(4) &= P_{1A} + P_{2A} + P_{3A} - P_{turbine\,A} \leq 0 \\
g_{constraint}(5) &= P_{1B} + P_{2B} + P_{3B} - P_{turbine\,B} \leq 0 \\
g_{constraint}(6) &= f_2(Q_{1A} + Q_{1B}) - p_{22} \leq 0, \\
g_{constraint}(7) &= f_3(Q_{1A} + Q_{1B}) - p_{23} \leq 0,
\end{aligned} \quad (36)$$

Where, $f_2 = 2100\,psi,\ f_3 = 4300\,psi$

Level 2 Machine Level Constraints:

$$\begin{aligned}
& S_{1A} = S_{2A} = S_{3A} \\
& S_{1B} = S_{2B} = S_{3B} \\
& Q_{1A} \geq Q_{1Amin}, Q_{2A} \geq Q_{2Amin}, Q_{3A} \geq Q_{3Amin} \\
& Q_{1B} \geq Q_{1Bmin}, Q_{2B} \geq Q_{2Bmin}, Q_{3B} \geq Q_{3Bmin} \\
& \dot{M}_{2B} = \dot{M}_{1A} + \dot{M}_{1B} - \dot{M}_{2A} - \dot{M}_{extraction\,flow1} \\
& \dot{M}_{3B} = \dot{M}_{2A} + \dot{M}_{2B} - \dot{M}_{3A} - \dot{M}_{extraction\,flow2}
\end{aligned} \quad (37)$$

Name	Subroutine function	Input Variables	Output Variables
opt_form	populates all unknown variables the output state matrix for all compressors	$Q_{1A}, Q_{1B}, Q_{2A}, Q_{3A}, S_A, S_B, p_{11A}, p_{11B}, T_{amb}$ *(all system constants)*	$p_{21A}, p_{21B}, p_{22A}, p_{22B}, p_{23A}, p_{23B}, T_{21A}, T_{21B}, T_{22A}, T_{22B}, T_{23A}, T_{23B}, f_{objective}, g_{constraint}$
opt_setup	provides initial guess for manipulated variables	*none*	*none*

Table 4. Level 3 Subroutines

Recapping the level 3 constraints, Eq. (36), the first three equations specify that discharge pressures of machines must be at equilibrium between trains A and B. The following two equations specify the requirement that absorbed compressor power be less than maximum available turbine power and finally the last two equations set minimum requirements for discharge pressures on the second and third stages. Examining these equations it is apparent that they have been written in open form so that the degree to which constraints are violated at intermediate stages of the optimization may be calculated.

It is also noteworthy that the decision to build level 2 constraints into the model and leave other constraints at the system level, where they eventually are resolved by the optimization program, was arbitrary. In fact all of the constraints could have been left for the optimizer to solve.

Once a model has been developed, a numerical optimization routine must be utilized with the model. In the author's work, the system model "opt_form" was passed to the optimization program using subroutine "opt_setup", where "opt_setup" provided initial guesses for manipulated variables and constants needed for the calculations to proceed, Table 4. The optimization program then used "opt_form" to generate numerical partial derivatives of the objective function with respect to each of the manipulated variables. These partial derivatives were used to estimate new values for the manipulated variables which would maximize the objective function. Clearly when constraints were encountered by the program, the direction of change in the manipulated variables was modified so that these constraints would be avoided. Many methods exist for performing these numerical procedures as discussed by Reklaitis (1983) and Grace et al. (1993).

Table 5. Sample Optimization at +50 F Ambient Temperature

OPERATION STATE						
train	A	A	A	B	B	B
stage	1	2	3	1	2	3
Tamb °F	50	50	50	50	50	50
speed (1=100%)	0.93	0.93	0.93	0.94	0.94	0.94
Qfeed *	154	201	145	184	128	133
Qrecycle *	27	0	0	0	58	0
Qinlet *	181	201	145	184	186	133
Qinlet ACFM	8029	2963	543	8355	2690	494
MW	23.6	22.5	22.5	23.6	22.5	22.5
p1 psig	210	606	2026	210	606	2026
p2 psig	656	2051	4142	656	2051	4142
t1 °F	75	72	85	84	65	82
t2 °F	238	261	191	249	267	192
Poly head(ft)	42217	47401	24056	43022	47384	23942
η polytropic	0.77	0.74	0.68	0.76	0.69	0.65
stage power(hp)	13149	16262	6460	13731	16112	6156
A total (hp)	35872	35872	35872	N/A	N/A	N/A
B total (hp)	N/A	N/A	N/A	35999	35999	35999

* Flows are expressed in million standard cubic feet or as otherwise noted.

RESULTS

The results of a typical optimization performed at 50 °F are given in Table 5. Similar optimizations performed over a range of ambient temperatures are also presented in Fig. 5. From Fig. 5 it is apparent that gas processing capacity rapidly deteriorates as ambient temperature increases and available turbine power decreases. Table 5 gives some insight into why this is the case. In Table 5 it is apparent that recycle valves are open in the first and second stages of the A and B trains respectively. As can be seen in Fig. 5 this leads to a steeper reduction in gas processing capacity as ambient temperatures increase than would be otherwise expected. This behavior begins to occur at approximately 0 °F which accnunts for the change in the slope of Fig. 5 at this point. Considering this system behavior further, it is apparent that absorbed power of the first and second stages cannot be reduced by decreasing throughput and can only be decreased by reducing speed. In actuality, both of these actions are taken by the optimizer in an effort to maximize gas processing rates while meeting system constraints.

The situation just described has been made considerably more complex by the activity of the recycle controllers and the fact the available gas turbine horsepower is insufficient to assure that these controllers are inactive at all ambient temperatures. It is noteworthy that this was, in fact, the original design intent for the system; however, upon start up it was discovered that the margins required for surge control were larger than expected. These margins were as much as 12% of the compressor inlet flow at surge and far greater than the 6-7% expected. This can, in part, be attributed to the multistage design and closely coupled operation of the two parallel trains.

Because of the sharp decline in gas processing rates observed with increasing ambient temperature, various operational scenarios were investigated for maximizing plant throughput. Interestingly, it was discovered that plant gas processing rates could be increased if the natural gas liquids extraction process operating between the first and second stage was turned off as is shown in Fig. 5. This result was counter intuitive and highlighted why such a model was useful.

In general, actual plant operation conformed quite closely to simulation predictions. This was particularly true when competing operational strategies were compared. In these cases, the overall calibration of the model was somewhat less important.

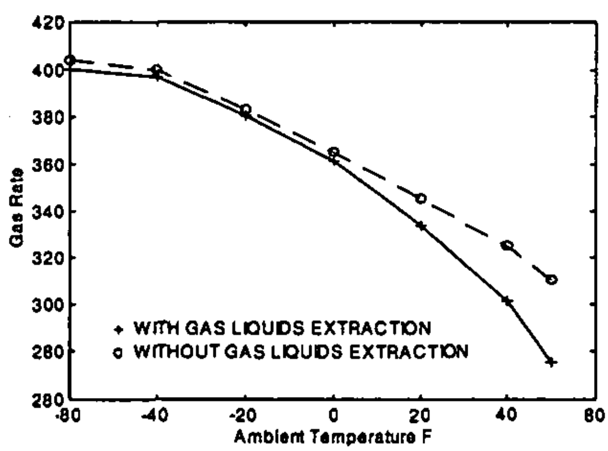

Figure 5. Third Stage Gas Rate (10^6 SCFD) versus Ambient Temperature (°F)

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