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REPRESENTING FLEXIBLE SUPPORTS BY POLYNOMIAL TRANSFER FUNCTIONS

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ABSTRACT

Flexible bearing supports may have a great influence in the calculation of forced response and stability of rotor systems. However, this effect is not always included in rotor analyses since an accurate model of the foundation and pedestals may be difficult and costly to obtain. It is common practice to use either a one degree of freedom model or a full modal analysis to represent the bearing supports. While the one degree of freedom model is easy to set up for computer calculations, it often requires experience to determine values for the stiffness, mass and damping of the model that will accurately represent the support under study. This model, however, fails to capture the dynamics of the system for stability analyses when more than one mode of the support structure is in the range of interest. On the other hand, modal representation provides much more information and can be measured experimentally, but requires measurement of the mode shapes. Even though modal representation can include all the dynamics of the system in the frequency range of interest, it provides much more information than is required for calculation of the rotor response and it is more difficult to use in calculation programs. This paper presents a procedure to include the support characteristics using transfer functions. Transfer functions permit modeling of multi-degree of freedom systems while maintaining the size of a one degree of freedom system (2x2 matrix if rotation at the bearing is not considered). Another advantage of transfer functions is that they can be obtained from existing discrete models, from modal information or can be measured directly. The fixed size of the transfer function matrix permits the method to be easily incorporated into rotor dynamic stability and forced response programs. The method is applied to stability calculations of models of typical industrial machines.

NOMENCLATURE

c, C	damping coefficient
f	force (scalar complex value)
F	force vector $\{f_x \quad f_y\}^T$
g	polynomial transfer function
G	transfer function matrix
k, K	stiffness coefficient
m	mass
r	number of mode shapes
s	complex frequency
u	displacement vector $\{x \quad y\}^T$
x	displacement in the horizontal direction (scalar complex value)
y	displacement in the vertical direction (scalar complex value)
ϕ	mode shape
ω_d	imaginary part of the complex frequency s
ω_k	frequency of the kth mode shape
ζ_k	modal damping of the mode k

indexes

eq	equivalent coefficient
m	denominator order (transfer functions)
n	numerator order (transfer functions)
n	number of degrees of freedom
o	magnitude value
r	rest degrees of freedom (not support degrees of freedom)
r	mode number
s	support degrees of freedom
x	x-direction in the x-z plane
y	y-direction in the y-z plane

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INTRODUCTION AND BACKGROUND

In many cases the influence of the foundations and casing of rotating machinery is neglected due to the difficulty in obtaining a representative model or due to its complexity. Bansal and Kirk (1975) presented a method to include the effects of a single degree of freedom foundation in the transfer matrix method. Gash (1976) used experimental data to determine the foundation characteristics for rotordynamic analyses using the finite element method. In his approach, the experimental data was used to modify the bearing coefficients when no cross-talk was considered between the pedestals. The global finite element matrix was augmented when cross-talk was considered. The foundation characteristics were modeled as rotational speed dependent and used for both forced response and stability analysis. This approach is correct for forced response analysis but it fails to capture the characteristics of the foundation for stability analysis.

Queitzsch (1985) presented a method to calculate the forced response of a multiple level rotor on flexible substructures. The system was divided into several substructures, analyzed independently and then coupled together. The different components of the system are characterized by impedance functions and then coupled at the bearings. Barrett, Nicholas and Dhar (1986) used experimental forced response functions of the casing of a turbine to modify the bearing stiffness and damping coefficients. Nicholas and Barrett (1986) developed expressions for equivalent bearing coefficients when the bearing is mounted on a flexible support. The supports are modeled as single degree of freedom systems, represented by a mass, a spring and a damper in both the vertical and horizontal directions. Nicholas, Whalen and Franklin (1986) used this method for the analysis of a steam turbine using the forced response function as the representation of the flexible supports. Rouch, McMains and Stephenson (1989) studied the same problem but used the finite element approach. In their case, the forced response data was included in the system point by point.

Fan and Noah (1989) presented a method that permitted the analysis of rotor systems using reduced subsystem models. Each subsystem is modeled separately, reduced either by modal reduction or dynamic reduction and then coupled together by means of the bearings, seals and other coupling mechanisms. Stephenson and Rouch (1992) presented a method to obtain the dynamic matrices of a system using experimental modal data. The matrices are calculated using a least square solution from a complete set of modal vectors. Oliveras (1995) used forced response functions to estimate the parameters of a multi-degree of freedom model for the supports of a flexible rotor-bearing system.

Wygant (1993) presented a method of including the effects of flexible supports and casings using modal information. This study added singularity cancellation and pedestal cross talk into the transfer matrix procedure. In his approach, modal information of the foundation was required to include its effects in the stability calculation. Brockett and Barrett (1995) included magnetic bearings with a single degree of freedom foundation in the transfer matrix method. Singularities arrived both from the magnetic bearings as well as from the flexible supports.

The contribution of this work is to show that an accurate representation of the supports and foundation can be included using polynomial transfer functions. This representation has the advantage that the model can be obtained from experimental data by the procedure published by Sanathanan and Korner (1963) and then used by Gähler and Herzog (1994); the support is always represented by a 2x2 matrix (when rotation at the bearings is not considered) no matter how complex the support model is. This representation is applicable to experimental and analytical analysis of the foundation without modification, making it very attractive for rotordynamic programs where experimental data may be introduced later to verify the initial studies.

FLEXIBLE SUPPORTS AND TRANSFER FUNCTIONS

Assume that a multi-degree of freedom model of a support (pedestal) is known and can be written as:

$$\begin{bmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_n \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \vdots \\ \ddot{u}_n \end{Bmatrix} + \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \cdots & C_{nn} \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \vdots \\ \dot{u}_n \end{Bmatrix} + \cdots$$
$$\cdots + \begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1n} \\ K_{21} & K_{22} & \cdots & K_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ K_{n1} & K_{n2} & \cdots & K_{nn} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{Bmatrix} = \{F\} \quad (1)$$

Grouping the degrees of freedom where the bearing is located, the equations can be re-arranged and written as:

$$\begin{bmatrix} m_s & 0 \\ 0 & m_r \end{bmatrix} \begin{Bmatrix} \ddot{u}_s \\ \ddot{u}_r \end{Bmatrix} + \begin{bmatrix} C_{ss} & C_{sr} \\ C_{rs} & C_{rr} \end{bmatrix} \begin{Bmatrix} \dot{u}_s \\ \dot{u}_r \end{Bmatrix} + \begin{bmatrix} K_{ss} & K_{sr} \\ K_{rs} & K_{rr} \end{bmatrix} \begin{Bmatrix} u_s \\ u_r \end{Bmatrix} = \begin{Bmatrix} f_s \\ 0 \end{Bmatrix} \quad (2)$$

where the subscript s refers to the degrees of freedom of interest of the support where the bearing is located. Subscript r refers to the rest of the degrees of freedom of the support.

Reducing the system in function of the degrees of freedom of interest at the bearing location we get:

$$\begin{aligned} &\left[s^2 m_s + sC_{ss} + K_{ss}\right] u_s \\ &-\left[sC_{sr} + K_{sr}\right]\left[s^2 m_r + sC_{rr} + K_{rr}\right]^{-1}\left[sC_{rs} + K_{rs}\right] u_s = f_s \end{aligned} \quad (3)$$

or:

$$u_s = [G(s)] f_s \quad (4)$$

where:

$$[G(s)] = \begin{bmatrix} \left[s^2 m_s + sC_{ss} + K_{ss}\right] \cdots \\ -\left[sC_{sr} + K_{sr}\right]\left[s^2 m_r + sC_{rr} + K_{rr}\right]^{-1}\left[sC_{rs} + K_{rs}\right] \end{bmatrix}^{-1} \quad (5)$$

$[G(s)]$ is the transfer function that relates the degrees of freedom of interest at the bearing location due to forces at the same location.

If the rotational degrees of freedom are reduced (most bearings do not provide restrictions in bending) the transfer function matrix can be written as:

$$[G(s)] = \begin{bmatrix} g_{xx}(s) & g_{xy}(s) \\ g_{yx}(s) & g_{yy}(s) \end{bmatrix} \quad (6)$$

By applying forces in the X direction and then in the Y direction and measuring the response in the X and Y directions, the elements of $[G(s)]$ can be measured for the frequency range of interest. A ratio of polynomials can be fit to the magnitude and phase information using Gähler and Herzog (1994) such that

$$g_{ij}(s) = \frac{a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0}{s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0} \quad (7)$$

Assuming some amount of modal damping, the transfer function matrix can be obtained from modal information as:

$$g_{ij}(s) = \sum_{k=1}^{r} \frac{\phi_i^k \phi_j^k}{\omega_k^2 + 2\zeta_k \omega_k s + s^2} \quad (8)$$

Once the matrix $[G(s)]$ has been calculated, the equivalent dynamic coefficients of the flexible support at the bearing location can be written as:

$$\left[K_{s\,eq}(s)\right] = [G(s)]^{-1} \quad (9)$$

and can be used directly in calculation tools that can use one degree of freedom models of bearing supports (Brockett and Barrett (1985)). A single degree of freedom system with equivalent stiffness and damping coefficient will have the following dynamic stiffness:

$$\begin{aligned} & s = p + i\omega_d \\ & \left[K_{dyn}(s)\right] = K_{eq} + sC_{eq} = K_{eq} + pC_{eq} + i\omega_d C_{eq} \end{aligned} \quad (10)$$

Then, comparing equations (9) and (10), the dynamic stiffness can be separated into equivalent damping and equivalent stiffness as:

$$s = p + i\omega_d \qquad \text{if } \omega_d \neq 0 \Rightarrow$$

$$\begin{aligned} C_{eq} &= \frac{1}{\omega_d} \operatorname{Im}\left(K_{s\,eq}(s)\right) \\ K_{eq} &= \operatorname{Re}\left(K_{s\,eq}(s)\right) - \frac{p}{\omega_d} \operatorname{Im}\left(K_{s\,eq}(s)\right) \end{aligned} \quad (11)$$

If $\omega_d = 0$ there is not enough information to perform the separation and the dynamic stiffness of the support should be used in its complex form.

The transfer function approach captures all of the dynamics of the system and can be used for both forced response and stability analysis. While for forced response analysis there are alternative methods to represent the support structure this approach is especially useful for stability analysis. One major improvement of using transfer function representation over modal representation is the savings in time to measure the data experimentally since the transfer function representation only requires the response at the bearing locations due to forces applied at such locations. One drawback is that it is not possible to calculate the response at other points of the support structure once the response of the rotor and bearing location is known, without additional information. However, this restriction is acceptable for rotor-bearing systems where what is desired is usually the response of the rotor and not the support structure. The response at other locations of the support structure can be calculated if the transfer functions between these locations and the bearing locations are known.

It is also worth noticing that the size of the transfer function matrix remains fixed. The difference in the order of the system will be reflected in the order of the polynomial representing the elements of the transfer function but the size of the transfer function matrix remains the same. This property makes this approach very suitable for calculation tools since the modifications are minimal to include multi-degree of freedom supports.

SINGULARITY CANCELLATIONS

The dynamic reduction used to calculate the transfer function matrix introduces some singularities into the problem. Singularities can be characterized as locations in the complex space where the value of the characteristic polynomial becomes very large. These singularities deform the solution space making it very difficult for determinant search algorithms to find solutions in the vicinity of the singularity points. In the transfer function approach, singularities are those values of the complex frequency s where the value of the determinant of the transfer function is zero or $\|[G(s)]\| = 0$ (Wygant (1993), Brockett and Barrett (1995)). Since these singularities are known exactly, they can be removed by a method very similar to the root cancellation procedure used in several root search methods (i.e. Newton-Raphson method with root cancellation).

EXAMPLE

To illustrate the use of this method, and example is presented. The example is an overhung compressor, running at 12000 rpm, supported by tilting pad bearings mounted on flexible supports. This example was presented by Wygant (1993). The flexible supports are modeled as multi-degree of freedom systems. Figure 1 shows a model of the rotor. The rotor data is given in Table 1. The tilting pad bearings are located at stations 6 and 19 respectively. Table 2 shows the tilting pad bearings parameters. The full set of bearing coefficients (Brockett and Barrett (1993)) was used. Table 3 shows the coefficients for both bearings reduced at 12000 rpm. Each support was represented using its modal information. This required the knowledge of its mode shapes, modal frequencies and modal damping. In this particular case the supports were represented by 12 planar mode shapes and 5% modal damping was assumed. From this modal information, it is possible to calculate polynomial transfer functions that represent the coefficients of those polynomial transfer functions (Tables 4 and 5). Figures 2 and 3 show the forced response functions (FRF) of the two supports. Table 6 shows the damped eigenvalues calculated for this problem. Table 7 shows the damped eigenvalues calculated neglecting the contribution of the supports.

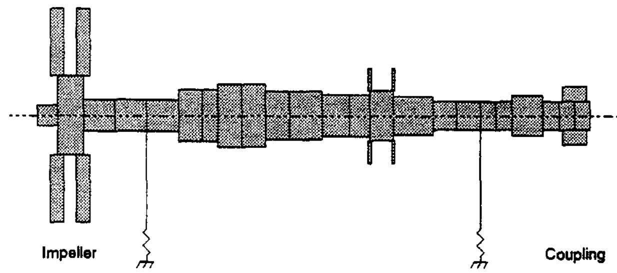

FIGURE 1. ROTOR MODEL

TABLE 7. ROTOR GEOMETRY EXAMPLE 2.

Stat.	Length (m)	Outer Dia (m)	Lumped Polar Moment of Inertia (N-m2)	Lumped Transversal Moment of Inertia (N-m2)	Lumped Weigth (N)
1	0	0.0635	3.741095	1.870548	0
2	0.06604	0.0635	9728.502	11879.67	8.0068
3	0.08636	0.254	1.84E+08	1.1E+08	1288.205
4	0.1016	0.1016	1.84E+08	1.1E+08	1311.781
5	0.1016	0.1016	196155.8	228837	63.16475
6	0.1016	0.1016	196155.8	228837	63.16475
7	0.0762	0.1651	611013.4	443746.6	94.30231
8	0.0508	0.1651	854949.9	521864.2	104.5332
9	0.0762	0.2032	1518915	891492.1	136.5604
10	0.0762	0.2032	2353870	1397567	189.9391
11	0.0762	0.1524	1549252	947339.6	148.1258
12	0.1016	0.1524	868739.4	643625.6	124.5502
13	0.08636	0.13208	734291.6	582262.2	116.5434
14	0.06604	0.13208	420166.5	308333.5	80.068
15	0.0762	0.16256	2042916	1122466	184.6012
16	0.127	0.12192	2115311	1312072	206.8423
17	0.0762	0.09144	302473	357493.2	76.0646
18	0.0762	0.09144	96526.6	92941.32	38.25471
19	0.0508	0.09144	80461.81	69154.41	32.0272
20	0.0508	0.09144	64348.77	45415.76	25.79969
21	0.1016	0.127	271653.4	244626	62.27511
22	0.0508	0.09144	271653.4	244626	62.27511
23	0.0508	0.09144	64348.77	45415.76	25.79969
24	0.0508	0.09144	1649915	1079719	149.4603
25	0	0.09144	32191.62	22718.22	12.89984

TABLE 2. BEARING PARAMETERS.

	Bearing 1	Bearing 2
Type	Tilt Pad/ 5 pads	Tilt Pad/ 5 pads
Journal Diameter (mm)	203.2	183.0
Pad Length (mm)	76.2	68.6
Bearing Clearance (mm)	0.3048	0.3048
Bearing Preload	0.367	0.367
Pad Arc Length (deg)	60.0	60.0
Pivot Offset Factor	0.5	0.5
Rotational Speed (rpm)	12000	12000
Static Load (N)	4070.12	538.23
Pad loading position	between pads	between pads
Inlet Pressure(kPa)	137.895	137.895
Inlet Temperature (°C)	71.11	71.11
Oil Viscosity (Pa.s)@ 48.88 °C	0.017924	0.017924
Oil Viscosity (Pa.s)@ 93.33 °C	0.004550	0.004550

TABLE 3. SYNCHRONOUSLY REDUCED BEARING COEFFICIENTS.

	Bearing 1 Station 6	Bearing 2 Station 19
Kxx (N/m)	3.728E+07	5.731E+07
Kxy (N/m)	91507.9	4893.0
Kyx (N/m)	91481.0	4892.9
Kyy (N/m)	5.933E+07	5.810E+07
Cxx (N-s/m)	28204.18	32543.82
Cxy (N-s/m)	16.06	1.14
Cyx (N-s/m)	16.07	1.15
Cyy (N-s/m)	34223.29	32774.99

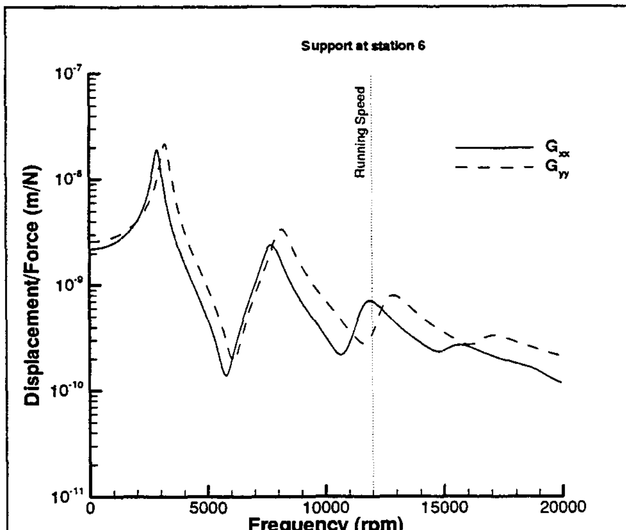

FIGURE 2. MAGNITUDE OF THE FORCED RESPONSE FUNCTION FOR THE SUPPORT AT STATION 6.

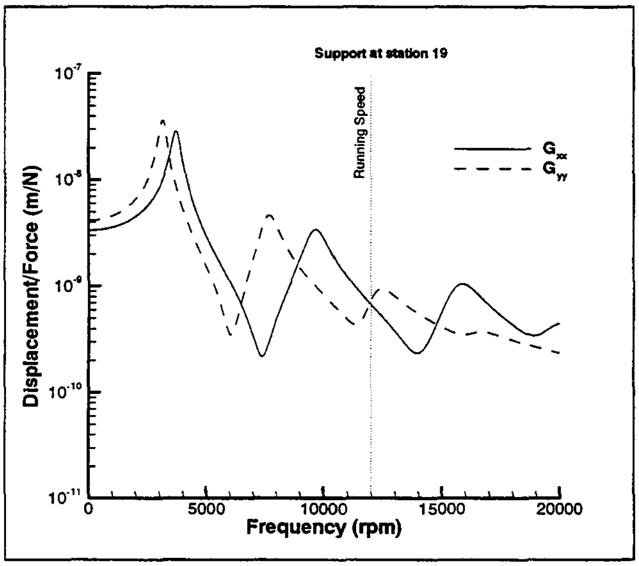

FIGURE 3. MAGNITUDE OF THE FORCED RESPONSE FUCNTION FOR THE SUPPORT AT STATION 19.

DISCUSSION OF RESULTS

Table 6 and 7 show the influence of the support structure on the damped eigenvalues. It can be shown from these tables that the support structure in this case has a strong effect on both the damped frequencies and logarithmic decrements. In order to compare these modes, the mode shapes were calculated. They are not presented here due to space constrains and because they do not convey much more information than to help identify the modes.

The support effects decreased the first damped frequency from 1207 rpm to 1181 rpm while the logarithmic decrement increased from 0.0368 to 0.0405. The second damped frequency was reduced by 22 %, from 3277 rpm to 2672 rpm. The logarithmic decrement decreased by 38 %, from 0.3968 to 0.2892. This is consistent with the finding of Nicholas and Barrett (1986) and Nicholas, Whalen and Franklin (1986) where the inclusion of the support flexibility tended to decrease the second critical frequency while the effects in the first critical frequency were small. Modes 3, 4 and 5 on Table 6 are introduced by the support characteristics. Mode 3 shows displacement mainly of the impeller side while modes 4 and 5 show displacement of the coupling side. Mode 6 in Table 6 corresponds to mode 3 in Table 7. Modes 10, 11 and 12 in Table 6 correspond very well to modes 6, 7 and 8 in Table 7. Modes 7, 8, 9 in Table 6 and modes 4 and 5 in Table 7 show displacement mainly in the coupling side. The support effects added the extra mode but it is not possible to determine which one it is from the calculated mode shapes.

These results were expected because the supports have several peak responses in the range of interest (Figures 2 and 3). As shown in this example, neglecting the effects of the supports tends to predict different modes while some others are missed completely. In some instances, the calculated mode shapes are different from those calculated without the support contribution, even though the damped frequency and logarithmic decrements are close (modes 7, 8 and 9 in Table 6, and 4 and 5 in Table 7).

CONCLUSIONS

Transfer function representation of bearing supports is an attractive method of including the dynamics of the support structure behind the bearings. The transfer function representation has the advantage of including the dynamics of model with several degrees of freedom while maintaining a fixed size matrix representation. Experimentally determined transfer functions increase the accuracy of the support model since no assumptions are needed (there is no need to assume modal damping as in the case of modal representation). The transfer function representation uses the minimum amount of information required for rotor analysis, therefore minimizing the calculation time and the acquisition time if they are measured experimentally. However, this makes it impossible to calculate the response of the support at other locations other than at the bearing location. This limitation is acceptable for rotordynamic analysis since the desired solution is the response of the rotor and not the support structure.

The example shows the importance of including the effects of the support structure. It was shown that neglecting the effects of the support structure could lead to substantially different eigenvalues while some others are missed completely.

Although cross-talk between supports has not been explicitly treated in this work the same principles and theory apply. It has been shown that using the transfer function representation of flexible supports introduces singularities in the search surface that should be removed, otherwise it may be very difficult for search algorithms to reach solutions near the singularity points. It has been assumed for this work that the bearings do not restrict rotational movement (the common practice). If this assumption does not apply to a particular case, the transfer function matrix should be expanded to include the rotational degrees of freedom but the method works in the same manner.

TABLE 4. TRANSFER FUNCTION FOR SUPPORT 1.

	Support 1 (Station 6)			
	Gxx		Gyy	
Power	Num.	Denom.	Num.	Denom.
S^0	1.44E+28	6.50E+36	4.50E+28	1.73E+37
S^1	5.53E+24	4.46E+33	1.62E+25	1.09E+34
S^2	6.25E+22	9.14E+31	1.72E+23	2.00E+32
S^3	1.66E+19	3.68E+28	4.17E+19	7.46E+28
S^4	7.47E+16	2.36E+26	1.76E+17	4.43E+26
S^5	1.33E+12	6.48E+22	2.87E+13	1.13E+23
S^6	3.50E+10	2.21E+20	6.96E+10	3.56E+20
S^7	3.82E+06	4.11E+16	7.02E+06	6.10E+16
S^8	6726.5	9.03E+13	1.14E+04	1.24E+14
S^9	0.3450	1.03E+10	0.5400	1.31E+10
S^{10}	4.39E-04	1.61E+07	6.34E-04	1.88E+07
S^{11}	0	858.4	0	928.8
S^{12}	0	1	0	1

TABLE 5. TRANSFER FUNCTION FOR SUPPORT 2.

	Support 2 (Station 19)			
	Gxx		Gyy	
Power	Num.	Denom.	Num.	Denom.
s^0	3.76E+29	1.13E+38	5.09E+28	1.23E+37
s^1	1.14E+26	6.06E+34	1.87E+25	7.92E+33
s^2	1.00E+24	9.68E+32	1.98E+25	1.50E+32
s^3	2.08E+20	3.06E+29	1.98E+23	5.81E+28
s^4	7.39E+17	1.54E+27	4.90E+19	3.61E+26
s^5	1.04E+14	3.31E+23	2.08E+17	9.46E+22
s^6	2.16E+11	8.80E+20	3.46E+13	3.10E+20
s^7	1.90E+07	1.29E+17	8.47E+10	5.44E+16
s^8	2.67E+04	2.22E+14	8.69E+06	1.14E+14
s^9	1.1146	2.03E+10	0.6863	1.22E+10
s^{10}	1.14E-03	2.49E+07	8.15E-04	1.81E+07
s^{11}	0	1078	0	907.4
s^{12}	0	1	0	1

TABLE 6. DAMPED EIGENVALUES, CONSIDERING SUPPORT FLEXIBILITY.

Eigenvalue Number	Logarithmic Decrement (dim)	Damped Frequency (RPM)
1	0.0405	1181
2	0.2892	2672
3	0.2970	2975
4	0.3044	3094
5	0.3161	3628
6	0.5256	4469
7	0.5785	7013
8	0.7844	7184
9	0.5695	7990
10	0.0398	9082
11	0.1337	9787
12	0.3605	20070

TABLE 7. DAMPED EIGENVALUES, NEGLECTING THE SUPPORT CONTRIBUTION.

Eigenvalue Number	Logarithmic Decrement (dim)	Damped Frequency (RPM)
1	0.0368	1207
2	0.3968	3277
3	0.5183	4256
4	0.8749	7137
5	0.7785	7168
6	0.0360	9084
7	0.1243	9847
8	0.3467	20080

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