VIBRATION CONTROL OF A ROTOR SYSTEM UTILIZING A BEARING HOUSING WITH CONTROLLABLE SPRING NONLINEARITY

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ABSTRACT

On the basis of characteristics of vibration in the rotor system with spring nonlinearity, a new method for vibration control has been developed. In the method, the spring characteristics of a bearing housing are controlled to be of softening nonlinearity when the rotor supported on it is accelerated and to be of hardening one when it is decelerated. So vibratory amplitudes of the rotor system always vary along the smallest solution curve in the whole operating process. A model of vibration of the rotor system supported on the controllable bearing housing is derived. Its dynamic behaviour is predicted and verified by experiments. Both theoretical and experimental results show that not only vibratory amplitudes and transmitted forces are suppressed significantly but also nonlinear vibration performance of the rotor supported on squeeze film dampers, such as 'lock up' at rotor pin-pin critical speeds and asynchronous vibration, can be avoided.

NOMENCLATURE

\[\begin{align*}
C & \text{ damping coefficient} \\
C & \text{ clearance} \\
e & \text{ unbalance mass radius} \\
F & \text{ force} \\
K & \text{ dimensionless stiffness of squeeze oil film} \\
K_T & \text{ stiffness of SMA wires} \\
L & \text{ axial width of the bearing housing} \\
M & \text{ mass} \\
R & \text{ radius of the inner race} \\
x & \text{displacement in } x \text{ direction} \\
y & \text{displacement in } y \text{ direction} \\
\alpha & \text{ acceleration} \\
\varepsilon & \text{ strain} \\
\theta & \text{ angle} \\
\mu & \text{ viscosity of lubricant} \\
\omega & \text{ rotating frequency} \\
\omega_0 & \text{ critical speed} \\
\varepsilon & \text{ eccentricity ratio}
\end{align*}\]

INTRODUCTION

Modern gas turbine engines typically utilize squeeze film dampers (SFDs) as a flexible damped support to attenuate vibration amplitudes and to reduce transmitted forces. Despite the successful applications of SFDs, investigators have widely recognized that because of hardening nonlinear characteristics of squeeze film forces, some remarkable nonlinear performance of a rotor system on SFDs, such as bistable operation and 'lock up' at pin-pin critical speeds, will appear when the system is accelerated through critical speeds to its operating speed. On the other hand, the unbalance response of a rotor system using SFDs can be markedly dependent on rotor unbalance and the SFD design. SFDs, which can attenuate vibratory amplitudes and reduce transmitted forces significantly under normal unbalance condition, will amplify vibration with large increase of unbalance in gas turbine operation.

Over the last two decades, many efforts have been made to develop more efficient technique for controlling vibration of a rotor system, which includes active and passive control. In the field of passive vibration control, some investigations improving stiffness and damping characteristics of SFDs and their effect on rotor dynamics have been made. San Andres (1988) investigations show that for unpressurized dampers operating at moderately large squeeze film Reynolds numbers, the possibilities of bistable operation and jump phenomena are reduced and virtually disappear. Although the effective direct damping coefficient increases in a wide eccentricity range, the effective cross-coupled damping coefficient (stiffness coefficient) remains slightly influenced when eccentricity \(\varepsilon > 0.75\). That means the high nonlinear

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THEORY

As shown in figure 1, hardening spring nonlinearity of supports bends the frequency response curves of the rotor system to the right and softening one bends the curves to the left. The bending of the frequency response curve leads to multivalued amplitudes and hence to a jump phenomenon. To see the jump phenomenon, let us suppose that an experiment is conducted for a rotor system with hardening spring nonlinearity in which the amplitude of the excitation is held constant while the frequency is varied very slowly. The excitation is referred as stationary. During the experiment, \( \omega \) starts far below \( \omega_p \) then \( \omega \) monotonically increases, the amplitude of the response increases slowly along the curve DCE and the amplitude varies smoothly through C. There is no downward jump to B. The amplitude of the response continues to increase smoothly until E is reached. At that point, any further increase in \( \omega \) precipitates a spontaneous downward jump from E to F. For further increases in \( \omega \), the amplitude continues to decrease along the curve from F toward A. If the experiment is started at \( \omega \) far above \( \omega_p \) and \( \omega \) is monotonically decreased, the amplitude of response increases slowly along the curve AFB (in Fig. 1) until B is reached. At that point, any slight decrease in \( \omega \) precipitates a spontaneous jump from B up to C. For further decreases in \( \omega \) the amplitude decrease slowly along the curve DCE. For a rotor system really rotating in a gas turbine, because of acceleration, amplitude jump phenomenon does not occur and amplitudes vary fast when decreases in vicinity of point E.

For a rotor system with softening spring nonlinearity, the jumps take place in the opposite directions as shown in figure 1. For frequency of the excitation in the interval between BF and CE in figure 1, there are three steady state solutions for each value of \( \omega \). The middle one is a saddle point, hence the response corresponding to it is unstable and unrealizable in any experiment. The other two are stable foci: hence both are realizable. The response amplitude and the transmitted force corresponding to the largest solution are high and those corresponding the smallest one are low. As described above, the initial conditions determine which of the possible responses actually develops.

Based on the characteristics of vibratory response in a rotor system with spring nonlinearity, a new method for vibration control has been developed. If support stiffness is adjusted to be of softening nonlinearity when a rotor system on the supports is accelerated through critical speeds to the operating speed or be of hardening one when it is decelerated through critical speeds to stop, the vibratory amplitudes will vary along the smallest solution curve. The vibratory amplitudes and transmitted forces can be significantly suppressed by controlling nonlinear characteristics of support stiffness, especially when the rotor system passes through critical speeds. In this paper, a bearing housing is designed for adjusting nonlinear characteristics of support stiffness, whose configuration is same as that of a SFD, but some shape memory alloy (SMA) wires are mounted between the inner race and a machinery casing. The adjustment of nonlinear characteristics is realized by controlling oil supplement into the bearing housing, which will be illustrated in detail in the following section.

DESIGN OF A BEARING HOUSING WITH CONTROLLABLE SPRING NONLINEARITY

Figure 2 shows a geometric configuration of a bearing housing with controllable spring nonlinearity. The outer race is inlaid in a machinery casing and the inner race is connected to the machinery casing with a squirrel cage spring. A journal of a rotor system is mounted in the rolling element bearing in the inner race. There is a clearance between the inner and outer...
machinery casing
squeeZe oil film
outer race
SMA wire
inner race
rolling element

Figure 2. Configuration of the bearing housing

Figure 3. Dimensionless stiffness of squeeze oil film

Figure 4. Constitutive relation of SMA

race, which is full of oil. Squeeze oil film exists in the clearance as the inner race and journal moves radially. Additionally, the machinery casing and the inner race are connected with some SMA wires in two directions which are mutually perpendicular. The wires are deformed while the journal moves radially. The whole support stiffness of the bearing housing consists of three parts: the squirrel cage spring stiffness, the squeeze oil film stiffness and the stiffness of SMA wires. They are analysed respectively as follows.

Squeeze oil film stiffness

It is assumed that (a) the inner race surrounding a rolling element bearing is centrally preloaded with constant symmetric radial support stiffness and prevented from rotating; (b) the short bearing approximation is valid; (c) the outer and the inner race are rigid; (d) unpressurized oil is supplied. The dimensionless radial stiffness of the squeeze oil film for circular centred orbits can be expressed as (Zhang, 1991)

\[ K_0 = \frac{r^2 \sin \theta \cos \theta}{(1 + \epsilon \cos \theta)^3} \, d\theta \]

Figure 3 shows the dimensionless stiffness for increasing orbit ratio \( e \). When \( e > 0.4 \) the dimensionless stiffness increases nonlinearly with increase of eccentricity ratio \( e \). Because \( e \) is the ratio of journal eccentricity to radial clearance, the oil film stiffness with small clearance is of high hardening nonlinearity. This means that the SFD with small clearance can be used in suppressing vibratory responses as a rotor system is decelerated through critical speeds.

Stiffness of shape memory alloy wires

Figure 4 illustrates a constitutive relation of SMA with deformation pseudoelasticity measured in our experiment (Liu, 1993). It is found that stress–strain curves formed a closed loop, which means that the all deformation can be recovered. When \( e < e_t \), stress increases linearly with increase of strain and stiffness is held constant \( K_r \). When \( e > e_t \), the stress increases very slowly along the curve AEB and its stiffness is much less than \( K_r \). If the load acting on SMA wires is reduced from point E, the stress will decrease along the curve EFCO. So, stiffness of shape memory alloy wires is of high softening nonlinearity. This means that the shape memory alloy wires can be used in suppressing vibratory responses of a rotor system supported on SMA wires as it is accelerated through critical speeds.

Squirrel cage stiffness

Squirrel cage stiffness is held constant in the whole process increasing or decreasing rotor speed. The control of characteristics of spring nonlinearity is realized by oil control. When the rotor system supported on the bearing housing with controllable spring nonlinearity is started to be driven to the operating speed, in other word, when it is accelerated through critical speeds, the oil supplied into the clearance between the outer and inner race is cut off. Because this clearance and an oil tank connect with each other, the surplus oil in the clearance will be squeezed into the oil tank quickly and oil film will not exist after oil supplement is cut off. So, the rotor system is supported only on the SMA wires and the squirrel cage spring. The characteristics of support stiffness are of softening nonlinearity and the vibratory response varies along the smallest solution curves. When the rotor speed is
The vibratory response is suppressed in whole operating.

**THEORETICAL MODEL AND COMPUTER SIMULATION**

A rigid rotor system mounted on the bearing housing with controllable spring nonlinearity is modeled as a single mass, two degree of freedom system (refer to figure 2). When the rotor system is accelerated or decelerated, the tangential force of unbalance mass should be added to equilibrium equations. The equations can be seen as

\[
\begin{align*}
\frac{d^2 x}{dt^2} + C\frac{dx}{dt} + G(x) &= M\omega^2 \cos \theta + a \sin \theta \\
\frac{d^2 y}{dt^2} + C\frac{dy}{dt} + G(y) &= M\omega^2 \sin \theta - a \cos \theta
\end{align*}
\]

where \( G(x), G(y) \) are resistant forces of the bearing housing, which are composed of the squirrel cage spring force, the squeeze oil film force and the deformed force of SMA wires. They may be written as

\[
G(x) = \begin{cases} 
\delta(x)F_x + K_x x + F_{T_x} & \text{for } \alpha \leq 0 \\
0 & \text{for } \alpha > 0 
\end{cases}
\]

\[
G(y) = \begin{cases} 
\delta(y)F_y + K_y y + F_{T_y} & \text{for } \alpha \leq 0 \\
0 & \text{for } \alpha > 0 
\end{cases}
\]

\[F_x \text{ and } F_y \text{ are resultants of the squeeze oil film force in X and Y directions respectively. They may be expressed as (Yan, 1992)}
\]

\[
F_x = - \mu RL \frac{\pi^2}{2} \left( \frac{x}{C^2 - x^2 - y^2} \right)^2 + \frac{3\pi x (2x + y)}{2(C^2 - x^2 - y^2)} + \frac{2\pi (x^2 + y^2)}{2(C^2 - x^2 - y^2)}
\]

\[
F_y = - \mu RL \frac{\pi^2}{2} \left( \frac{y}{C^2 - x^2 - y^2} \right)^2 + \frac{3\pi y (2x + y)}{2(C^2 - x^2 - y^2)} + \frac{2\pi (x^2 + y^2)}{2(C^2 - x^2 - y^2)} - \frac{4\pi (2x^2 + y^2)}{2(C^2 - x^2 - y^2)}
\]

\[F_{T_x} \text{ and } F_{T_y} \text{ represent deformed forces of SMA wires in X \ and Y \ directions respectively. They are complex to calculate and dependent on not only the strain itself but also the deformation history. As shown in figure 4, as the load acting on the SMA wires is decreased, the stress will vary along the curve EF. It is assumed that stiffness of the SMA wires corresponding to the curve EF is equal to that corresponding to the curve CO. For example, considering the resistant force in X direction, we define that the } F_{T_x} \text{ denotes deformed force corresponding to deformation value } x_1 \text{ and } F_0 \text{ denotes that corresponding to } x_0. \text{ When deformation value is increased from } x_0 \text{ to } x_1, \text{ the deformed force } F_{T} \text{ can be determined from } F_0 \text{ according to four following cases}
\]

**case I**: \( x_1 \geq x_0 \text{ and } x_0 \geq x_c \)

\[F_{T_x} = K_x (x - x_0) + F_0\]

\[F_{T_x} = \begin{cases} 
K_x (x_1 - x_0) + F_0 & \text{for } x_1 \geq x_0 \text{ and } x_0 \geq x_c \\
K_x (x_1 - x_0) + F_0 & \text{for } x_1 \geq x_0 \text{ and } x_0 \geq x_c
\end{cases}
\]

**case II**: \( x_1 \leq x_0 \text{ and } x_1 \geq x_c \)

\[F_{T_x} = K_x (x - x_0) + F_0\]

\[F_{T_x} = \begin{cases} 
K_x (x_1 - x_0) + F_0 & \text{for } x_1 \leq x_0 \text{ and } x_1 \geq x_c
\end{cases}
\]

**case III**: \( x_1 \leq x_0 \text{ and } x_0 \geq x_c \)

\[F_{T_x} = K_x (x - x_0) + F_0\]

\[F_{T_x} = \begin{cases} 
K_x (x_1 - x_0) + F_0 & \text{for } x_1 \leq x_0 \text{ and } x_0 \geq x_c
\end{cases}
\]

**case IV**: \( x_1 \leq x_0 \text{ and } x_1 \geq x_c \)

\[F_{T_x} = K_x (x - x_0) + F_0\]

\[F_{T_x} = \begin{cases} 
K_x (x_1 - x_0) + F_0 & \text{for } x_1 \leq x_0 \text{ and } x_1 \geq x_c
\end{cases}
\]

Considering a constant acceleration process, we may obtain

\[\begin{cases} 
\theta = \frac{1}{2} \omega^2 + \omega t + \theta_0 \\
\omega = \omega + \theta_0
\end{cases}
\]

Equation (1) through (6) were solved numerically to obtain vibratory response of the rotor system. The vibratory response is dependent on initial conditions which include starting speed \( \omega_0 \), acceleration \( a \) and phase angle \( \theta_0 \). The acceleration value was selected small so that it had no effect on vibratory response amplitude. During acceleration, the rotor was assumed to start to be driven from zero. So, all initial values were...
Without control 

with control using a SFD

with control utilizing the bearing housing

Figure 6. Vibration response for \( \epsilon = 0.1 \text{mm} \) and \( \epsilon = 0.35 \text{mm} \)

Figure 7. Vibration response for \( \epsilon = 0.2 \text{mm} \) and \( \epsilon = 0.8 \text{mm} \)

equal to zero. During deceleration, the computation began at a very large rotating speed and carried out for sufficient number of "cycles" so that initial transients were damped out.

**NUMERICAL RESULTS**

Engineers recognize the efficient effect of SFDs on vibration suppression in gas turbines and aeroengines. But the greatest disadvantage of SFDs is their undesirable nonlinear performance such as bistable operation and "lock up" phenomena, which not only limits their effectiveness in controlling vibration but also amplifies vibration under large unbalance conditions. If the ability of the bearing housing with controllable spring nonlinearity in suppressing vibratory response is same as or better than that of SFDs and the bearing housing is able to avoid occurrence of the nonlinear vibration phenomena existing in the rotor system on SFDs, the validity of the developing method utilizing the bearing housing on suppressing vibration of a rotor system will be verified very well. The validity will be illustrated in following comparisons of the numerical results.

Figure 5 through figure 7 show the predicted vibration responses of a rotor system in three cases for different unbalance conditions and clearances. The three cases are that vibration of the rotor system is controlled using a SFD and the bearing housing with controllable spring nonlinearity and is not controlled. The system without control is linear and its all dynamic parameters, such as stiffness and damping etc., are equal to those of the nonlinear system on the bearing housing in static equilibrium position. Vertical axis represents amplitude ratio which is equal to the ratio of vibratory amplitude to unbalance mass radius. Horizontal axis represents the rotating speed. The vibratory responses of the system with control using a SFD and the bearing housing and without control are presented by the lines marked by symbols ○, □ and * respectively.

For small unbalance \( \epsilon = 0.05 \text{mm} \) (as shown in figure 5), the response amplitudes of the system with control using both a SFD and the bearing housing are effectively suppressed compared with those without control. The maximum amplitude with control utilizing the bearing housing are 4.4, which appears while the system is accelerated and its rotating speed reached 182rad/s. The amplitude peak value with control utilizing the bearing housing is near to that using a SFD in acceleration but 25% less than that using a SFD in deceleration.

In figure 6, as unbalance radius is increased from 0.05mm to 0.1mm, bistable operation exists in the rotor on a SFD in a rotating frequency range from 350rad/s to 430rad/s. In practice, while the system supported on a SFD is accelerated through critical speeds, the vibratory response amplitude will be held in the largest solution curve for a wide rotating frequency range from 350rad/s to 430rad/s and its force transmissibility should be much larger than that of linear system according to previous investigation (figure 5 in Zhang 1991). Under this circumstance, SFDs can not be used in vibration suppression and there is a probability to amplify vibration. However the system with control utilizing the bearing housing will pass the critical speed easily with small amplitudes and transmissibility no matter when it is accelerated or decelerated. In addition, the vibration is suppressed significantly compared with that without control.

Moreover, figure 7 shows that permanent bistable solution exists in the system using a SFD for \( \omega < 375 \text{rad/s} \) under increased unbalance condition \( \epsilon = 0.2 \text{mm} \). In practice, under such unbalance condition, "lockup" phenomenon may occur and high amplitudes and high force transmissibility may remain to the operating speed. A SFD really amplifies vibration. But, the system with control utilizing the bearing housing can be accelerated or decelerated through critical speeds with very
small vibratory amplitudes, which are much smaller than those without control.

The other benefits of the vibration control utilizing the bearing housing with controllable spring nonlinearity can be avoiding subharmonic vibration appearance, which frequently occurs in the system on a SFD rotating at a high speed. Figure 8(a) shows the trajectory of the rotor system on a SFD for e = 0.2mm and C = 0.8mm rotating at 800rad/s. The vibration is composed of harmonic vibration at rotating frequency and 1/3 subharmonic vibration at 1/3 rotating frequency. Figure 8(b) shows the trajectory of the system on the bearing housing with same dynamic parameters. The trajectory is a circle, which indicates that the vibration is only harmonic at rotating frequency.

EXPERIMENTS

In order to examine the benefits utilizing the bearing housing with controllable spring nonlinearity, experiments have been conducted. The fundamental test rig was well documented by Liu(1993) and is described here briefly. Figure 9 is a schematic drawing of an experimental rotor system supported on the bearing housing with controllable spring nonlinearity. Figure 2 is its side view from A—A direction. The experiment rotor is rigid, mounted on the bearing housing at the left side and on a rigid casing at the right side. Oil is pressured into the clearance in the bearing housing through an electromagnetic valve by a pump. The valve is controlled by a voltage put on the valve coil. The vibration response at the left side is measured by probe A and B and sent to a computer. The computer computes the acceleration value and controls the valve to cut off or resupply the oil into the clearance according to the acceleration value. The weight of equivalent mass lumped at the bearing housing station W = 7.8kg; the clearance between the inner race and the outer race C = 0.35mm; the 8 wires' diameter = 0.5mm and length = 10mm.

The vibratory responses were measured in three experimental cases. In case I, the vibratory responses with control utilizing the bearing housing were measured for different unbalance conditions. In case II, the bearing housing was replaced by a SFD and the responses with control using a SFD were measured for same unbalance conditions so as to compare with those utilizing the bearing housing. In case III, a squirrel cage spring replaced the bearing housing in order to measure the response without control. The other dynamic and geometric parameters, such as acceleration, initial speed, diameter and width of oil film, viscosity of lubricant and so on, remained unchanged in the three cases.

Figure 10 shows a comparison of vibratory amplitudes with control utilizing the bearing housing and that without control. The system without control supported only on a squirrel cage spring is linear, which dynamic parameters such as stiffness, damping and mass are same as those of the nonlinear system on the bearing housing in static equilibrium position. ◇ and * depict the vibratory amplitudes with and without control respectively. The amplitudes with control in acceleration do not coincide with those in deceleration due to changing stiffness characteristics. The maximum amplitude of the system without control appearing at 140Hz is 1.06mm. However the maximum amplitude with control is only 0.22mm, which appears at 116Hz in acceleration. The amplitudes are suppressed significantly. Therefore, the ability of the bearing housing in vibration control has been verified by the experiment.

In order to know the ability of the bearing housing with controllable spring nonlinearity in avoiding appearance of harmful nonlinear vibration, experiments for suppressing vibration using a SFD and the bearing housing have been carried out so as to compare results of them. Figure 11 and figure 12 illustrate the vibration amplitudes of the rotor system with control utilizing the bearing housing for increasing unbalance mass. Figure 13(a) shows the vibratory amplitudes using a SFD. Figure 11 shows that the maximum amplitude appearing as the rotating speed is increased to 116Hz in acceleration is 0.22mm. When the rotating speed reaches 170Hz, the rotor system starts to be slowed down. The amplitude peak value appearing at 116Hz in deceleration is 0.17mm. Compared with the curve at same unbalance condition in figure 13(a), the maximum amplitude utilizing the bearing housing is equal approximately to that using a SFD.

In figure 13(a), when additional mass = 2.5g, there are two response curves between rotating speed 120Hz and 135Hz. The higher is the response in acceleration and the lower is in deceleration. This means that bistable operation exists in the
system on a SFD in frequency range from 120Hz to 135Hz. The vibratory amplitude is held in high level in this frequency range. This phenomenon does not appear in the system with control utilizing the bearing housing under the same unbalance condition as shown in figure 12.

Figure 13(b) is spectrums of vibratory response using a SFD at rotating speed 120Hz and 155 Hz. At 120Hz, the vibratory frequency component is single, which is at the rotating speed. But there are two main frequency components at 155Hz. One was at the rotating speed and the other is at half rotating speed. So, if rotating speed is greater than 138Hz, the subharmonic vibration will appear in the system on a SFD. The amplitude peak value of subharmonic vibration is much higher than that of harmonic vibration. For the smaller unbalance condition ie. no additional unbalance mass, the amplitude peak of harmonic vibration is 0.17mm, but the amplitude of subharmonic one has reached 0.30mm when \( \omega = 170\text{Hz} \) and will increase continuously with increase of rotating frequency. For the larger unbalance condition ie. additional unbalance mass= 2.5g, the amplitude peak value of harmonic vibration is 0.26mm, but the subharmonic amplitude has reached 0.31mm and is increasing continually. However, figure 11 and 12 show that the vibration of the rotor system utilizing the bearing housing is harmonic and its amplitudes remain small until rotating speed reaches 170Hz. Subharmonic vibration results in alternating stress in a shaft and fatigue damage. It should be avoided in an operating rotor. Harmful nonlinear vibration such as bistable operation and subharmonic vibration, which frequently occurs in the rotor system using SFDs, can be avoided by the control utilizing the bearing housing.
CONCLUSION

The method for vibration control developed in this paper is significant effective in suppressing the vibration and can avoid the occurrence of harmful nonlinear vibration such as subharmonic vibration, bistable operation and "lock up" phenomenon which frequently appears in the system using SFDs to attenuate vibration.

The configuration of the bearing housing with controllable spring nonlinearity is very similar to that of a SFD. So, advantages of a SFD such as structure simplicity and small space requirement remain with the bearing housing.

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Reference

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