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THE TRANSFER MATRIX — COMPONENT MODE SYNTHESIS FOR ROTORDYNAMIC ANALYSIS

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ABSTRACT

The transfer matrix-component mode synthesis has been developed for the analysis of critical speed, response to imbalance and rotordynamic optimal design of multi-spool rotor system. This method adopted the advantages of the transfer matrix method for the train structure and the component mode synthesis for reducing degrees of freedom. In this method, the whole system is divided into several subsystems at the boundary coordinates. The constrained vibration modes and the static deflection curves of the constrained rotor subsystems are analysed by the improved transfer matrix method. The whole system is connected together by the component mode synthesis in accordance with the coordinate transformation. Numerical examples show that this method is superior to the traditional transfer matrix method and the component mode synthesis by FEM. This method has been successfully used for the rotordynamic analysis and optimal design of the compressors and the gas turbine aeroengines.

key words: transfer matrix-component mode synthesis, rotordynamics, critical speed, response to imbalance, optimal design.

NOMENCLETURES

C = damping
Cg = gyroscopic moment
e = eccentricity
Ip = polar moment of inertia
J = area moment of inertia
K = stiffness
L = length of segment
M = mass, bending moment
Q = shear force, generalized force
q = sectional variable
R = reaction force
s = complex eigenvalue
Y = modal coordinate
y = leteral deflection
Z = physical coordinate
δ = static deflection curve
φ = constrained vibration mode
θ = sectional slope
ω = spin velocity
Ω = vibration frequency

Subscript
b = boundary
i = inner

INTRODUCTION

The methods for the rotordynamics analysis have been developed for 50 years. Primarily, there are transfer matrix method and finite element method. The transfer matrix method is based on Prohl-Myklestad method and has been improved by Urban, Lund and others [1][2][3][4]. It is a powerful tool for the rotordynamic analysis, especially for the single rotor system. The finite element method has been widely used in mechanical engineering because of its effectiveness and suitability. When the system has a lot of degrees of freedom, the matrix size will be very much large and it needs much more computer storage and spends

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much more computer time. The component mode synthesis is a good idea to reduce matrix size for the complex systems. The component mode synthesis using the finite element method for solving the rotordynamic problems has been developed by Gunter, Nelson and others [5][6][7][8].

In order to take the advantages of the transfer matrix method for the single rotor system and the component mode synthesis for the complex rotor system, these two methods are jointed together, the transfer matrix – component mode synthesis (TMCMS), for the analysis of the complex rotor systems [9]. This method has been used for analysing the critical speeds and responses to imbalance, as well as for the optimal design of rotordynamics [10][11].

THE TRANSFER MATRIX — COMPONENT MODE SYNTHESIS

The TMCMS is based on the concept of the superposition of two kinds of mode shapes. One of them is constrained vibration mode shape and the other is the static deflection curve. The constrained vibration mode shape is the free vibration of the constrained undamped subsystem when all of the boundary coordinates are fixed. The static deflection curve is obtained when the boundary coordinate is released in turn.

The equation of motion for free vibration of a rotor system is as follows,

$$M\ddot{Z} \mp i\omega C_g \dot{Z} + KZ + C\dot{Z} = 0 \qquad (1)$$

where the sign "−" in front of the second term refers to the forward whirl and the sign "+" the backward whirl.

For the use of the component mode synthesis, the whole system is divided into several subsystems at some special points where the subsystems are coupled or the characteristics is complicated such as bearing and squeeze film damper etc. as shown in FIG. I. These special poits are defined as boundary coordinates. The other degrees of freedom are defined as inner coordinates. Then, equation of motion (1) will be expressed as,

$$\begin{bmatrix} M_i & 0 \\ 0 & M_b \end{bmatrix}\begin{Bmatrix} \ddot{Z}_i \\ \ddot{Z}_b \end{Bmatrix} \mp i \begin{bmatrix} \omega_i C_{gi} & 0 \\ 0 & \omega_b C_{gb} \end{bmatrix}\begin{Bmatrix} \dot{Z}_i \\ \dot{Z}_b \end{Bmatrix} + \begin{bmatrix} K_{ii} & K_{ib} \\ K_{bi} & K_{bb} \end{bmatrix}\begin{Bmatrix} Z_i \\ Z_b \end{Bmatrix} + \begin{bmatrix} C_{ii} & C_{ib} \\ C_{bi} & C_{bb} \end{bmatrix}\begin{Bmatrix} \dot{Z}_i \\ \dot{Z}_b \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (2)$$

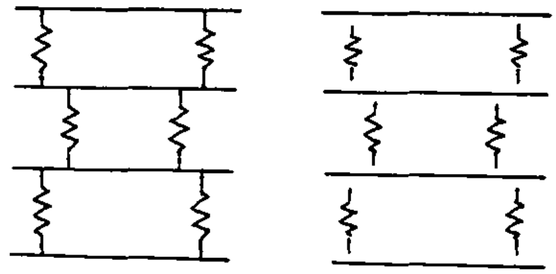

FIG. 1 Scheme of the rotor system.

For the constrained undamped subsystem, as shown in FIG. 2, the equation of motion for free vibration will be:

$$M_i\ddot{Z}_i \mp i\omega_i C_{gi}\dot{Z}_i + K_{ii}Z_i = 0 \qquad (3)$$

The eigensolution will be as the form of

$$Z_i = \varphi e^{i\Omega_c t} \qquad (4)$$

The boundary conditions of the scheme are:

$$\{q_o\} = [y_o, \theta_o, M_o, Q_o]^T = [y_o, \theta_o, 0, 0]^T$$
$$\{q_n\} = [y_n, \theta_n, M_n, Q_n]^T = [y_n, \theta_n, 0, 0]^T$$
$$y_1 = y_2 = \ldots = y_i = \ldots = y_k = 0 \qquad (5)$$

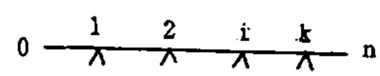

FIG. 2 The constrained subsystem.

Then, the following equations can be obtained:

$$\begin{cases} y_1 = f_1(y_o, \theta_o, \ldots\ldots) = 0 \\ y_2 = f_2(y_o, \theta_o, R_1, \ldots) = 0 \\ \quad \cdots\cdots\cdots\cdots \\ y_k = f_k(y_o, \theta_o, R_1, R_2, \ldots) = 0 \\ M_n = f_n(y_o, \theta_o, R_1, R_2, \ldots) = 0 \\ Q_n = f_{n+1}(y_o, \theta_o, R_1, R_2, \ldots) = 0 \end{cases} \qquad (6)$$

where f_i are the function of the transfer matrices of the segments. Equation (6) can be expressed as

$$\begin{cases} A(1,1)y_o+A(1,2)\theta_o+A(1,3)R_1+\ldots+A(1,n+1)R_k=0 \\ A(2,1)y_o+A(2,2)\theta_o+A(2,3)R_1+\ldots+A(2,n+1)R_k=0 \\ \ldots\ldots \\ A(k,1)y_o+A(k,2)\theta_o+A(k,3)R_1+\ldots+A(k,n+1)R_k=0 \\ A(n,1)y_o+A(n,2)\theta_o+A(n,3)R_1+\ldots+A(n,n+1)R_k=0 \\ A(n+1,1)y_o+A(n+1,2)\theta_o+A(n+1,3)R_1+\ldots \\ \qquad\qquad +A(n+1,n+1)R_k=0 \end{cases} \tag{7}$$

where $A(i,j)$ is the function of the transfer matrices of the relative segments and the rotating frequency. The constrained vibration mode shape φ and eigenvalue Ω_c can be obtained by solving these equations.

Let $yi=1\ (i=1,\cdots k)$ in turn for each time, equation (7) will become nonhomogeneous, solving corresponding equations for non—rotating condition, the static deflection curves δ can be obtained.

The modal transformation matrix is introduced to create the relation between the physical and modal coordinates:

$$\begin{Bmatrix} Z_l \\ Z_b \end{Bmatrix} = \begin{bmatrix} \varphi & \delta \\ 0 & I \end{bmatrix} \begin{Bmatrix} Y_l \\ Y_b \end{Bmatrix} e^{st} \tag{8}$$

Substituting the modal transformation matrix (8) into equation of motion (2), then

$$\left(s^2 \begin{bmatrix} M_l & 0 \\ 0 & M_b \end{bmatrix} \mp is \begin{bmatrix} \omega_l C_{gl} & 0 \\ 0 & \omega_b C_{gb} \end{bmatrix} + \begin{bmatrix} K_{ll} & K_{lb} \\ K_{bl} & K_{bb} \end{bmatrix} + s \begin{bmatrix} C_{ll} & C_{lb} \\ C_{bl} & C_{bb} \end{bmatrix} \right) \begin{bmatrix} \varphi & \delta \\ 0 & I \end{bmatrix} \begin{Bmatrix} Y_l \\ Y_b \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{9}$$

By premultiplying the transfer of modal transformation matrix, equation of motion (9) can be transformed into the following final form:

$$\begin{bmatrix} A_{ll}+B_{ll} & B_{lb} \\ B_{bl} & A_{bb}+B_{bb} \end{bmatrix} \begin{Bmatrix} Y_l \\ Y_b \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{10}$$

where

$$A_{ll}=\varphi^T(\Omega_c^2 M_l \mp \Omega_c \omega_l C_{gl})\varphi$$

$$A_{bb}=K_b+sC_b$$

$$B_{ll}=s^2\varphi^T M_l \varphi \mp is\varphi^T \omega_l C_{gl}\varphi$$

$$B_{lb}=s^2\varphi^T M_l \delta \mp is\varphi^T \omega_l C_{gl}\delta$$

$$B_{bl}=s^2\delta^T M_l \varphi \mp is\delta^T \omega_l C_{gl}\varphi$$

$$B_{bb}=s^2(\delta^T M_l \delta + M_b) \mp is(\delta^T \omega_l C_{gl}\delta + \omega_b C_{gb})$$

Equation (10) is the eigenvalue problem expression for axi—symmetrical rotor system.

CRITICAL SPEEDS

From the determinant of the coefficients of equation (10), the eigensolutions can be obtained by a iterative procedure.

$$\begin{vmatrix} A_{ll}+B_{ll} & B_{lb} \\ B_{bl} & A_{bb}+B_{bb} \end{vmatrix} = 0 \tag{11}$$

Then, the vibration mode shapes can be obtained by substibuting the eigenvalue into equation (10).

FIG. 3 shows the scheme of a dual rotor system with one intershaft bearing and the calculation model. The data of this rotor system are listed in Table 1. Some of the calculation results are listed in Table 2 compared with the exact solutions obtained by Largrange's equation.

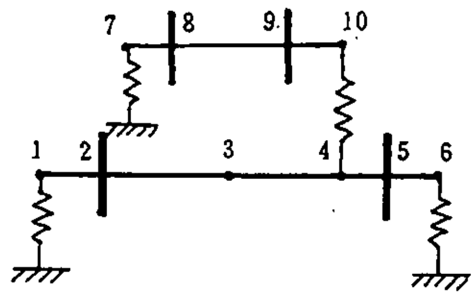

FIG. 3 The scheme of a dual rotor system

Table 1 Data of the dual rotor system.

Rotor No.	ω (rad/s)	J (m^4)	No.	M (kg)	Ip ($kg \cdot m^2$)	Kc^{-7} (N/m)	L (m)
1	1047.2	$2.6467e^{-8}$	1	0.0577	0	2.6269	0.0762
			2	10.7023	0.0859	—	0.1778
			3	0.2499	0	—	0.1524
			4	0.1538	0	—	0.0508
			5	7.0869	0.0678	—	0.0508
			6	0.0385	0	1.7513	0
2	1570.8	$2.1935e^{-8}$	7	0.047	0	1.7513	0.508
			8	7.202	0.0429	—	0.1524
			9	3.692	0.0271	—	0.0508
			10	0.047	0	0.8756	0

$E = 2.068\ e^{11}\ N/m^2$

Table 2 The calculation results ω(rad/s).

Mode	Exact solution	TMCMS	Error %
B	31.536	31.424	0.362
B	150.535	150.545	0.0065
F	469.315	469.341	0.0054
B	638.798	638.783	0.0024
B	677.979	678.000	0.0030
F	727.428	727.561	0.0184
B	949.970	949.855	0.0120
F	1414.835	1415.098	0.0187
B	1523.667	1523.753	0.0060
B	1781.283	1781.739	0.0260
F	2119.448	2119.944	0.0234
F	2330.428	2330.448	0.0009

RESPONSE TO IMBALANCE

For the calculation of response to imbalance, the equation of motion (1) should be changed into nonhomogeneous form:

$$M\ddot{Z} \mp i\omega C_g \dot{Z} + C\dot{Z} + KZ = F \qquad (12)$$

where F is the imbalance force and can be expressed as

$$\{F\} = [M]\{e\}\Omega^2 \qquad (13)$$

By the same dirivation, the following equations can be obtained for rotating speed Ω:

$$\begin{bmatrix} A_{ii} + B_{ii} & B_{ib} \\ B_{bi} & A_{bb} + B_{bb} \end{bmatrix} \begin{Bmatrix} Y_i \\ Y_b \end{Bmatrix} = \begin{Bmatrix} Q_i \\ Q_b \end{Bmatrix} \qquad (14)$$

where Q_i and Q_b is the generalized force exerted on inner and boundary coordinates respectively, due to the imbalance and expressed as

$$\begin{Bmatrix} Q_i \\ Q_b \end{Bmatrix} = \begin{bmatrix} \varphi & \delta \\ 0 & I \end{bmatrix} \begin{Bmatrix} F_i \\ F_b \end{Bmatrix} = \begin{bmatrix} \varphi & \delta \\ 0 & I \end{bmatrix} \begin{Bmatrix} M_i e_i \\ M_b e_b \end{Bmatrix} \Omega^2 \qquad (15)$$

For convenience, equation (14) can be rewritten as the real number form by extending the matrix size:

$$\begin{bmatrix} A & B & 0 & 0 \\ C & D & 0 & -E \\ 0 & 0 & A & B \\ 0 & E & C & D \end{bmatrix} \begin{bmatrix} ReY_i \\ ReY_b \\ ImY_i \\ ImY_b \end{bmatrix} = \begin{Bmatrix} ReQ_i \\ ReQ_b \\ ImQ_i \\ ImQ_b \end{Bmatrix} \qquad (16)$$

where

$A = -\Omega^2\varphi^T M_i \varphi \pm \Omega\varphi^T \omega_i C_{gi} \varphi + \varphi^T(\Omega_c^2 M_i \mp \Omega_c \omega_i C_{gi})\varphi$;

$B = -\Omega^2\varphi^T M_i \delta \pm \Omega\varphi^T \omega_i C_{gi} \delta$;

$C = -\Omega^2\delta^T M_i \varphi \pm \Omega\delta^T \omega_i C_{gi} \varphi$;

$D = -\Omega^2\delta^T M_i \delta - \Omega^2 M_b \pm \Omega\delta^T \omega_i C_{gi} \delta \pm \Omega\omega_b C_{gb} + K_b$;

$E = \Omega C_b$;

$Y_i = ReY_i + iImY_i$;

$Y_b = ReY_b + iImY_b$;

$Q_i = ReQ_i + iImQ_i$;

$Q_b = ReQ_b + iImQ_b$.

When the eccentricity e has been given, the response to imbalance can be calculated by the equation (16). The steady response to imbalance in physical coordinate will be obtained by the modal transformation matrix (8). FIG. 4 shows the steady response to imbalance of a gas turbine rotor by TMCMS.

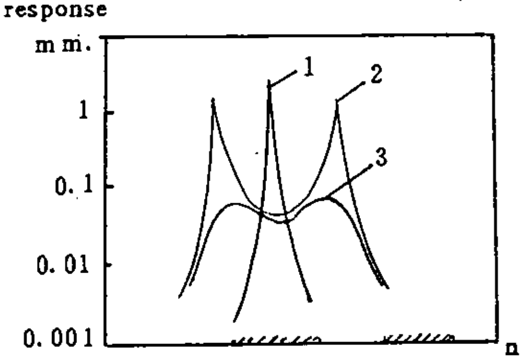

FIG. 4 Response to imbalance of a gas turbine

OPTIMAL DESIGN

The most important things in the optimal design of the rotordynamics are to keep the critical speeds far away from the operating ranges and to minimize the response to imbalance at any operating ranges. The most effective way to these ends is the optimal design of the stiffness of the support and the damping coefficient of the damper. The rotor system has many supports and maybe also many dampers. So that, the sensitivity analysis of the stiffnesses of the supports to the critical speeds and the damping coefficients of the dampers to the responses to imbalance is necessary.

The CMS method is the best way for the optimal design because all the parameters which will be optimized are the boundary coordinates. The recirculation calculation is very easy and much more saving the computer time.

For the sensitivity analysis of the damping coefficients of the dampers, the derivative equation can be obtained from equation (16) as follows,

$$\begin{bmatrix} A & B & 0 & 0 \\ C & D & 0 & -E \\ 0 & 0 & A & B \\ 0 & E & C & D \end{bmatrix} \begin{Bmatrix} \frac{\partial ReY_i}{\partial C_j} \\ \frac{\partial ReY_b}{\partial C_j} \\ \frac{\partial ImY_i}{\partial C_j} \\ \frac{\partial ImY_b}{\partial C_j} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \Omega ImY_{bj} \\ 0 \\ -\Omega ReY_{bj} \end{Bmatrix} \quad (17)$$

The sensitivity to the physical coordinate will be:

$$\begin{Bmatrix} \frac{\partial Z_i}{\partial C_j} \\ \frac{\partial Z_b}{\partial C_j} \end{Bmatrix} = \begin{Bmatrix} \varphi \frac{\partial ReY_i}{\partial C_j} + \delta \frac{\partial ReY_b}{\partial C_j} + i(\varphi \frac{\partial ImY_b}{\partial C_j} + \delta \frac{\partial ImY_b}{\partial C_j}) \\ \frac{\partial ReY_b}{\partial C_j} + i \frac{\partial ImY_b}{\partial C_j} \end{Bmatrix} \quad (18)$$

The sensitivity of the j damper S_j is the sum of all of the sensitivities of the coordinates:

$$S_j = \sum_m \frac{\partial Z_{im}}{\partial C_j} + \sum_n \frac{\partial Z_{bn}}{\partial C_j} \quad (19)$$

where m, n is the number of inner and boundary coordinate respectively.

The stiffness coefficients of the supports and the damping coefficients of the dampers will be changed by a optimal procedure in accordance with the results of the sensitivity analysis. The optimal goal is to get the critical speeds far away form the operating range and to obtain the minimum vibration response during passing through the critical speeds and in the normal operating range.

The comparison of the response to imbalance before and after the optimal design is shown in FIG. 4, where curve 1 represents the original response, curve 2 the peak response remove away from the operating ragion by stiffness optimal design and curve 3 the response after stiffness and damping optimal design.

CONCLUSIONS

The transfer matrix-component mode synthesis presented in this paper is a suitable method for calculating the critical speed and response to imbalance as well as for the optimal design of the complex multi-spool rotor system. The static deflection curve and the constrained vibration mode is analysed by the improved transfer matrix method. The component mode synthesis is employed for analyzing the dynamic characteristics of the whole rotor system and the sensitivity of the parameters of the supports and the dampers to the critical speeds and responses to imbalance for the optimization of rotordynamics.

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