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## PROBABILISTIC ASSESSMENT OF COMBUSTOR LINER DESIGN

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### ABSTRACT

A typical hot structural component within an engine such as composite combustor liner is computationally simulated and probabilistically evaluated in view of the numerous uncertainties associated with the structural, material, and thermo-mechanical load variables (primitive variables) that describe the combustor. The combustor is evaluated for buckling (eigenvalue) loads, vibration frequencies, and local stresses. Results show that the scatter in the combined stress is not uniform along the length of the combustor. Furthermore, coefficient of thermal expansion, hoop modulus of the liner material, and the thermal load profile dominate stresses near the support and the intermediate location of the combustor liner. However, the liner thickness, the liner material hoop modulus, and pressure load profile have significant impact on stresses near the free end of combustor.

### INTRODUCTION

Aerospace propulsion systems are a complex assemblage of structural components that are subjected to a variety of cyclic and transient loading conditions. Inherent variability of material properties and fabrication processes, as well as geometrical tolerances introduce additional uncertainties. Deterministic structural analysis methods are not adequate to properly evaluate the design parameters variability. Furthermore, these methods lead to conservative designs due to safety factors.

As an alternative to the deterministic approach, Probabilistic Structural Analysis Methods (PSAM) are being developed at NASA Lewis Research Center (ref. 1) which enable a user to assess the effects fluctuating loads, variable material properties, and uncertain geometries have on the scatter of structural responses (eigenvalues, frequencies, effective stresses, etc.). PSAM provides

a more reliable and systematic way to quantify sensitivities associated with the corresponding uncertainties in the design variables, and the significance of the design variables to the system model. PSAM is embedded in a computer code NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) (refs. 2 and 3).

In the recent past, a methodology has been developed to perform probabilistic progressive buckling assessment of space type trusses using the NESSUS computer code (ref. 4). In addition, the computational simulation of probabilistic evaluation for adaptive/smart/intelligent behavior of space type truss structures was demonstrated and step-by-step procedures were outlined (ref. 5). The objective of this paper is to demonstrate the use of the NESSUS computer code to computationally simulate and probabilistically evaluate a hot structural component within an engine such as a ceramic composite combustor liner. The combustor is evaluated for frequencies, buckling, and local stresses. Furthermore, this paper determines the uncertainties associated with the dominant parameters for reliable/robust combustor liner design. Due to the fact that this is a finite element based simulation where the physics are properly represented, the authors expect that the experimental data for these responses will verify or nearly so to their respective predicted distributions.

### FUNDAMENTAL APPROACH AND CONSIDERATIONS

One of the major problems encountered in the analysis of the hot section components in an engine, such as a combustor liner is that it has to be economically viable inclusive of life-cycle costs and reliability as well as environmentally acceptable. Furthermore, high temperature ceramic matrix composites that might be used should possess material characteristics to provide long term durability as well as resistance to possible through thickness high

thermal gradients. In addition, the recommended ceramic matrix composite materials should have properly connected economical fabrication processes to minimize overall manufacturing costs. The presently available methods/programs do not allow us easily to statistically quantify the uncertainties in the above described design parameters. Therefore, using the NESSUS code, dominant parameters can be identified to achieve acceptable vibration frequency values, buckling loads, and combined stress values for an effective configuration of combustor liner with appropriate support boundary conditions.

### FINITE ELEMENT MODEL

A three dimensional preliminary configuration of a typical hot section component such as combustor liner is computationally simulated by using bilinear isoparametric variable-thickness shell element based on Reissner-Mindlin plate and shell theories (see Fig. 1). The element is a four-noded quadrilateral in three-dimensional space. The liner is composed of a typical ceramic matrix composite material (SiC/SiC). As shown in Fig. 1, the combustor outer liner is exposed to uniform pressure loading and variable thermal loading. In the initial finite element analysis it was assumed that the all the nodes at the support locations were constrained in all six directions. However, all other nodes along its length were assumed to rotate freely and displace in all directions.

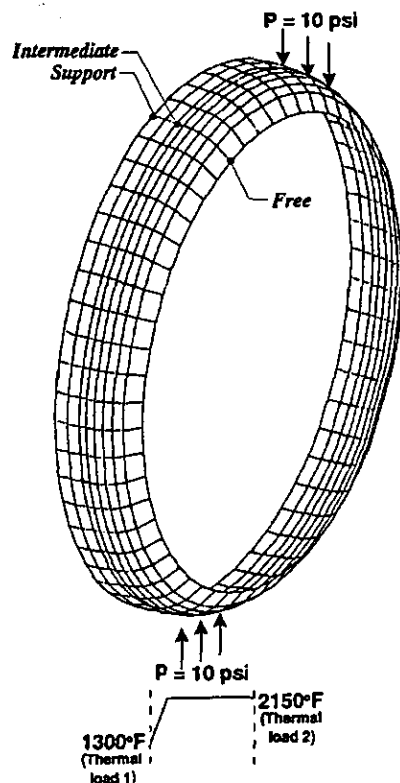


Figure 1.—Typical combustor liners for wide body commercial jet engines.

### PROBABILISTIC MODEL

The following primitive variables were considered in the probabilistic analysis:

- (1) Density of the liner material ( $\rho$ )
- (2) Coefficient of thermal expansion ( $\alpha$ )
- (3) Thickness of the liner ( $t$ )
- (4) Pressure load ( $P$ )
- (5) Thermal load at the support ( $T_1$ )
- (6) Thermal load along the length of the combustor ( $T_2$ )
- (7) Flexible support conditions
- (8) Material properties

In the case of the material properties, the axial modulus, the hoop modulus, three directional shear moduli, and Poisson's contribution were considered for the analyses. In the present probabilistic analysis these material property variables are assumed to be independent of each other. The normal distribution is assumed to represent the uncertainties in the above discussed primitive variables. Normal distribution assumption is justified because all those properties vary equally on either side about the assumed mean values. Nessus, however, has 10 (ten) different types of distributions. When the influence of distribution-type is considered important, sensitivity studies can be performed readily to assess their respective effects. Initially, the NESSUS/FEM (Finite Element Methods) module was used to analyze deterministically the combustor liner for mean values of each of these primitive variables. In the subsequent probabilistic analyses, each primitive variable is perturbed independently and by a different amount. Usually, the perturbed value of the primitive variable is obtained by a certain factor of the standard deviation on either side of the mean value.

In general, the finite element equation for motion is written as:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = F(t) \quad (1)$$

where  $[M]$ ,  $[C]$ , and  $[K]$  denote the mass, damping, and stiffness matrices respectively. These matrices are calculated probabilistically in the NESSUS code. Furthermore,  $\{\ddot{u}\}$ ,  $\{\dot{u}\}$ , and  $\{u\}$  are the acceleration, velocity and displacement vectors at each node, respectively. The forcing function vector,  $F(t)$ , is time independent at each node.

In this paper, the static case is considered by setting the mass and damping matrices to zero and considering the forcing function being independent of time in equation (1) such that

$$[K]\{u\} = F \quad (2)$$

Using the NESSUS code, a linear buckling analysis is carried by making use of the subspace iteration technique to evaluate the probabilistic buckling load. The matrix equation for the buckling

(eigenvalue) analysis for a linear elastic structure is as follows:

$$([K] - \lambda[K_g]) \{\phi\} = \lambda\{\phi\} \quad (3)$$

In the above equation,  $[K]$  is the standard stiffness matrix,  $[K_g]$  is the geometric stiffness matrix,  $\lambda$  is the eigenvalue, and  $\phi$  are the eigenvectors.

Furthermore, the vibration frequency analysis is also carried by setting only the matrix to zero and using the following equation:

$$([K] - \omega^2[M]) \{\phi\} = \omega^2\{\phi\} \quad (4)$$

Finally, the NESSUS/FPI (Fast Probability Integration) module extracts the response variables (buckling loads, vibration frequencies and combined stresses) to calculate respective probabilistic distributions and respective sensitivities associated with the corresponding uncertainties in the primitive variables. The mean, distribution type and percentage variation for each of the primitive variables are given in Table I.

TABLE I.—PRIMITIVE VARIABLES AND UNCERTAINTIES FOR PROBABILISTIC STRUCTURAL ANALYSIS OF COMBUSTOR LINER (RANDOM INPUT DATA)

Primitive variables	Distribution type	Mean value	Scatter, $\pm$ percent
Density	↓	Normal	0.0003 lbs/in.-sec <sup>2</sup>
Coefficient of thermal expansion		2.3 × 10 in./in./°F	
Thickness		0.189 in.	
Pressure load		10 psi	
Thermal load (T <sub>1</sub> )		1300 °F	
Thermal load (T <sub>2</sub> )		2150 °F	
Axial modulus		20.44 ksi	
Hoop modulus		15.18 ksi	
Poisson's contribution		0.87 ksi	
Shear modulus		2.97 ksi	
Shear modulus	2.31 ksi		
Shear modulus	2.53 ksi	↓	

## DISCUSSION OF RESULTS

The combustor material has to satisfy all the structural reliability and safety requirements against the thermo-mechanical loadings. Therefore initially, the combustor was analyzed to obtain the cumulative distribution functions (CDF) of the probabilistic buckling loads (see Fig. 2). The sensitivity factors from Fig. 3 show that the uncertainties in the combustor thickness had the highest impact on the probabilistic distribution of the buckling load followed by the coefficient of thermal expansion and the thermal load (T<sub>2</sub>). Furthermore, the hoop modulus and through thickness shear modulus had reasonable impact on the probabilistic buckling loads. The cumulative distribution functions (CDF) for the probabilistic vibration frequencies (see Fig. 4) showed a wide scatter between the lowest and the highest vibration frequencies. According to Fig. 5, the probabilistic vibration frequencies were very sensitive to the changes in the density of the material, the thickness of the material, and the shear/hoop/axial moduli of the material. However at higher probability level, the uncertainties in the density of the material showed the highest sensitivity on the probabilistic vibration frequencies.

In addition to satisfying the design requirements with allowable vibration frequencies as well as buckling loads, the stress level in the combustor liner should be below some critical stress level. Therefore, the probabilistic combined stress level was determined for thermo-mechanical loads, initially with support nodes completely fixed. According to the Fig. 6, the cumulative distribution functions (CDF'S) for the combined stress at the supported end, the intermediate location and free end clearly exhibited a non-uniform stress distribution along the length of the combustor. Figures 7, 8, and 9 respectively show sensitivities associated with primitive variables at the above mentioned three locations. At the supported end, the coefficient of thermal expansion, the thickness, the thermal loads, and axial and hoop moduli have considerable impact on combined stress with the coefficient of thermal expansion having most significant impact (see Fig. 7). In the case of the intermediate location, the coefficient of thermal expansion, the hoop modulus, and the thermal load (T<sub>2</sub>) had equally significant impact on the combined stress distribution. Finally at the free end, the changes in the thickness, the pressure load, and the hoop modulus were very sensitive to the combined stress distribution.

According to the Fig. 6 (fixed supports), the combined stress magnitude at the 50% probability level near the support and intermediate location were considerably high. Therefore, the support conditions were altered to allow the unrestrained radial growth of the combustor (see Fig. 10). Succeeding probabilistic analyses indicated that the combined stress magnitude at 50% probability level reduced the stress magnitude by more than 50% to that of fixed supports. According to Fig. 11, the thermal loads were the most significant contributors to the combined stress magnitude. Next were the coefficient of thermal expansion, the axial and hoop moduli. Figure 12 displays a summary of the dominant parameters for reliable/robust combustor liner designs. The summary in Fig. 12 is a ranking of the sensitivities associated with the primitive variables with respect to probabilistic buckling (eigenvalue), vibration frequency, and the combined stress magnitudes.

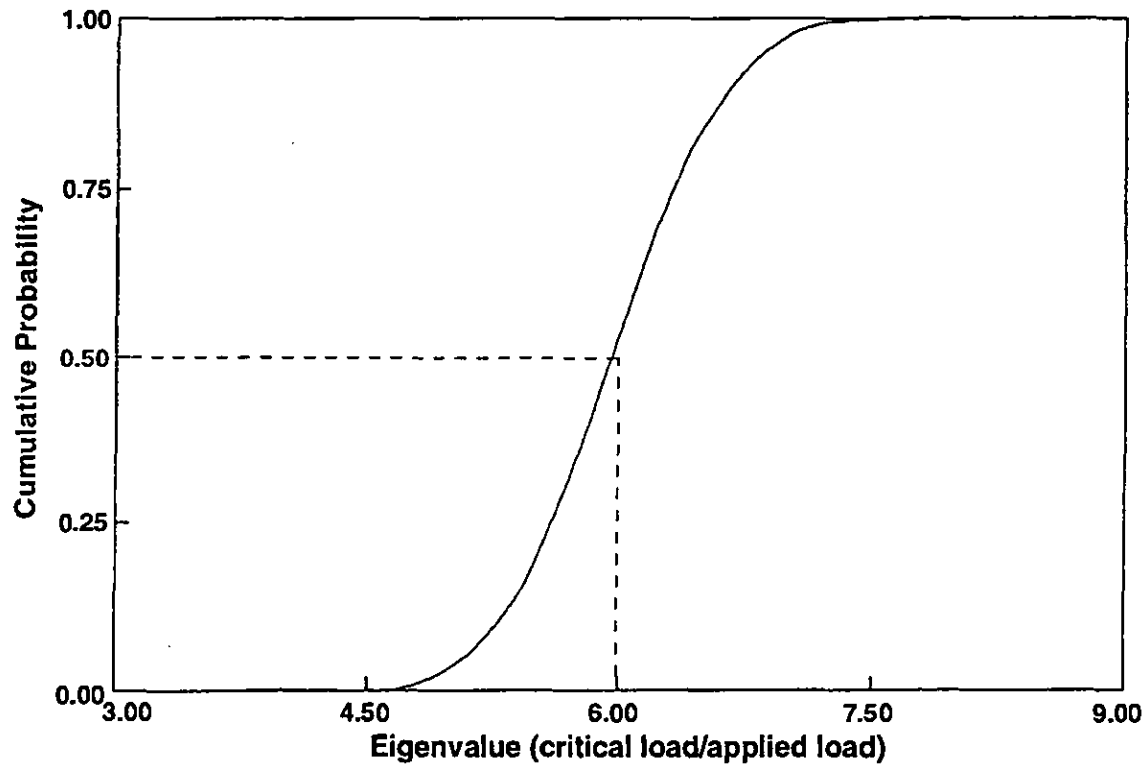


Figure 2.—Probabilistic buckling load of combustor liners.

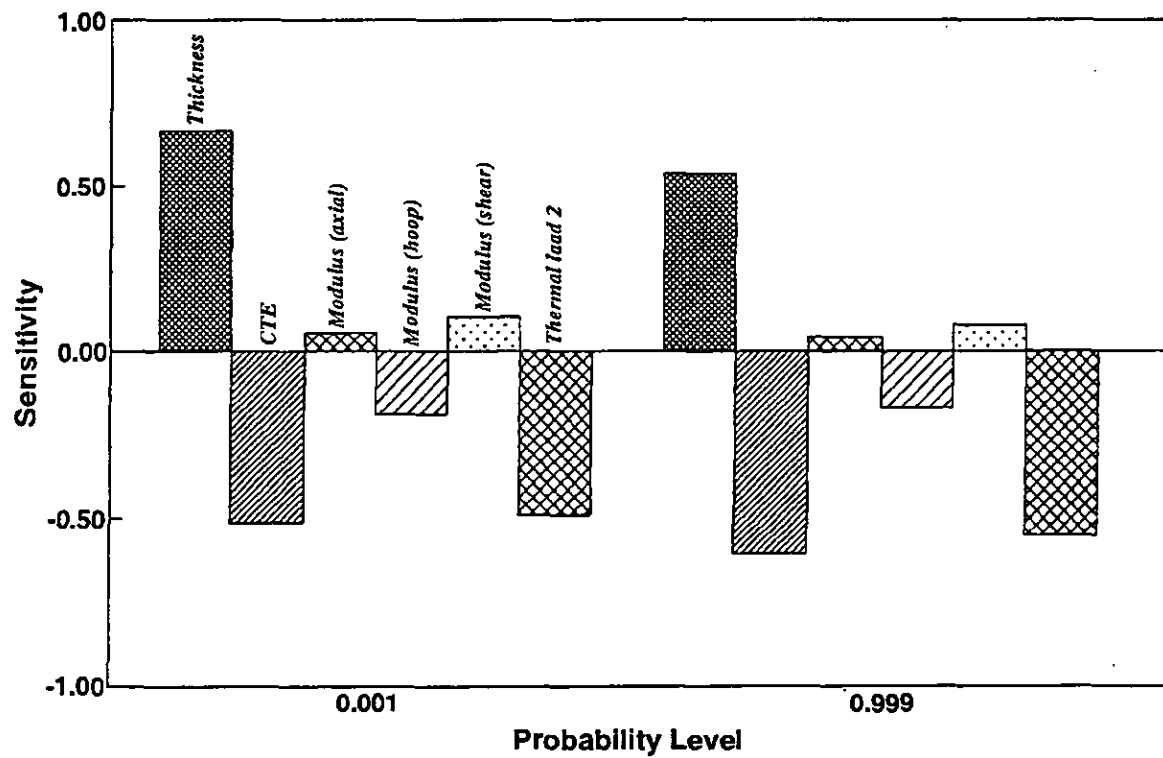


Figure 3.—Sensitivities for probabilistic buckling load of combustor liners.

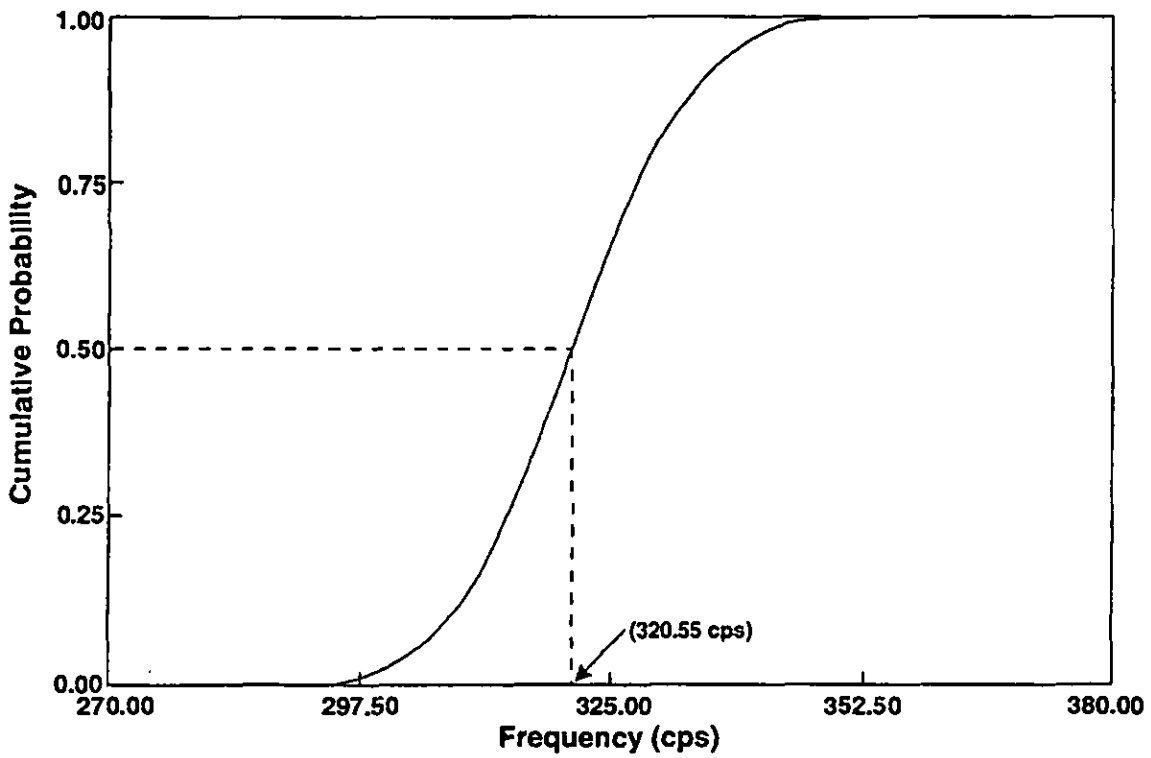


Figure 4.—Probabilistic vibration frequency of combustor liners.

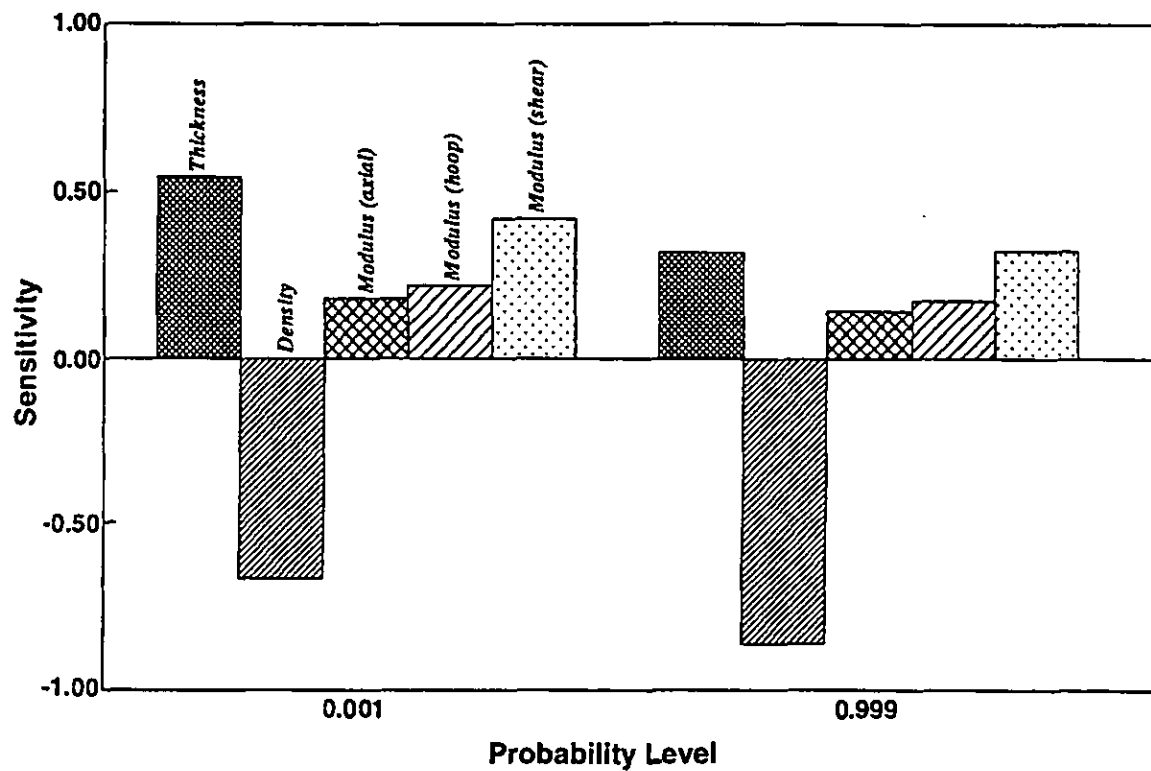


Figure 5.—Sensitivities for probabilistic vibration frequency of combustor liners.

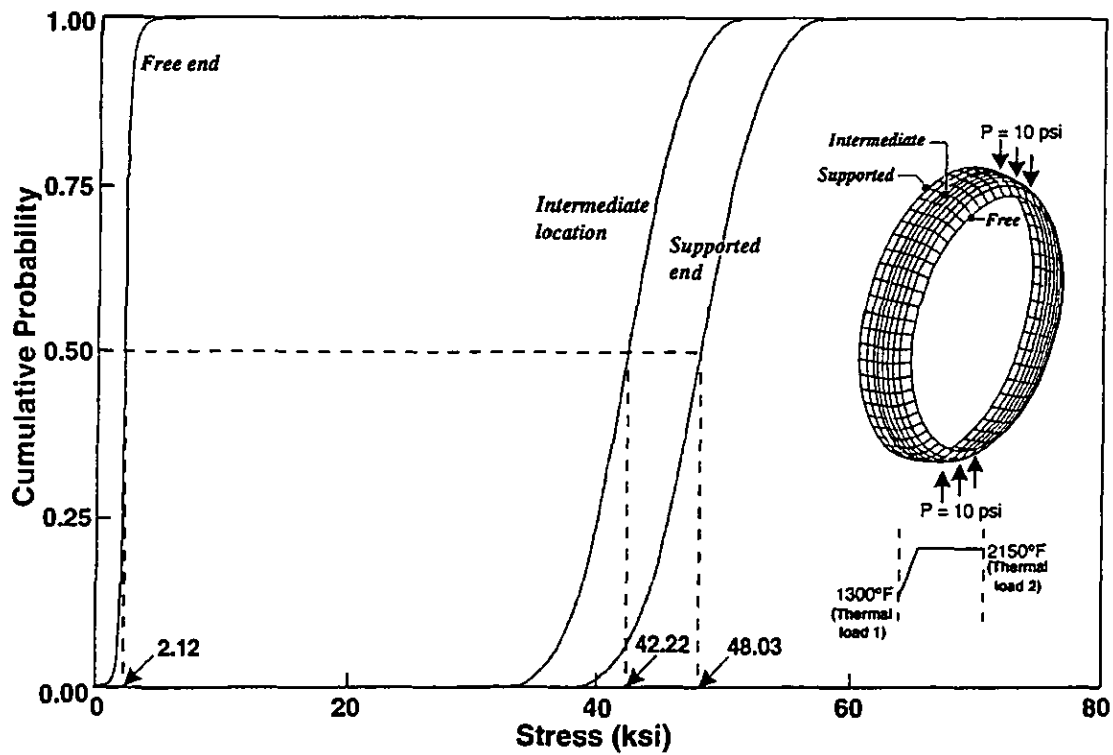


Figure 6.—The scatter range in combined stress of combustor liners.

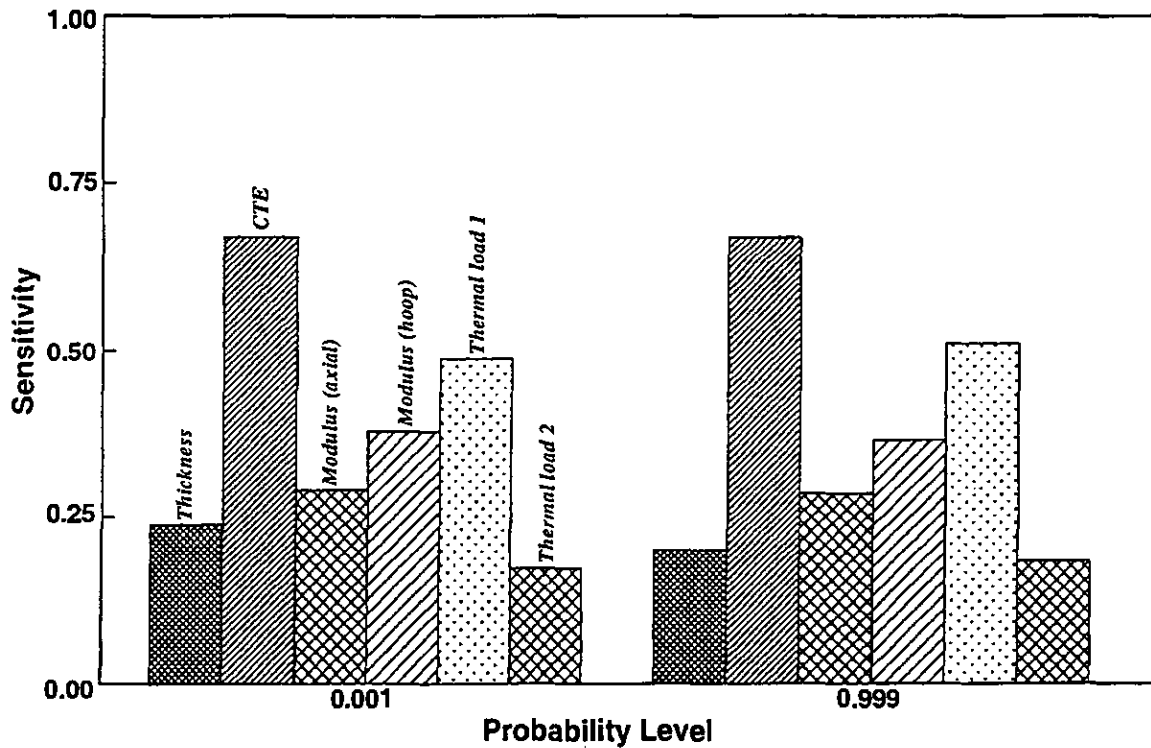


Figure 7.—Thermal expansion coefficient and temperature profile dominate stresses near the support of combustor liners.

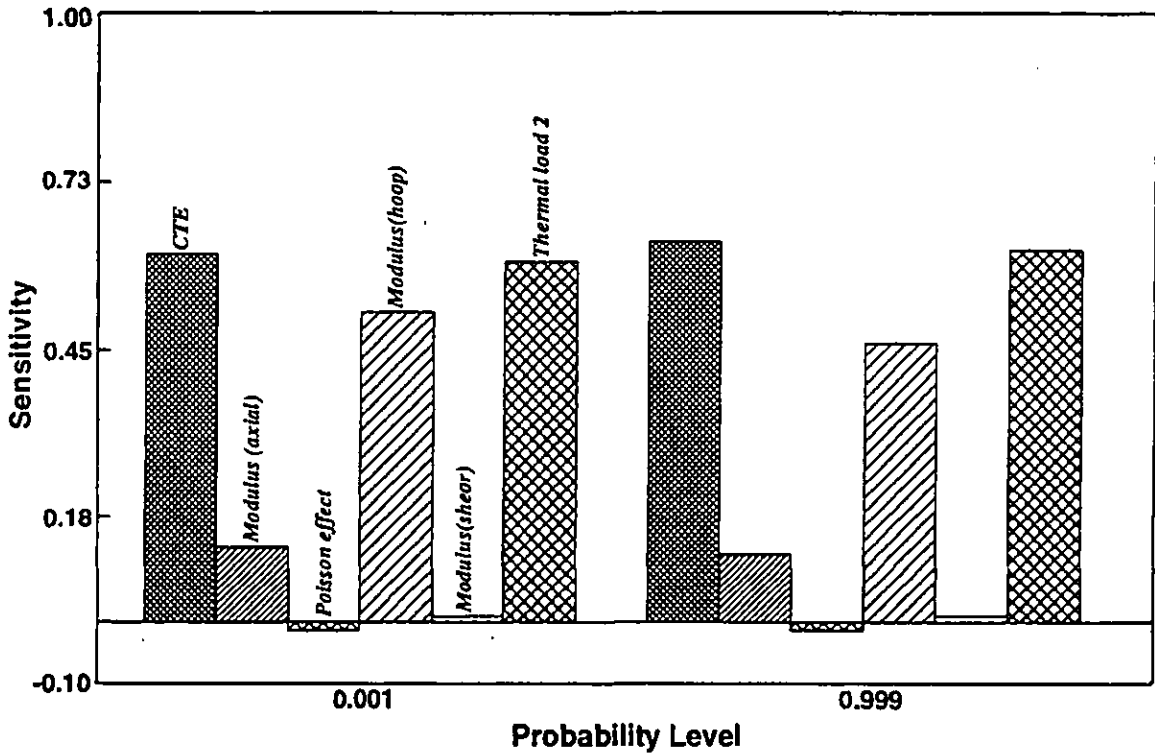


Figure 8.—Thermal expansion coefficient, hoop modulus and temperature profile dominate stresses near intermediate location of combustor liners.

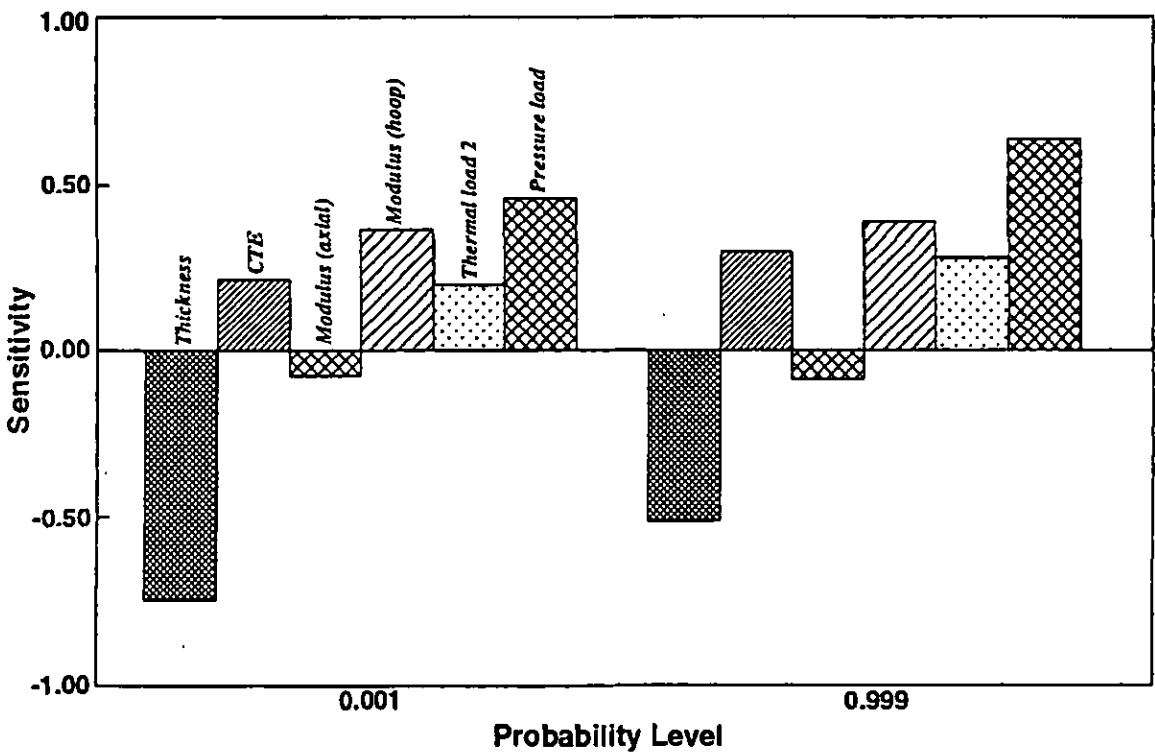


Figure 9.—Thickness, hoop modulus and pressure load dominate stresses near free end of combustor liners.

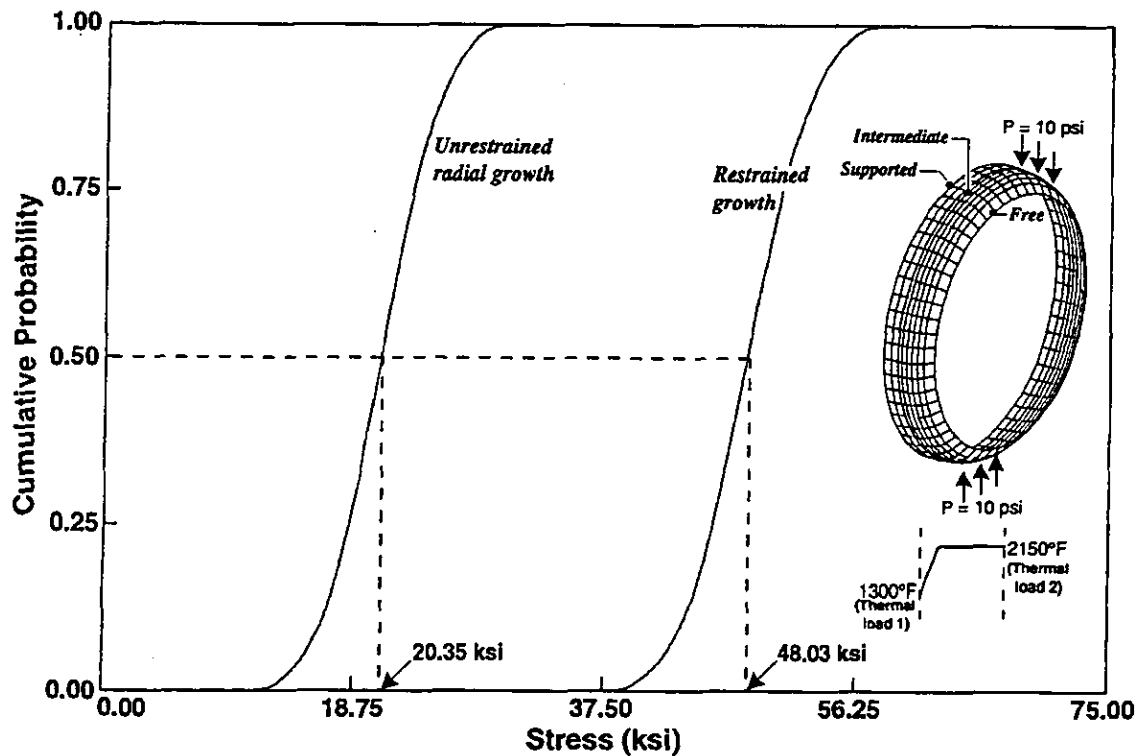


Figure 10.—Support compliances substantially influence stresses near the support of combustor liners.

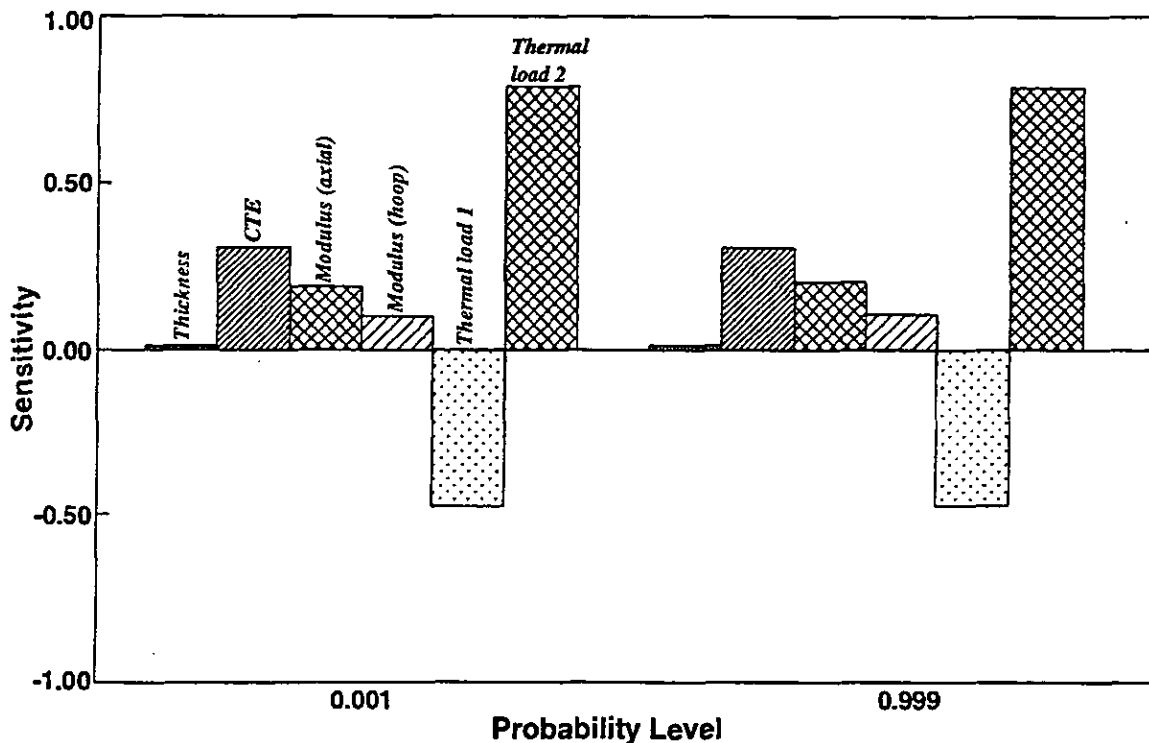


Figure 11.—Support compliances influence the ranking of stress sensitivity factors for combustor liners (unrestrained radial growth).



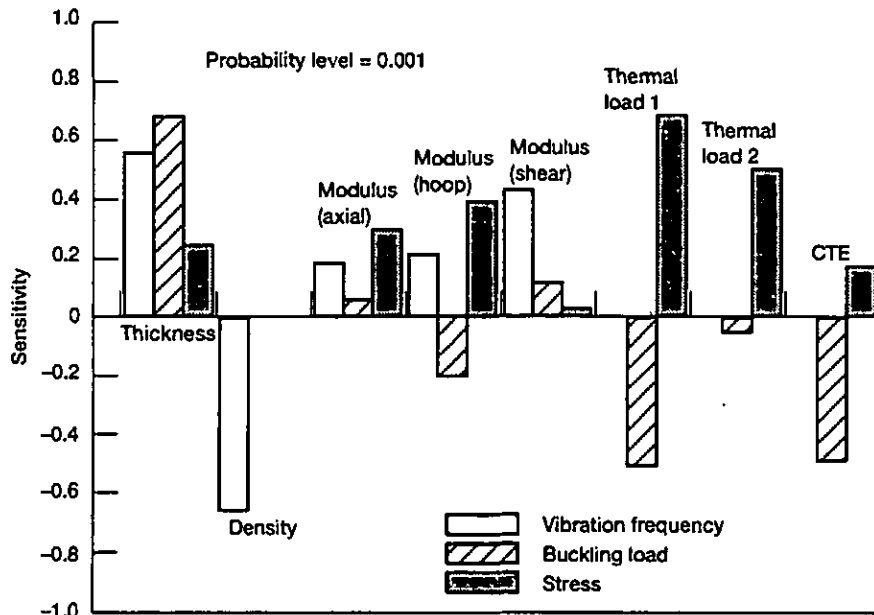


Figure 12.—Dominant parameters for reliable/robust combustor liner designs.

## CONCLUSIONS

The computational simulation of probabilistic evaluation of a typical hot structural component within an engine such as a ceramic composite combustor liner is demonstrated using the NESSUS computer code. The combustor is analyzed for compressive pressure loading and nonuniform thermal loading along its length. The cumulative distribution functions (CDF's) for buckling (eigenvalues), vibration frequencies, and combined stresses are evaluated. The results indicate that: (1) the variations in the thickness, the coefficient of thermal expansion and thermal loads have significant impact on probabilistic buckling loads; (2) the scatter in the thickness, density, and shear/hoop/axial moduli of the liner material have significant impact on probabilistic vibration frequencies; (3) the uncertainties in the combined stress magnitudes is not uniform along the length of the combustor; (4) by altering the support conditions to allow unrestrained radial growth, the stress level near the support location reduced by more than 50% to that of fixed support conditions at 50% probability level. Collectively, the results provide quantifiable guidance on dominant parameters for reliable/robust combustor liner design and optimum support condition.

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