Dynamic Analysis Technique of Rotating Centrifugal Impeller

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ABSTRACT

A dynamic analysis technique which can be employed in rotating centrifugal impeller is presented in this paper. It shows that multi-component partition can be made in repetitive sector region of the centrifugal impeller. The basic repetitive sector region of the centrifugal impeller is divided into three substructures: the full blade, the short blade and the sectorial part of the disc. By using Benfield mode substitution combined with group transformation successfully, the Hermite generalized mass and stiffness matrices under the reduced coordinates are derived. From this, the natural frequencies and the corresponding modal shapes of the bladed disc coupled system can be solved.

The comparison of the analytical results obtained by using this method, other methods and the experimental data of models verifies the reliability, practicability and considerable economic benefits of the method presented in this paper.

NOMENCLATURE

\( a \)  acceleration
\( b_i \)  interior degree of freedom (DOF) of full blade b
\( b_j \)  interface DOF of full blade b
\( C_{b_i} \)  constraint modes
\( D_{b_i} \), \( D_{b_j} \)  loaded constraint modes
\( d_s \)  non-wave propagation DOF of sectorial disc d
\( d_i \)  interior DOF of sectorial disc d
\( d_j \)  interface DOF between substructures b and d
\( d_l \)  interface DOF between substructures s and d
\( d_r \), \( d_c \)  wave propagation DOF of sectorial disc d
\( f_{s_b} \)  equivalent nodal force vector of short blade s
\( f_{s_b} \)  equivalent nodal force vector of short blade s
\( h \)  force vector between substructure interfaces

\( K_{s_i} \)  centrifugal stiffness matrix
\( K_{s_j} \)  condensed stiffness matrix of substructure b
\( K_{s_k} \)  condensed stiffness matrix of substructure s
\( M \)  mass matrix
\( M_{O_r} \)  gyro matrix
\( N \)  shape function matrix
\( n \)  number of iteration time
\( P \)  centrifugal force vector
\( R_{b_s} \)  displacement compatibility matrix between substructures b and d
\( R_{s_s} \)  displacement compatibility matrix between substructures s and d
\( S_{b_s} \)  interior DOF of short blade s
\( S_{b_s} \)  interface DOF of short blade s
\( \delta \)  displacement vector
\( \delta' \)  displacement vector at new equilibrium position
\( \Delta \delta \)  displacement increment vector
\( \psi \)  inequilibrium force vector of the impeller system
\( \Lambda_{s_i} \), \( \psi_{s_b} \)  loaded normal modes
\( \rho \)  material density
\( \Pi_{s_i} \), \( \gamma_{s_b} \)  fixed interface normal modes of short blade s
\( \alpha_{s_b} \), \( \phi_{s_b} \)  fixed interface normal modes of full blade b

INTRODUCTION

The centrifugal compressor had received many years of intensive development, due to its long use in industry and as a blower for pressure-charging reciprocating-type aircraft engines. It is also adopted for gas turbine aero-engines, especially for turboprop and turboshaft engines which always have a combined compressor, i.e. it has several axial stages and a single centrifugal stage. The inducer, which induces the air into the impeller vanes smoothly, is no longer a separate component, but integrates with the impeller. The modern design is an integrated impeller with full blades and short blade as shown in Fig 1a.
To perform a dynamic analysis for this centrifugal impeller, usually group theory or wave propagation technique is used first to limit the analytical region in a repetitive sector region (a full blade, a short blade and the corresponding sector region of impeller disc) of the cyclo-symmetric structure. (Fig. 1b). But there are still too many degrees of freedom (DOF) in a sector region. It is effective to employ dynamic substructure technique combined with group theory algorithm. Henry and Farrar applied Craig’s constraint substructure method to the basic sector region successfully, but the proper selection above can only eliminate the interface degrees of freedom between one branch component (just one blade) and one main component (corresponding sector impeller disc). Ref.[4] extends the Benfield-Hruda substructure technique that has a high computational efficiency to the extent which can be used simultaneously with the constraint mode substitution transformation of the boundary DOF at the basic repetitive sector region. It can eliminate all the interface DOF between the substructures, when the repetitive sector region is selected arbitrarily. So, it is applicable to the case of multi-component partition in the basic repetitive sector region. In Ref.[3], using the substructure simultaneous iteration, the rotating modes of an axial bladed disc coupled system are solved and the geometric nonlinear deformation effect under centrifugal field is considered.

The purpose of this paper is to extend the substructure simultaneous iteration into the case of multi-component partition of the basic repetitive sector region and the new equilibrium position of rotating normal mode analysis is obtained too.

Figure 1  Centrifugal Impeller

NON-ROTATING NORMAL MODE ANALYSIS

Assuming the number of full blades and short blades on the centrifugal impeller is N respectively, the investigated repetitive sector region is 1/ N impeller. One full blade, one adjacent short blade and the corresponding sector of the impeller disc are included in the region (Fig. 1b). Three parts of the basic repetitive sector region are the three substructures. 1/N impeller disc is the main component fixed at the inner diameter boundary and two blades are both branch components. 8 nodes, 40 DOF thick shell superparametric element is the discrete model of the full blade which has large twist angle and large variation of thickness from the tip to the root of the blade. The short blade has the same discrete model. Brick element is selected as the discrete model of the impeller disc.

According to the above modelling state, the interior and interface DOF of full blade are $b_i$ and $b_j$, respectively and $S_i$ and $S_j$ as the interior and interface DOF of short blade. The degrees of freedom of the sector region of the disc are divided into several parts: The wave propagation DOF $d_i$ and $d_j$, and the non-wave propagation DOF $d_k$ as the interface DOF connecting with the short blade and the interface DOF $d_l$ connecting with the short blade. The consistent partition processes are performed to the stiffness and mass matrices of the main component and branch components. Then, in accordance with standard procedures, the fixed interfaces ($b_j = 0$ and $S_j = O$) normal modes ($A_{kk}$, $P_{n}$) and ($H_{kk}$, $Y_{n}$) of the full blade and the short blade are obtained. The constraint modes ($C_{ii}$, $E_{ii}$) and the mass and stiffness matrices of the two blades (full blade and short blade) condensed to the interface sets are abstracted. From these, the interface loaded mass and stiffness matrices of the impeller disc (with full blade and short blade) are obtained too. The compatibility of displacements between the interface DOF of disc and blades should be considered. Consequently, the loaded normal modes ($A_{kk}$, $P_{n}$) and the loaded constraint modes ($D_{kk}$, $D_{n}$) determined by wave propagation sets are abstracted under the conditions that the sector region of the disc is constrained ($d_i = d_j = O$). Usually, first 8 orders fixed interface normal modes of the blades (guide blade and splitter blade) and first 6 orders fixed loaded normal modes of the impeller disc are abstracted by authors’ experiences.

After Benfield mode substitution transformation is performed, the kinetic energy and strain energy expressions of three components are set up and are superposed to form the total kinetic energy and the total strain energy of the complete centrifugal impeller (with blades) system. By using Hamilton principle and through detailed derivation the generalized mass matrix and the generalized stiffness matrix of the system is obtained under the reduced coordinates. Finally, using wave propagation technique the reduced coordinates just have the wave propagation DOF of one boundary side. The corresponding mass and stiffness matrices are gotten and the Hermitian matrix pairs are solved. The so-called eigenvalues are the coupled vibration natural frequencies of the nonrotating complete centrifugal impeller system, whereas the eigenvectors solved out still need to be transformed individually according to sub-regions and sub-spaces to get the modal shapes of the system. Detailed expressions are shown in Ref.[4]. The first four orders nonrotating normal modes of a centrifugal impeller system of a real aero-engines are calculated in Table 1. They have good agreement with the experimental data.

Table 1. Nonrotating normal modes with different nodal diameters of a real centrifugal impeller system

<table>
<thead>
<tr>
<th>Diameter</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
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<td>4</td>
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<td>5902</td>
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<td>8303</td>
</tr>
</tbody>
</table>

A — analytical results  E — experimental data

First 3 orders modal shapes with 2, 4, 8 nodal diameters at the back surface of the impeller disc of the above system (Table 1 expressed) are shown in Fig.2.

In order to analyse the dynamic behaviours and to compare the differences of bladed impeller disc and non-bladed impeller disc. The dynamic analysis of non-bladed impeller disc is made. Table 2 gives the nonrotating fundamentally normal modes with different nodal diameters of a
non-bladed impeller disc. The disc body is the same one as shown in Table 1.

\[ \psi = B^T \sigma dV + M \delta + M_c \delta - K, \delta - P = 0 \]  

where \( M, M_c, K_c \) and \( P \) are mass matrix, gyro matrix, centrifugal stiffness matrix and centrifugal force vector respectively. In order to solve the new equilibrium position and the corresponding stiffness matrix under large displacements of the centrifugal impeller system, let \( \delta = \delta^* \) so, while \( \delta = \delta^* \) the new equilibrium position is obtained. Newton–Raphson iteration is used to get the incremental solution.

For the centrifugal impeller system described above, the full blade short blade and the corresponding sectorial disc are included in the basic repetitive sector region. The DOF of these three components are partitioned as follows:

\[ \begin{align*}
\delta &= (\delta_i^T \delta_s^T \delta_d^T)^T \\
\sigma &= (\sigma_i^T \sigma_s^T \sigma_d^T)^T \\
d &= (d_i^T \delta_s^T \delta_d^T)^T
\end{align*} \]

\[ d_s^* = (d_i^T \delta_s^T \delta_d^T)^T \]

The displacement increment of the nth iteration is

\[ \Delta \delta_{n+1} = (\delta_i^T \delta_s^T \delta_d^T)^T \]

\[ f_s = \sigma - K \Delta \delta_{n+1} - P \]

The force vector between substructures' interfaces \( h \) is expressed as

\[ h = (h_i^T h_s^T h_d^T)^T \]

The mass matrices, stiffness matrices and load force vectors of substructures are partitioned as the above DOF partition. For full blade (substructure \( b \))

\[ h_b = (0 h_b^T) \]

\[ f_b = (f_b^T f_b^T) \]

Making transformation

\[ \begin{align*}
\{ b \} &= \begin{bmatrix} I & C \end{bmatrix} \{ e \} \\
\{ b^* \} &= \begin{bmatrix} O & I \end{bmatrix} \{ e \}
\end{align*} \]

where \( C \) is the constraint mode matrix of full blade. Then

\[ \begin{align*}
\{ e \} &= \{ b \} \{ e \} \\
\{ e^* \} &= \{ b^* \} \{ e \}
\end{align*} \]

\[ f_b^* = f_b^T + h_b^T C f_b \]

Eq.(15) and Eq.(16) are the condensed stiffness matrix and the equivalent nodal force vector of substructure \( b \) respectively. The same analysis is made for the short blade (substructure \( s \)) and the corresponding condensed stiffness matrix \( K_s^* \) and the equivalent nodal force vector \( f_s^* \) is obtained too.

For sector impeller disc

\[ \begin{align*}
\{ e \} &= \begin{bmatrix} h_s^T & h_s^T & h_s^T \end{bmatrix} \\
\{ f_s \} &= \begin{bmatrix} f_s^T & f_s^T & f_s^T \end{bmatrix}
\end{align*} \]

Its equilibrium equation is

\[ \{ e \} = \{ f_s \} \]
The compatibility of interface forces and interface displacements between blade and disc should be considered

\[ h_{n} = -R_{b}^{*} h_{a}, \quad h_{n} = -R_{d}^{*} h_{a}, \]

\[ d_{i} = R_{b}^{*} b_{i}, \quad d_{i} = R_{d}^{*} s_{i}, \]

where \( R_{b} \) or \( R_{d} \) is the displacement compatibility matrix between substructure \( b \) (or substructure \( s \)) and substructure \( d \) and \( R_{b} \) and \( R_{d} \) are both orthogonal matrices.

From \( h_{b} \) and \( h_{d} \) expressions and the compatible relations, the following equations are obtained:

\[ f_{d} = f_{b}, \quad f_{d} + R_{b}^{*} f_{b}, \]

\[ f_{d} = f_{d}, \quad f_{d} + R_{d}^{*} f_{d}. \]

As centrifugal forces acting on all sectors of the impeller system also have the rotationally periodic property, the displacements and outer forces of the two wave propagation boundaries \( t \) and \( t' \) at the basic repetitive sector region totally satisfy the relation of cyclosymmetry. Finally get the equation as follows:

\[ \text{(19)} \quad d^{*} - f_{d}^{*} \]

\[ \text{(22)} \quad \begin{bmatrix} K_{b} + K_{d} \\ K_{b} + K_{d} \\ K_{b} + K_{d} + K_{d} \end{bmatrix} \]

\[ d^{*} - (d^{*} - d^{*})^{T} \]

\[ f^{*} - (f^{*} - f^{*})^{T} \]

\[ \text{(23)} \quad \begin{bmatrix} \end{bmatrix} \]

\[ \text{(24)} \quad \begin{bmatrix} \end{bmatrix} \]

\[ \text{(25)} \quad \begin{bmatrix} \end{bmatrix} \]

\[ \text{(26)} \quad \begin{bmatrix} \end{bmatrix} \]

\[ \text{(27)} \quad \begin{bmatrix} \end{bmatrix} \]

\[ \text{(28)} \quad \begin{bmatrix} \end{bmatrix} \]

\[ \text{(29)} \quad \begin{bmatrix} \end{bmatrix} \]

The displacement increment of the \( n \)th iteration is

\[ d = (d_{i}, d_{i}, d_{i})^{T}, \quad (d_{i} = d_{i}) \]

And then \( D_{i} \) and \( S_{i} \) are solved out. Now, the total displacement increment \( \Delta t_{n-1} \) of the \( n \)th iteration is obtained and

\[ \Psi_{n-1} = [B_{n-1}, \sigma_{n-1}, \delta_{n-1}, K_{n-1}, \delta_{n-1} - P] \]

The displacement and stress vectors of the \( n \)th iteration at the basic sector region are also solved out,

\[ \delta_{n} = \delta_{n-1} + \Delta \delta_{n-1} \]

\[ \sigma = \sigma_{n-1} + DB_{n-1} \delta_{n-1} \]

Through many times of iteration, until \( \Psi_{n} \) is sufficient small or is approximately equal to zero. When \( \delta_{n} = \delta \) the new equilibrium position of the basic repetitive sector region is determined and all of the modal informations at the sector region have been used and can be used at the dynamic analysis later.

It is well known that using experimental way to get the rotating normal modes of a structure in detail is very complicated and it will be unreasonable in finance. So, the theoretical and numerical methods have its obvious practicability. In order to check the accuracy of the method presented in this paper, the comparison between the methods of Ref.[3] and this paper was made. For a real 24 axially bladed disc system, 1/24 structure and 1/12 structure are analysed as examples of the methods of Ref[3] and this paper respectively. Although axially bladed disc and centrifugal impeller are different in structures, the basic principle of mechanics is the same. The good agreement of the results of these two methods is shown in Table 3, when the axially bladed disc system rotates at RPM 9000.

<table>
<thead>
<tr>
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<tbody>
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<tr>
<td>4</td>
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<td>571.9</td>
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</table>

The rotating normal modes of a real centrifugal impeller system (with full blades and short blades) of an aero-engine compressor (the same model with Table 1) are shown in Table 4, when it rotates at RPM 37700.

<table>
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<tr>
<td>4</td>
<td>4683</td>
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</table>

For further investigating the dynamic characteristics of the centrifugal impeller system, nonrotating normal modes of the separate inducer blade fixed at its root as a cantilever beam are calculated in Table 5 (with the experiment data).

<table>
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<table>
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<th>Modal Shapes</th>
<th>Relative Errors</th>
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<td>B^+ — Bending mode</td>
<td>+3.8%</td>
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<tr>
<td>T^+ — Tension mode</td>
<td>+4.2%</td>
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DISCUSSION

Through the comparison of the analytical results of the methods presented in this paper, the analytical results of other methods and the experimental data, the reliability and practicability of the method presented in this paper are verified. Dynamic substructure method combined with wave propagation technique and Benfield-Hruda mode transformation method, which has high efficiency of modal synthesis among substructures, is performed in this paper. Consequently, all the interface degrees of freedom between full blade and short blade with impeller disc respectively are totally eliminated in the final coupled equations and boundaries of wave propaga-
tion can be chosen arbitrarily. Therefore, the higher analytical efficiency is obtained from the new method of this paper. In this aspect, it is superior to the method presented in Ref.[6].

It is obvious that under the precondition of ensuring sufficient analysing accuracy, the CPU time and storage capacity will be saved remarkably. The benefit of the method presented in this paper is shown in Table 6.

Table 6 The comparison of benefit with different analysing methods for calculating the rotating normal modes of a centrifugal impeller system

<table>
<thead>
<tr>
<th>Item</th>
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<th>Wave propagation</th>
<th>Ref.[6]</th>
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<td>8</td>
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<td>42</td>
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</tbody>
</table>

It must be pointed out from the calculated results of this paper that the coupled vibration characteristics for the centrifugal impeller system and the axially bladed disc system still have several differences:

1. From the calculating results of Table 1 and Table 2 it is seen that the blades (full blades or short blades) strengthen the stiffness of the impeller disc. So the normal modes of the impeller with full blades and short blades are higher than the normal modes of the non—bladed impeller disc. The full blades and the short blades just like the reinforce ribs distributed around the impeller disc. But for the axially bladed disc system it has the opposite results. The normal modes of the axially bladed disc are lower than the normal modes of the non—bladed disc.

2. The blade (full blade or short blade) vibration of the centrifugal impeller is mostly consisted of torsion modes and it is shown in Table 5. So, the normal modes of centrifugal impeller will raise smoothly while the rotating speed or nodal diameter is increased. But the blade vibration of axially bladed disc is mostly consisted of bending modes, so, the normal modes will raise abruptly while the rotating speed or nodal diameter is increased.

3. For the modal shapes of vibration, the centrifugal impeller system also has its own characteristics. Owing to the torsion modes of the full blade or short blade mostly appear and they form the couple vibration between blades and impeller disc, the nodal lines of the vibration at the back surface of the impeller disc are basically radiused, but they have different shapes with different orders.

The method presented in this paper can be further extended. It can be applied to general structures in which the basic repetitive sector region with one main component and multi—branch components are partitioned. Actually, it only needs to extend the generalized mass matrix and the corresponding stiffness matrix under the reduced coordinates.

REFERENCES