THE EFFECT OF FLUID INERTIA IN SQUEEZE FILM DAMPER BEARINGS: A HEURISTIC AND PHYSICAL DESCRIPTION

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ABSTRACT

Fluid inertia forces are comparable to viscous forces in squeeze film dampers in the range of many practical applications. This statement appears to contradict the commonly held view in hydrodynamic lubrication that inertia effects are small. Upon closer inspection, the latter is true for predominantly sliding (rather than squeezing) flow bearings.

The basic equations of hydrodynamic lubrication flow are developed, including the inertia terms. The proper orders of magnitude of the viscous and inertia terms are evaluated and compared, for journal bearings and for squeeze film dampers. Exact equations for various limiting cases are presented: low eccentricity, high and low Reynolds number. The asymptotic behavior is surprisingly similar in all cases. Due to inertia, the damper force may shift 90° forward from its purely viscous location. Inertia forces are evaluated for typical damper conditions.

The effect of turbulence in squeeze film dampers is also discussed. On physical grounds it is argued that the transition occurs at much higher Reynolds numbers than the usual lubrication turbulence models predict.

NOMENCLATURE

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\[ \begin{align*}
\text{c} &= R_o - R_i \\
\varepsilon &= \text{eccentricity, see Figs.1 and 2} \\
f_i, f_t &= \text{radial and tangential forces on shaft, see Fig.2} \\
F &= \frac{f}{c_0 L} \\
J &= \left| \frac{f_i}{f_t} \right| \\
\dot{\epsilon} &= \sqrt{\omega^2 + \epsilon^2} \\
\dot{h} &= e + \epsilon \cos \delta' \\
L &= \text{bearing length normal to plane} \\
\phi &= \text{phase angle, see Fig.1} \\
\omega &= \text{rotation rate of inner cylinder, see Fig.1, or precession rate of inner cylinder, see Fig.2} \\
\text{r} &= \text{radial coordinate} \\
R_i, R_o &= \text{inner, outer cylinder radius} \\
R &= \text{cylinder radius} \\
\text{Re}^* &= \text{modified Reynolds number, see Eq.(14)} \\
\text{Re} &= \text{Reynolds number, see Eq.(15)} \\
\text{t} &= \text{time} \\
U &= \text{reference sliding velocity} \\
U_{\text{U}, \text{v}} &= \text{fluid velocities along and across the film (i.e., in \( \theta \)- and \( \gamma \)-directions), respectively} \\
\gamma &= \text{boundary layer thickness} \\
\varphi &= \text{eccentricity ratio} \\
\phi' &= \text{bearing angular coordinate with respect to fixed axis, see Figs.1 and 2} \\
\tau &= \text{bearing angular coordinate with respect to maximum film thickness point, see Fig.2} \\
\mu &= \text{viscosity} \\
\rho &= \text{fluid density} \\
\tau &= \text{fluid shear stress} \\
\phi &= \frac{\tau}{2} + \tan^{-1}\left(\frac{-\frac{\tau}{2}}{\tau}\right) \\
\text{tr} &= \text{transition to turbulence}
\end{align*} \]

INTRODUCTION

The interested worker who seeks to investigate the possible effect of fluid inertia in fluid film bearings and consults the classical textbooks in this field encounters statements such as "(the) assumption, that of negligible fluid inertia, yields erroneous results only in special cases ..., and can be left intact" [1],
and "other errors ... easily outweigh such a small correction," [2] etc. It can come as a great surprise that in squeeze film damper bearings, in the range of most practical applications this "small correction" is generally 30 - 200% of the lubrication theory prediction, especially at high speeds (say > 10,000 cpm). In addition, the damper force undergoes a significant phase shift from its purely viscous location. The existence of such inertial phenomena is no longer open to controversy, having been well established by analysis of the governing differential equations, and recent experiments.

Reinhart and Land [3] use a low Reynolds number perturbation method to solve for the inertia effect. Szeri et al. [4] have developed an analysis for dampers based on the averaged fluid inertia forces across the film. There have been a number of papers on squeezing flow between parallel disks or plates but these are not directly applicable to dampers, e.g. [5,6]. Tichy has solved the problem of low eccentricity ratio but arbitrary Reynolds number for short and long dampers [7,8]. The curious (and fortunate) fact has evolved, as referred to by Jones and Wilson [9] and Tichy [10] that low Reynolds number perturbations are highly accurate even for Re = 100. Recent experiments also have demonstrated the striking effect of fluid inertia and the wide discrepancy from lubrication theory. A group from Mechanical Technology Incorporated (MTI) intends to present such data at the 1983 Gas Turbine Conference. The author's own experiments, which will be published shortly, also tend to corroborate the theoretical findings.

The reason for the slow acceptance of these results, and for statements such as those quoted above is that conclusions correctly drawn for steady journal bearing application have been applied by inference to dampers. By simple physical reasoning the correct order of magnitude for squeeze film damper pressures and forces due to fluid inertia is evaluated in this study. The present paper reviews the physical differences between journal and damper bearing fluid flow. Simple formulae are presented for the forces, pressure and velocity field. Limitations to the laminar inertia theory, and the transition to turbulence are discussed. Turbulence is thought to play a much smaller role in damper bearings due to a delay of transition, which is supported substantially by theory and experiment.

THE BASIC EQUATIONS

The basic force balance equation for hydrodynamic lubrication laminar flows in circular geometries, in the absence of fluid inertia effects is:

$$\tau = \frac{1}{R} \frac{\partial p}{\partial \theta} + \frac{3}{2} \frac{\partial v}{\partial y}.$$  

(1)

The symbols used are explained in the Nomenclature.

For purposes of illustration, let us restrict ourselves to an idealized case, although the results obtained apply to more general circumstances. We consider only two-dimensional flows (i.e., one-dimensional or "long" bearings), with full Sommerfeld $2\pi R$ boundary conditions (i.e., no cavitation), and steady operating conditions at constant eccentricity ratios. The long bearing case applies quite well to the sealed end bearing even if it is not physically long. For squeeze film dampers, by "steady" we mean that the inner journal performs a centered orbit of constant eccentricity at a constant rate of precession. This configuration is steady with respect to a centered coordinate system which rotates with the journal precession. Equation (1) states that pressure on a fluid particle is in balance with the viscous shear stress. Since the net force is zero there is no acceleration of the particle, i.e., no inertia.

If the forces are not in balance the fluid particle will accelerate according to

$$\frac{d\mathbf{v}}{dt} = \frac{\partial p}{\partial \theta} R + \frac{3}{2} \frac{\partial v}{\partial y}.$$  

(2)

In Eq. (2), $a_0$ represents the acceleration of the fluid particle in the film-wise direction. The symbol $du/dt$, denotes the time rate of change of velocity as seen by an observer who moves with the particle. This derivative differs from the partial derivative $\partial u/\partial t$, which is the time rate of change of $u$ as seen by a stationary observer. Equation (2) is nothing more than Newton's second law, $ma = F_{net}$, on a per unit volume basis.

The shear stress is related to the velocity through the well-known Newton's law of viscosity

$$\tau = \mu \frac{\partial u}{\partial y}.$$  

(3)

An additional relationship is needed which governs the mass balance on a small fixed control volume of fluid (not a fluid particle):

$$\frac{1}{R} \frac{\partial u}{\partial \theta} + \frac{v}{R} = 0.$$  

(4)

This is the continuity equation for an incompressible fluid and states that the net mass flow rate to or from the control volume is zero.

ORDER-OF-MAGNITUDE ANALYSIS

We seek now to assess the orders-of-magnitude of the various terms in Eqs. (1) - (4), for both squeeze film dampers and journal bearings. One can get away with what seems to be a lot of loose mathematics in order-of-magnitude analysis because we do not worry about little things like 100 - 500% errors. We try only to be accurate to within a factor of ten or so.

Derivatives with respect to the film coordinate $R$, $\partial/\partial R$, are approximately of order $1/R$ denoted $O(R^{-1})$. The entire fluid film in a damper or journal bearing extends a distance $2\pi R$. If a particular variable changes along the film, it changes on an approximate length scale $R$. The factor $2\pi$ is too small to bother with. Similarly, $\partial/\partial y \sim O(c^{-1})$. Again, if a variable changes with $y$ (across the film) the approximate length scale is $c$.

For journal bearings the film-wise velocity $u$ is $O(uR)$, see Fig.1. For squeeze film dampers, the squeezing velocity is $O(uec)$, see Fig.2. At any instant the squeezing motion is purely linear and normal to the line of centers. From the continuity equation,

$$\nu = O(uR \cdot \frac{1}{R} \cdot c) \sim O(uec),$  

(5)

* Called, interchangeably, the material derivative, substantial derivative, or Lagrangian derivative.
SFD: \( \frac{1}{R} \frac{D u}{D \theta} = \frac{2w}{R} \cos (\varepsilon + \frac{1}{c}) \)

Thus the order of the SFD film-wise velocity component is smaller by a factor \( \varepsilon \). In fact the actual velocity profiles are given by

\[
JB: u = \frac{2w}{R} \left( \frac{y}{h} \right) + O(\varepsilon) R
\]

SFD: \( u = \frac{2w}{R} \left[ \left( \frac{y}{h} \right)^2 \right] \cos (\omega t - \theta) \) \( O(\varepsilon) \)

Let us now assess the order of magnitude of the viscous shear-stress term in the two cases is then:

\[
JB: \tau = O\left( \frac{2w}{c} \right) \text{ SFD: } \tau = O\left( \frac{2w}{c} \right)
\]

The order of magnitude of the viscous shear-stress term is then

\[
JB: \frac{\partial \tau}{\partial y} = O\left( \frac{2w}{c^2} \right) \text{ SFD: } \frac{\partial \tau}{\partial y} = O\left( \frac{2w}{c^2} \right)
\]

In the journal bearing, the particles move purely in the circumferential direction at the velocities given by Eq. (7). Particles near to the inner journal \( y/h \approx 1 \) move faster than those near the outer cylinder, but all particles move at nearly constant speed, i.e., the accelerations are small. In fact, it turns out that

\[
JB: \frac{Du}{Dt} \approx \frac{2w}{c^2} \approx O(\varepsilon^2 R^2)
\]

The squeeze film damper flow is somewhat more complicated. At a given angular location, setting say \( \theta = 0 \) in Eq. (8), the velocity field oscillates in a sinusoidal manner. For each time period \( \Delta t = 1/2\omega \) the velocity completely changes direction - clockwise to counterclockwise and back again.

These surges in one direction and then the other are the basic cause of the inertia effect in squeeze film dampers. The velocities are surprisingly high for the relatively low squeezing rate. For the following conditions

\[
R = 7.5 \text{ cm } (\sim 3 \text{ in.}) \quad \omega = 1500 \text{ r.p.m. } (15,000 \text{ cpm})
\]

\[
c = 0.03 \text{ cm } (\sim 0.012 \text{ in.}) \quad \varepsilon = 0.2 \quad L = 2.5 \text{ cm } (\sim 1 \text{ in.})
\]

the squeezing velocity \( v \approx O (10 \text{ cm/s}) \), but the sliding velocity \( u \approx O (20 \text{ m/s}) \). This velocity completely changes direction every \( \sim 0.6 \text{ ms} \). Seen in this context, the large inertia forces are not surprising at all. Thus

\[
SFD: \frac{Du}{Dt} \approx \left( \frac{2w}{R} \right) = O(\varepsilon^2 R^2)
\]

Thus the order of magnitude of the fluid particle accelerations are the same in the squeeze film damper and the journal bearing, even though the SFD velocities are less.

Let us consider the pressure field as a superposition of the viscous stress term and an inertia stress term, although this is strictly correct only in certain limiting cases which result in linear mathematics. From Eq. (2)

\[
\frac{\partial P}{\partial \theta} = R \frac{\partial \tau}{\partial y} + \varepsilon R \frac{Du}{Dt}
\]

\[
JB: p \sim O(\varepsilon w R^2) + O(\varepsilon \omega R^2)
\]

SFD: \( p \sim O(\omega w R^2) + O(\omega w R^2) \)

\[
(13a)
\]

\[
(13b)
\]

These equations could also be written in terms of the force magnitude by multiplying pressure by the projected area \( RL \):

\[
JB: F = O(1) + (\varepsilon \omega R^2)
\]

SFD: \( F = O(1) + (\varepsilon \omega R^2) \)

\[
(14)
\]

If pressure is nondimensionalized by dividing Eq. (13a) by the factor \( (\omega w R^2/c^2) \) and (13b) by \( (6\omega w R^2/c^2) \), and similarly for Eq. (14) we obtain

\[
JB: F or P \sim O(1) + (\varepsilon \omega R^2)
\]

SFD: \( F or P \sim O(1) + (\varepsilon \omega R^2) \)

\[
(15)
\]

where the appropriate Reynolds number is

\[
Re^* = \frac{c R^2}{\mu}.
\]

The dimensionless group \( Re^* \) is sometimes called the modified or reduced Reynolds number. The conventional Reynolds number of fluid mechanics is defined in terms of sliding velocity and gap size:

\[
\text{Re} = \frac{\text{sliding velocity} \times \text{gap height}}{\mu} = \frac{\varepsilon Re}{\mu} = \text{Re} (R/c)
\]

(17)

The latter Reynolds number will be of primary importance, regarding the transition to turbulence. Note that the value of \( e Re \) (JB) or \( e Re \) (SFD) specifically defines the order of magnitude of the ratio of inertia to viscous forces. In this context "small" \( Re \) means \( << 1 \) and vice versa. This clear relationship is not
always the case. In pipe flows, for example, a Reynolds number of 100 would be considered "small", i.e., producing very stable laminar flow. The actual value of Re in pipe flow has no particular physical significance per se.

Finally, and most importantly, observe that for a given Reynolds number Re*, the relative value of the inertia force (i.e., relative to the lubrication theory viscous force) for the journal bearing is much less than that of the damper. Since e is presumed to be a small number, eRe* \ll Re*. It turns out that this conclusion is true for all practical values of e, for the same sort of physical reasons, but it is more difficult to show. Studying Eqs.(15) to (16), it seems physically strange that the relative value of the inertia force does not depend on the bearing radius, while the absolute value does not depend on the film thickness.

**PHASE RELATIONSHIPS FOR SFD FORCES**

The above discussion was only concerned with absolute values of the damper forces. It turns out that the viscous pressure peak and the inertia pressure peak occur at different times during the orbit cycle. Viscous forces are maximum in the region where u is maximum while the inertia forces are maximum where the rate of change of u is maximum.

Note on Fig. 2 that the shaft velocity is always in the direction \( \theta' = \pi/2 \). The angle \( \theta' \) is measured with respect to a fixed axis while \( \theta' \) is measured from the position of maximum film thickness. The fluid is squeezed out in both directions away from this point. The velocity u is positive maximum (counterclockwise) near \( \theta' = 0 \) and negative maximum near \( \theta' = \pi \). The shear stress and the viscous pressure gradient, cf. Eq.(13), take their extremal values in this region. The viscous pressure itself is maximum between these extremal values, when \( \partial p/\partial \theta' = 0 \), in the direction of the shaft velocity \( \theta' = \pi/2 \). Since pressures on both sides of this peak along the shaft tend to cancel each other out, the viscous force on the outer cylinder is also maximum in this direction.

Since the velocity completely changes direction between \( \theta' = 0 \) and \( \theta' = \pi \), it stands to reason that the rate of velocity change is maximum near \( \theta' = \pi/2 \) and \( \pi/2 \). Hence the inertia pressure gradient is maximum near these points and the inertia pressure and inertia force are maximum near \( \theta' = 0 \).

The damper pressure and forces (on the shaft) can be expressed as

\[
p - p_a = \frac{12 \pi \mu w L}{c^2} \sin \theta' + \frac{6}{5} \rho c \omega R^2 \cos \theta', \tag{18}
\]

viscous term \hspace{1cm} inertia term

and

\[
\frac{f}{f} = \frac{12 \pi \mu w L}{c^2}, \tag{19}
\]

viscous term \hspace{1cm} inertia term

\[
\frac{f}{f} = \frac{6}{5} \rho c \omega R^2 \frac{12 \pi \mu w L}{c^2} \left( \frac{1}{10} \text{Re}^* \right). \tag{20}
\]

Compare these equations to the order-of-magnitude forms, Eqs.(13) - (15). Equation (19) can be expressed in the equivalent form

\[
\beta = \left[ 1 + \left( \frac{\text{Re}^*}{10} \right)^{2/3} \right]^{1/2},
\]

\[
\phi = \frac{\pi}{2} + \cot^{-1} \left( \frac{\text{Re}^*}{10} \right), \tag{20}
\]

in which \( \beta \) is a correction factor to lubrication theory. From lubrication theory, the total fluid film force on the shaft leads the maximum film thickness location by 90°. The inertia effect has the tendency to move the total force further in this direction up to 180° past the maximum film thickness point. In the presence of cavitation and other effects, such clear isolation of the viscous and inertia forces is not possible - the radial force has a viscous component and vice versa.

**LIMITING CASES**

The above equations (8) and (18) - (20) are strictly speaking valid asymptotically in the combined limit \( e \ll 1 \), \( \text{Re}^* \ll 1 \), for the long bearing without cavitation. It is very curious that this equation is accurate to within several percent for \( e \ll 1 \) to \( \text{Re}^* = 100 \), from Ref.[8]; and for \( e \approx 0.3 \) to \( \text{Re}^* \approx 50 \) from the author's unpublished numerical studies. The equations can be derived as a low Reynolds number small perturbation to the lubrication theory solution, but near \( \text{Re}^* \approx 50 \) the "small perturbation" is four times larger than the lubrication solution.

Other limiting cases can be developed: (1) small \( \text{Re}^* \), arbitrary \( e \), due to Reinhardt and Lund [3]; (2) small \( e \), arbitrary \( \text{Re}^* \) [8]; and (3) large \( \text{Re}^* \), arbitrary \( e \). The closed form expressions for cases (1) and (2) are too complicated to reproduce here. The large \( \text{Re}^* \) case has recently been obtained by this writer and is reproduced here for the first time without proof:

\[
p_i = p_a + \frac{C \rho c \omega R^2}{c} \left( \beta - \beta^*-1 \right)^2, \tag{21}
\]

\[
= \frac{2}{c^2} \left( \beta - \beta^*-1 \right)^2 \sin \theta', \tag{21}
\]

A comparison of dimensionless inertia pressure profiles for the three cases is shown in Fig.3. It seems incredible that they are so close to one another, especially the low \( \text{Re}^* \) and high \( \text{Re}^* \) cases. The structure of the flow field is entirely different in the two cases. For low \( \text{Re}^* \), viscous effects dominate across the entire film producing the parabolic profile of Fig.4. For high \( \text{Re}^* \), a boundary layer structure develops where viscous effects are important only near the bearing surfaces and inertia effects dominate in the core or "free stream." However, the "bottom line" - the damper forces - come out almost identical in both cases. There may be an underlying, but yet undiscovered, physical reason for this peculiarity, as discussed by Jones and Wilson [9] and Tichy [10].

**ORDER OF MAGNITUDE OF THE SFD FORCES**

Consider now the kinematic conditions stated earlier for the typical high speed damper. In addition, let us assume typical lubricant properties:

\[
\mu = 10 \frac{\text{cp}}{\text{cm}^2}, \quad \rho = 0.8 \frac{\text{gm}}{\text{cm}^3}.
\]
For this case the viscous pressure maximum is $2.25 \times 10^6$ N/m$^2$ (325 psi) and the inertia pressure maximum is $2.44 \times 10^6$ N/m$^2$ (352 psi). The Reynolds number $Re^* = 10.8$ and the lubrication theory correction factor $\tau = 1.46$. The total force is 19350 N (4350 lbf), the phase angle $\phi = 97^\circ$, and the viscous and inertia forces are 9090 N (2046 lbf) and 10250 N (2307 lbf), respectively. It is rather astounding that such a small volume of fluid ($\approx 3.4$ cm$^3$) can create such large inertia forces, but recall that the accelerations are very high. There also is a physical leverage effect in the fluid film wedge. Stresses acting along the film cause pressures which act across the film, which are larger by a factor $\delta R/c$.

**TURBULENCE IN SQUEEZE FILM DAMPERS**

There are a number of models for turbulence in hydrodynamic lubrication which apply data from pure Couette flow to lubrication geometries, e.g., Ng-Pan [11], Black [12], Nelson [13] has attempted to do the same sort of thing for squeeze film dampers and he predicts the onset of turbulence at $Re \approx 2000$. This would appear to contradict the NTI data which follow the laminar fluid inertia theory over the speed range. A transition to turbulence would surely appear as a sudden change in the damper forces at a certain speed.

We move now from fairly well established fact to speculation. This worker proposes that the transition to turbulence in SFD's is governed by boundary layer transition fluid mechanics, rather than Couette flow fluid mechanics from which come the lubrication turbulence models. There is a key difference. In the latter (Couette flow) case the fluid inertia force ($du/dt$) is exactly zero and the velocity field does not change at all until turbulent transition, in the former case the velocity field constantly changes and adjusts itself in response to the inertia force, as in Fig. 4. The hypothesis is put forth that this adjustment promotes flow stability and the transition to turbulence is delayed greatly from the Couette or Poiseuille flow value of 2000.

A very crude analysis is proposed here to predict the onset on turbulence, which will be developed in later work. From standard fluid mechanics textbooks, for boundary layers with no pressure gradient (this is not strictly true for dampers, of course), turbulent transition occurs when

$$\frac{\mu}{C} = 2800, \quad (22)$$

where the subscript $tr$ denotes transition conditions. For small $Re^*$, nearly Poiseuille flow results and roughly $\delta \approx C/2$, i.e., the boundary layers fill the gap. For large $Re^*$ the following empirical formula is used for the boundary layer thickness:

$$\frac{\delta}{C} = 2 Re^*-1/2. \quad (23)$$

This result was achieved by plotting numerous velocity fields based on the formulae of Ref.[8] and is consistent with traditional boundary layer theory. Hence we have for small $Re^*$, using $W = \delta W$:

$$\frac{Re^*}{\mu} \frac{\delta W}{C} = \frac{\delta W}{C} = 5600, \quad (24)$$

and for large $Re^*$,

$$\frac{Re^*}{\mu} \frac{\delta W}{C} = \frac{\delta W}{C} = 1400 Re^* \frac{1}{2}. \quad (24)$$

These results are plotted in Fig.5.

For the example presented above, using Fig. 5, turbulent transition would occur at $Re^* \approx 6000$. Thus turbulence would not be expected until $w \approx 2600$ $\psi$ (26000 rpm).

**DISCUSSION AND CONCLUSIONS**

The existence of large inertia forces, at least as large as viscous forces, in squeeze film damper bearings is no longer open to controversy. There now exists a substantial body of theoretical work, and most researchers have reached the same qualitative conclusions. It now remains for these notions to be incorporated more thoroughly into rotordynamic analysis. It has been mentioned that low and high Reynolds number asymptotic behavior, at least in the case of the pressure field, is nearly identical. This is most fortunate because the methods of Refs.[3] and [4] are strictly dependent on the presumption of a small inertia correction to the viscous solution. In most applications the correction is not small at all. At low Reynolds number, $Re^* \ll 1$, the diffusion of momentum is mostly cross-film, while for $Re^* > 10$, the momentum transfer is mostly film-wise. In the latter case, an assumed velocity field at any point along the film can dominate the solution because it is swept along the film with no change except in boundary layers very near the wall. This means that an initial assumption of nearly viscous flow will insure that result in the end. It is most fortunate and unusual that this turns out to be the case.

Some speculative discussion on turbulence effects in squeeze film dampers has been presented. Again the common notion in hydrodynamic lubrication of a small brief laminar inertia region followed shortly by transition to turbulence does not appear to be true. For squeeze film dampers, simple analysis based on boundary layer considerations yields quite different conclusions, as supported by recent experiments.

A great deal of theoretical and experimental research remains on both academic and practical issues of the hydrodynamics of squeeze film dampers. Simple squeeze film flow has been studied since 1850, prior to Lord Reynolds' work, but still many intriguing and difficult issues remain.

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**REFERENCES**


Fig. 4 Typical Velocity Profiles - Low and High Reynolds Number

Fig. 5 Turbulent Transition Reynolds Number